GENERATION OF MAGNETIC AND CHROMOMAGNETIC FIELDS AT HIGH TEMPERATURE IN THE STANDARD MODEL

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The spontaneous generation of the magnetic and chromomagnetic fields at high temperature is investigated in the Standard Model. The consistent effective potential including the one-loop and the daisy diagrams of all boson and fermion fields is calculated. The mixing of the generated fields due to the quark loop diagram is studied.
PACS: 13.40.Ks

1. INTRODUCTION

One of interesting problems of nowadays high-energy physics is generation of strong magnetic fields in the early universe. Different mechanisms of producing the fields at different stages of the universe evolution have been proposed (see, for instance, the surveys [1,2,3]) and the influence of fields on various processes was discussed. In particular, the primordial magnetic fields, being implemented in cosmic plasma, may serve as the seed source of the present extra galaxy fields.

One of the mechanisms is a spontaneous vacuum magnetization at high temperature. It was investigated already for the case of pure *SU(2)* gluodynamics in Refs. [4,5,6] where the possibility of this phenomenon was shown. The stability of the magnetized vacuum was also studied [6]. As it is well known, the magnetization takes place for the non-abelian gauge fields due to a vacuum dynamics [7]. In fact, this is one of the distinguishable features of asymptotically free theories. In the papers mentioned the fermions were not taken into consideration. However, they may affect the vacuum state due to loop corrections in strong magnetic fields at high temperature.

In the present report the spontaneous vacuum magnetization is investigated in the standard model (SM) of elementary particles. All boson and fermion fields are taken into consideration. In the SM there are two kind of non-abelian gauge fields - the *SU(2)* weak isospin gauge fields responsible for weak interactions and the *SU(3)* gluons mediating strong interactions. The quarks possess both the electric and colour charges. So, they have to mix the chromomagnetic and the ordinary magnetic fields due to vacuum loops. Because of this mixing some specific configurations of the fields must be produced at high temperature. To elaborate this picture quantitatively, we calculate the effective potential (EP) including the one-loop and the daisy diagram contributions in the constant abelian chromomagnetic and magnetic fields, $H_c=const$ and $H=const$, at high temperatures. This type of resummations guarantees a vacuum stability if one takes into consideration the one-loop temperature masses of the transversal gauge fields in the external fields considered [6]. So, we will use this approximation in what follows. Since an abelian magnetic hypercharge field is not generated spontaneously, in what follows we shall consider the non-abelian component of the magnetic field. The mechanisms of hypermagnetic field generation have been discussed in Refs. [8,9]. It will be shown that at high temperatures either the strong magnetic or the chromomagnetic fields are generated.

They are stable in the approximation adopted due to the magnetic masses of $m^2_{transversal} \sim (gH)^{1/2} T$ of the gauge field transversal modes [10]. In this way the consistent picture of the magnetized vacuum state in the SM at high temperature can be derived.

2. BASIC FORMULAE

The one-loop contribution into EP is following

$$V^{(1)} = -\tfrac{1}{2} Tr \log G^{ab}, \qquad (1)$$

where G^{ab} stands for the propagators of all quantum fields $W^{\pm}$, $\boldsymbol{A}$, … in the background fields H and $\boldsymbol{H_3}$. In the proper time formalism, *s*-representation, the calculation of the trace can be carried out in accordance with the formula [15]

$$Tr \log G^{ab} = -\int_0^{\infty} \frac{ds}{s}\, tr \exp(-is\, G^{-1}_{ab}). \qquad (2)$$

Details of calculations based on the *s*-representation and formula (3) can be found in Refs. [16-18].

We make use the method of Ref. [16] allowing in a natural way to incorporate the temperature into this formalism. A basic formula of Ref. [16] connecting the Matsubara Green functions with the Green functions at zero temperature is needed,

$$G_k^{ab}(x,x';T) = \sum_{-\infty}^{+\infty} (-1)^{(n+[x])\sigma_k} G_k^{ab}(x-[x]\beta u, x'-n\beta u), \qquad (3)$$

where G_k^{ab} is the corresponding function at *T=0*, $\beta = 1/T$, *u=(0,0,0,1)*, *[x]* denotes an integer part of x_4/β, $\sigma_k = 1$ in the case of physical fermions and $\sigma_k = 0$ for boson and ghost fields. The Green functions in the right-hand side of (3) are the matrix elements of the operators G_k computed in the states $|x', a\rangle$ at *T=0*, and in the left-hand side the operators are averaged over the states with $T \neq 0$. The corresponding functional spaces U^0 and U^T are different but in the limit of $T \to 0$ U^T transforms into U^0.

The terms with *n=0* in Eqs. (3), (1) give the zero temperature expressions for the Green functions and the effective potential V', respectively. So, we can split it into two parts:

$$V'(H,\boldsymbol{H_3},T) = V'(H,\boldsymbol{H_3}) + V'_\tau(H,\boldsymbol{H_3},T). \qquad (4)$$

The standard procedure to account for the daisy diagrams is to substitute the tree level Matsubara Green functions in (1) $[G_i^{(0)}]^{-1}$ by the full propagator $G_i^{-1} = [G_i^{(0)}]^{-1} + \Pi(H,T)$ (see for details [6,13,14]), where the last term is polarization operator at finite temperature in the field taken at zero longitudinal momentum $k_l=0$.

Passing the detailed calculations we can notice that the exact one-loop EP will transform into EP, which contains the daisy diagrams as well as one-loop diagrams, by adding term contained the temperature dependent mass of particle to the exponent.

It is convenient for what follows to introduce the dimensionless quantities: $x = H/H_0$ ($H_0 = M_W^2/e$), $y = \boldsymbol{H_3}/\boldsymbol{H_3^0}$ ($\boldsymbol{H_3^0} = M_W^2/g_s$), $B = \beta M_W$, $\tau = 1/B = T/M_W$, $v = V/H_0^2$.

The total EP in our consideration consists of the several terms

$$v' = \frac{x^2}{2} + \frac{y^2}{2} + v'_{leptons} + v'_{quarks} + v'_{W-bosons} + v'_{gluons}. \quad (5)$$

These terms can be exactly written for SM fields (in dimensionless variables):

- leptons

$$v'_{leptons} = -\frac{1}{4\pi^2}\sum_{n=1}^{\infty}(-1)^n\int_0^{\infty}\frac{ds}{s^3}e^{-(m^2_{leptons}s+\frac{\beta^2 n^2}{4s})}\cdot$$
$$\cdot\left(xs\,Coth(xs)-1\right); \quad (6)$$

- quarks

$$v'_{quarks} = -\frac{1}{4\pi^2}\sum_{f=1}^{6}\sum_{n=1}^{\infty}(-1)^n\int_0^{\infty}\frac{ds}{s^3}e^{-(m_f^2 s+\frac{\beta^2 n^2}{4s})}\cdot$$
$$\cdot\left(q_f xs\,Coth(xs)\cdot ys\,Coth(ys)-1\right); \quad (7)$$

- W-bosons (see [19])

$$v'_W = -\frac{x}{8\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{ds}{s^2}e^{-(m_W^2 s+\frac{\beta^2 n^2}{4s})}\cdot$$
$$\cdot\left(\frac{3}{Sinh(xs)}+4Sinh(xs)\right); \quad (8)$$

- gluons (see [6])

$$v'_{gluons} = -\frac{y}{4\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{ds}{s^2}e^{-(m^2_{gluons}s+\frac{\beta^2 n^2}{4s})}\cdot$$
$$\cdot\left(\frac{1}{Sinh(ys)}+2Sinh(ys)\right). \quad (9)$$

Here, $m_{leptons}$, m_f, m_W and m_{gluons} are the temperature masses of leptons, quarks, W-bosons and gluons, respectively; $q_f = \left(\frac{2}{3},-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\frac{1}{3},\frac{2}{3}\right)$ - the charges of quarks.

Since we investigate the dynamics of high-temperature effects connected with the presence of external fields, we used only the leading in temperature terms of the Debye masses of the particles ([6,19]).

The temperature masses of leptons and quarks are

$$m^2_{leptons} = \left(\frac{e}{\beta}\right)^2,\ m_f^2 = \left(\frac{e}{\beta}\right)^2. \quad (10)$$

As it is known [6], the transversal components of the charged gluons and W-bosons have no temperature masses of order $\sim g_s\boldsymbol{H_3}$ and $\sim gH$. Only the longitudinal components have the Debye masses, but they are H- and $\boldsymbol{H_3}$-independent, therefore, they can be omitted in our consideration. Instead, the transversal component masses, which depend on the Landau level number, must be used. So, the transversal temperature masses of W-bosons and charged gluons

$$m_W^2 = 15\alpha_{e.w.}\frac{h^{1/2}}{\beta},\ m^2_{gluons} = 15\alpha_s\frac{h^{1/2}}{\beta} \quad (11)$$

are to be substituted. Here, $\alpha_{e.w.}$ and α_s are the electroweak and the strong interaction couplings, correspondingly.

In the approximation adopted in the present investigation we take as the masses the ground state energies of the transversal modes [10].

In the one-loop order the neutral gluon contribution is trivial $\boldsymbol{H_3}$-independent constant, which can be omitted. However, these fields are long-range states and they do give $\boldsymbol{H_3}$-dependent EP through the correlation corrections depending on the temperature and field. We included only the longitudinal neutral modes because their Debye's masses $\Pi^0(y,\beta)$ are nonzero. The corresponding EP is [6]

$$v_{ring} = \frac{1}{24\beta^2}\Pi^0(y,\beta) - \frac{1}{12\pi\beta}\left(\Pi^0(y,\beta)\right)^{3/2}$$
$$+\frac{\left(\Pi^0(y,\beta)\right)^2}{32\pi^2}\left[\log\left(\frac{4\pi}{\beta\,(\Pi^0(y,\beta))^{1/2}}\right)+\frac{3}{4}-\gamma\right], \quad (12)$$

γ is Euler's constant, $\Pi^0(y,\beta) = \Pi^0_{00}(k=0,y,\beta)$ is the zero-zero component of the neutral gluon field polarization operator calculated in the external field at finite temperature and taken at zero momentum [6]

$$\Pi^0(y,\beta) = \frac{2g^2}{3\beta^2} - \frac{y^{1/2}}{\pi\beta} - \frac{y}{4\pi^2}. \quad (13)$$

Equations (5)-(9), (12) will be used in numeric calculations.

3. GENERATION OF MAGNETIC AND CHROMOMAGNETIC FIELDS

In order to find the strengths of generated magnetic and chromomagnetic fields we have to find the minima of the EP in the presence both of them. First of all we will find the strengths x and y of fields, when the quark contribution is divided in two parts

$$v'_{quarks}(x,\beta) = v'_{quarks}\Big|_{y\to 0} = -\frac{1}{4\pi^2}\sum_{f=1}^{6}\sum_{n=1}^{\infty}(-1)^n\cdot$$
$$\cdot\int_0^{\infty}\frac{ds}{s^3}e^{-(m_f^2 s+\frac{\beta^2 n^2}{4s})}\cdot\left(q_f xs\,Coths(q_f xs)-1\right) \quad (14)$$

and

$$v'_{quarks}(y,\beta) = v'_{quarks}\Big|_{x\to 0} = -\frac{1}{4\pi^2}\sum_{f=1}^{6}\sum_{n=1}^{\infty}(-1)^n\cdot$$

$$\cdot\int_0^{\infty}\frac{ds}{s^3}e^{-(m_f^2 s+\frac{\beta^2 n^2}{4s})}\cdot(ys\,Coths(ys)-1)\,, \qquad (15)$$

where $v'_{quarks}(x,\beta)$ is that one in the magnetic field, $v'_{quarks}(y,\beta)$ - in the presence of the chromomagnetic field.

Table 1. *The strength of generated magnetic field*

β	x	$\delta\ x$	δx/x, %	$\bar{x}$
0.1	0.7	0.0000165	0.002	0.7000165
0.2	0.2	0.000745	0.373	0.200745
0.3	0.07	-0.0000549	-0.079	0.0699451
0.4	0.04	-0.0000358	-0.090	0.0399642
0.5	0.03	-0.0000467	-0.156	0.0299533
0.6	0.02	-0.0000492	-0.246	0.0199508
0.7	0.01	-0.0000380	-0.380	0.0099620
0.8	0.01	-0.0000619	-0.619	0.0099381
0.9	0.01	-0.0000241	-0.241	0.0099759
1.0	0.01	-0.0000357	-0.357	0.0099643

Table 2. *The strength of generated chromomagnetic field*

β	y	$\delta\ y$	δy/y, %	$\bar{y}$
0.1	0.8	0.000301	0.038	0.800301
0.2	0.2	-0.000239	-0.119	0.199761
0.3	0.09	-0.0000988	-0.110	0.0899012
0.4	0.05	-0.0000884	-0.177	0.0499116
0.5	0.04	-0.0000112	-0.280	0.039888
0.6	0.03	-0.0000982	-0.327	0.0299018
0.7	0.02	-0.0000442	-0.221	0.0199558
0.8	0.02	-0.0000733	-0.367	0.0199267
0.9	0.01	-0.000117	-1.166	0.009883
1.0	0.01	-0.000175	-1.749	0.009825

Let us rewrite the v' in (5) as follows

$$v'(\bar{x},\bar{y}) = v_1(\bar{x}) + v_2(\bar{y}) + v_3(\bar{x},\bar{y})\,, \qquad (16)$$

where $\bar{x} = x+\delta x$, $\bar{y} = y+\delta y$, δx and δy are the field corrections connected with the effect of fields interfusion in the quark sector.

Since the mixing of fields due to a quark loop is weak (that will be justified by numeric calculations) we can assume that $\delta x << 1$ and $\delta y << 1$, and write

$$\begin{cases} v_1(\bar{x}) = v_1(x+\delta x) = v_1(x) + \dfrac{\partial v_1(x)}{\partial x}\delta x, \\ v_2(\bar{y}) = v_2(y+\delta y) = v_2(y) + \dfrac{\partial v_2(y)}{\partial y}\delta y, \\ v_3(\bar{x},\bar{y}) = v_3(x+\delta x, y+\delta y) = v_3(x,y). \end{cases} \qquad (17)$$

After simple transformations we can find the δx and δy

$$\begin{cases} \delta x = \dfrac{\dfrac{\partial v_3(x,0)}{\partial x} - \dfrac{\partial v_3(x,y)}{\partial x}}{\dfrac{\partial^2 v_1(x)}{\partial x^2}}, \\ \delta y = \dfrac{\dfrac{\partial v_3(0,y)}{\partial y} - \dfrac{\partial v_3(x,y)}{\partial y}}{\dfrac{\partial^2 v_2(y)}{\partial y^2}}. \end{cases} \qquad (18)$$

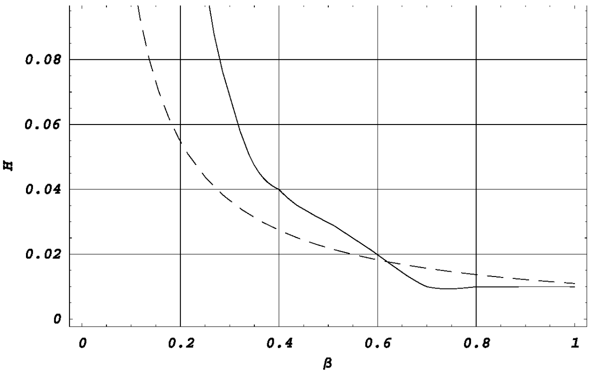

Fig. 1*. The dependence of the strengths of generated magnetic field (H) on inverse temperature (β). The dashed line is the theoretical position in the case of single magnetic field and the solid is calculated one in presence of both field*

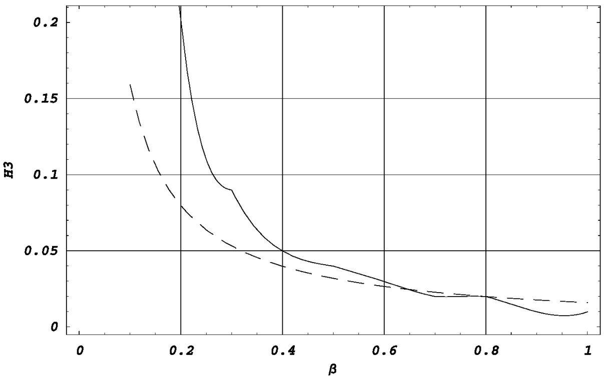

Fig. 2. *The dependence of the strengths of generated chromomagnetic field (H3) on inverse temperature (β). The dashed line is the theoretical position in the case of single chromomagnetic field and the solid one is calculated in the presence of both fields*

Hence we may obtain $\bar{x} = x+\delta x$ and $\bar{y} = y+\delta y$.

The results on the field strengths determined by numeric investigation of the total EP are summarized in Tables 1,2.

In the first column of Tables 1 and Table 2 we show the inverse temperature. In the second ones the strength of magnetic and chromomagnetic fields are adduced in the case of quark EP, which describes each field separately. Third columns give the field corrections in the case of total quark EP. The fourth columns present the relative value of corrections. And the last columns give the resulting strength of magnetic and chromomagnetic fields, correspondingly.

As it is seen, the increase of inverse temperature leads to decreasing the strengths of generated fields.

This dependence is well accorded with the picture of the universe cooling.

From the above analysis it follows that at high temperatures the value of the each type magnetic field is increased when other one is taken into account. With temperature decreasing this effect becomes less pronounced and disappears at comparably low temperatures $\beta \sim 1$.

4. DISCUSSION

Let us discuss the results obtained. As it was elaborated in the approximation to the EP including the one-loop and the daisy diagrams, in the SM at high temperatures both the magnetic and chromomagnetic fields have to be generated. These states are stable, as it follows from the absence of the imaginary terms in the EP minima.

If the quark loops are discarded, both of the fields can be generated in the system, separately. All these states are stable, due to magnetic mass $\sim g^2 (gH)^{1/2} T$ of transversal gauge field modes. Here it worth to mention that the one-loop transversal gauge field mass is of order $\sim g^4 T^2$ as nonperturbative calculations predict. This estimate is because the magnetic field strength of the spontaneously generated fields is of order $(gH)^{1/2} \sim g^2 T$ [5,6]. The possibility to calculate the magnetic mass in perturbation theory is due to the approach when an external field is taken into consideration exactly when the polarization operator of gauge field is calculated [10]. If one accounts for the magnetic field perturbatively, zero value will be obtained [20].

As it is seen from the Figs. 1,2, presenting the results of numeric computations within the exact EP, the strengths of generated fields are increasing with the temperature rising. It is also found that the curves obtained in high temperature expansion of the EP [6] are in good agreement with our numeric calculations.

The ground state possessing the magnetic and chromomagnetic fields makes advantage for existing of these fields in the electroweak transition epoch. The state with the fields is stable in the whole considered temperature interval. The imaginary part in the EP exists for the external fields much stronger then the strengths of the spontaneously generated ones. The interfusion of magnetic and chromomagnetic fields arisen from the quark sector of the EP is weak. The change of the field minima in inclusion of the fields mixing does not exceed 2 percents.

During the cooling of the universe the strengths of generated fields are decreasing that is in an agreement with what is expected in cosmology.

One of the consequences on the results obtained is the presence of strong chromomagnetic field in the early universe, in particular, at the electroweak phase transition and, probably, till the deconfinement temperature. Influence of this field on the transitions may bring new insight on these problems. As our estimate showed, the chromomagnetic field is as strong as magnetic one. So, the role of strong interactions in the early universe in the field presence needs more detailed investigations as compared to what is usually assumed [3].

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