

TRANSITION AMPLITUDE FOR FREE MASSLESS PARTICLE

V.G. Zima¹, S.A. Fedoruk²

¹Kharkov National University, Kharkov, Ukraine

e-mail: zima@postmaster.co.uk

²Ukrainian Engineering Pedagogical Academy, Kharkov, Ukraine

e-mail: fed@postmaster.co.uk

The propagator for massless particle of arbitrary spin is represented as BFV-BRST path integral in index spinor formalism. The classical formulation of the theory is investigated and it is carried out its Hamiltonization procedure. The structure functions are obtained. The BRST-charge of the model is calculated and it is shown, that it has the first rank. The expression for transition amplitude is transformed to the form of amplitude for a system with only the first class constraints. It is shown, that complexification of some phase variable results in the Gupta-Bleuler formalism. In these frameworks it is considered quantization procedure.

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1. INTRODUCTION

Calculation of the propagator for a particle is an important part of the quantization problem. The most powerful modern method for solving this problem, as well as the problem of quantization in general, is the Batalin-Fradkin-Vilkovisky Becchi-Rouet-Stora-Tyutin (BFV-BRST) approach [1]. Massive particle with spin has been in details considered in the paper [2]. However till now calculation of transition amplitude for massless particle with spin in the BFV-BRST approach is not brought to the desirable level of transparency both computations in definite approach to description of spin and for their connection in various approaches (pseudoclassical mechanics, twistorial formalism, index spinor etc.) [3-5]. It justifies further efforts on finding the propagator of particle with spin by the BFV-BRST method with using of various sorts of spinning variables.

In this paper the propagator of massless particle with arbitrary spin in the usual space-time dimension $D=4$ is represented as the BFV-BRST path integral. For description of spin it is used the index spinor formalism [6], which is seemed sufficiently general and convenient and having clear physical sense.

Section 2 presents Hamiltonian analysis of the massless particle with spin in index spinor formulation. Covariant and irreducible separation of the constraints in the classes is fulfilled and we prove the first rank of the BRST charge. It is analyzed the gauge transformations of the phase variables and corresponding variation of the action. In section 3 we construct the transition amplitude for the massless spinning particle as integral in BFV-BRST approach. Quantum theory is formulated in relativistic gauges with derivatives for the Lagrange multipliers. The presence of the second class constraints in the theory leads to modification of the integral measure and to some complication of the formulation. We transform the expression for transition amplitude to the form of amplitude for a system with only first class constraints. The half of the initial second class constraints plays the role of gauge fixing conditions for the other second

class constraints. Section 4 presents the complexification procedure for the ghost variables and the corresponding constraints in the path integral for transition amplitude. As result we obtain path integral in the Gupta-Bleuler approach. Section 5 describes general features of the BFV-BRST quantization with using Gupta-Bleuler procedure. Section 6 contains some concluding comments.

2. HAMILTONIAN ANALYSIS OF MASSLESS PARTICLE IN INDEX SPINOR FORMULATION

A spinning particle of arbitrary mass can be described in terms of commuting variables $z = (x, \zeta, \bar{\zeta})$, where x^μ is a four-vector of the space-time coordinate and ζ^a is a Weyl index spinor [6]. In the first-order formalism the Lagrangian for such a particle has the form

$$L = p\dot{\omega} - \frac{e}{2}(p^2 + m^2) - \lambda(\zeta \hat{p} \bar{\zeta} - j), \quad (1)$$

where

$$\omega = \dot{\omega} d\tau = dx - i d\zeta \sigma \bar{\zeta} + i \zeta \sigma d\bar{\zeta}$$

is bosonic 'superform', which is invariant with respect to the transformations of the N=1 'bosonic supersymmetry'

$$\delta x = i\varepsilon \sigma \bar{\zeta} - i\zeta \sigma \bar{\varepsilon}; \quad \delta \zeta = \varepsilon; \quad \delta \bar{\zeta} = \bar{\varepsilon}$$

with commuting Weyl parameter ε^a . Massive ($m > 0$) particle has been considered in details in [2]. Here we deal with the massless ($m = 0$) case.

On the constraint surface for massless particle the classical Pauli-Lyubanskii vector

$$w = (\zeta \hat{p} \bar{\zeta})p - p^2 (\zeta \sigma \bar{\zeta})$$

is proportional on shell to particle momentum, $w = j p$, thus the constant j has a sense of 'classical helicity'.

The primary constraints of the model (1) are

$$d_\zeta \equiv ip_\zeta - \hat{p}\bar{\zeta} \approx 0, \quad \bar{d}_{\bar{\zeta}} \equiv -i\bar{p}_{\bar{\zeta}} - \zeta \hat{p} \approx 0; \quad (2)$$

$$p_x - p \approx 0, p_p \approx 0; \quad (3)$$

$$p_e \approx 0, p_\lambda \approx 0. \quad (4)$$

The mass condition for massless particle

$$T \equiv p^2/2 \approx 0 \quad (5)$$

and the spin constraint

$$h \equiv \zeta \hat{p}_\zeta^\bar{} - j \approx 0, \quad (6)$$

which explicitly enter the Lagrangian (1), appear as the secondary constraints for preservation of constraints (4) in the Dirac procedure [7].

The constraints (3) are a pair of the self-conjugate second-class constraints. By introducing of Dirac brackets [7] for them one can identifies p_x and P in all other expressions and excludes P and p_p from the consideration. For all remaining canonical variables these Dirac brackets (DB) are coincided with initial Poisson brackets (PB). So the index indicating type of brackets can be omitted.

On the surface of spinor constraints (2) the spin constraint (6) is equivalent to the constraint

$$S \equiv \frac{i}{2} (\zeta p_\zeta - \bar{p}_\zeta \bar{\zeta}) - j \approx 0 \quad (7)$$

which will be called the spin constraint also.

Nonzero PB of the constraint algebra are

$$\{d_\zeta, \bar{d}_\zeta\} = 2i\hat{p},$$

$$\{S, \bar{d}_\zeta\} = -\frac{i}{2}\bar{d}_\zeta, \quad \{S, d_\zeta\} = \frac{i}{2}d_\zeta.$$

Thus, the constraint T and S belong to the first class. Spinor constraints d_ζ and \bar{d}_ζ are mixes of constraints of the first and second classes because of singularity of a matrix \hat{p} , $\det \hat{p} = -2T \approx 0$.

On a constraint surface at $j \neq 0$ spinors ζ and $\hat{p}_\zeta^\bar{}$ form a basis in the space of spinors with nondotted indexes,

$$(\zeta \hat{p}_\zeta^\bar{}) \delta_\alpha^\beta = \zeta^\beta (\hat{p}_\zeta^\bar{})_\alpha - \zeta_\alpha (\hat{p}_\zeta^\bar{})^\beta.$$

Covariant and irreducible separation of the spinor constraints (4) in the classes is achieved by projection on these spinors and conjugated ones. For the projections of spinor constraints,

$$\phi \equiv d_\zeta \hat{p}_\zeta^\bar{}, \quad \chi \equiv \zeta d_\zeta, \quad \bar{\phi} \equiv \zeta \hat{p}_\zeta^\bar{d}_\zeta, \quad \bar{\chi} \equiv \bar{d}_\zeta \bar{\zeta},$$

we have algebra

$$\{\phi, \bar{\phi}\} = -4iT(S + j), \quad \{\chi, \bar{\chi}\} = 2i(h + j), \\ \{S, \phi\} = \{\chi, \phi\} = \{\phi, \bar{\chi}\} = i\phi, \quad \{S, \chi\} = 0.$$

Hence, the projections ϕ and $\bar{\phi}$ are independent constraints of the first class. The projections χ and $\bar{\chi}$ are the second-class constraints. The PBs of the first-class constraints $F_a = (T, S, \phi, \bar{\phi})$ are the same as the Dirac brackets

$$\{A, B\}_D = \{A, B\} + i(2\zeta \hat{p}_\zeta^\bar{})^{-1} [\{A, \bar{\chi}\} \{\chi, B\} - \{B, \bar{\chi}\} \{\chi, A\}]$$

introducing for the second-class constraints $\chi, \bar{\chi}$.

In the algebra

$$\{F_a, F_b\} = 2U_{ab}^c F_c$$

of the first class constraints

$$F_a, \quad a = T, S, \phi, \bar{\phi};$$

$$F_T \equiv T, F_S \equiv S, F_\phi \equiv \phi, F_{\bar{\phi}} \equiv \bar{\phi},$$

the first rank structural functions U_{ab}^c are differ from zero only. Already the second rank structural functions $U_{b_1 b_2 b_3}^{a_1 a_2}$, which are determined in general with ambiguity by the equality

$$D_{[b_1 b_2 b_3]}^{a_1} = 2U_{b_1 b_2 b_3}^{a_1 a_2} F_{a_2},$$

where

$$D_{b_1 b_2 b_3}^{a_1} = \{U_{b_1 b_2}^{a_1}, F_{b_3}\} + U_{b_1 b_2}^k U_{b_3 k}^{a_1},$$

one can take equal to zero.

Due to reparametrization invariance of the action the Hamiltonian is a linear combination of the first class constraints

$$H = eT + \lambda S + \kappa \phi + \bar{\kappa} \bar{\phi} \equiv \lambda^a F_a.$$

As the generating functions of gauge transformations the first class constraints F_a generate the following transformations for the coordinates $\delta z = \{z, F_a \xi^a\}$, the momenta $\delta p_z = \{p_z, F_a \xi^a\}$ and the Lagrange multipliers $\delta \lambda^a = \dot{\xi}^a + \xi^b U_{bc}^a \lambda^c$ with local parameters $\xi^a(\tau)$. The last equality is a necessary condition for invariance of the Hamiltonian action with respect to the gauge transformations. For considered model of massless particle we have

$$\delta x = \varepsilon p + i\kappa \zeta \bar{\phi} p_\zeta - i\bar{\kappa} \bar{p}_\zeta \bar{\phi} \bar{\zeta}, \quad \delta p = 0,$$

$$\delta \zeta = \frac{i}{2} \phi \zeta + i\kappa \zeta \bar{\phi} \tilde{p}, \quad \delta p_\zeta = -\frac{i}{2} \phi p_\zeta - i\bar{\kappa} \hat{p} \bar{p}_\zeta,$$

$$\delta e = \dot{\varepsilon} - 4i(S + j)(\kappa \bar{\lambda}_\phi - \bar{\kappa} \lambda_\phi), \quad \delta \lambda = \dot{\phi},$$

$$\delta \lambda_\phi = \dot{\kappa} + i(\phi \lambda_\phi - \kappa \lambda). \quad (8)$$

The corresponding variation of the Hamiltonian action

$$A = \int_{t_i}^{t_f} d\tau (p_{zA} \dot{z}^A - \lambda^a F_a)$$

is equal to

$$\delta A = \left(\varepsilon \frac{p^2}{2} + j\phi + \kappa \phi + \bar{\kappa} \bar{\phi} \right) \Big|_{t_i}^{t_f}$$

and vanishes (outside of the constraint surface) only if

$$\varepsilon(\tau_i) = \varepsilon(\tau_f) = 0, \quad \phi(\tau_i) = \phi(\tau_f),$$

$$\kappa(\tau_i) = \kappa(\tau_f) = 0, \quad \bar{\kappa}(\tau_i) = \bar{\kappa}(\tau_f) = 0.$$

This circumstance makes directly admissible only 'relativistic gauges' [1], i.e. the gauges with derivatives,

which impose restrictions on \dot{e} , $\dot{\lambda}_\phi$, $\dot{\bar{\lambda}}_\phi$, expressing them in terms of the other phase space variables. It should be stressed that the last conditions on ε and κ 's are not necessary on the constraint surface in contrast with the rest parameter φ .

In some relations it will be convenient to pass from the complex second class constraints $\chi, \bar{\chi}$ to real constraints $h \approx 0$ and

$$g \equiv \frac{1}{2}(\zeta p_\zeta + \bar{p}_\zeta \bar{\zeta}) \approx 0,$$

which are equivalent to complex ones at the account of spin constraint (7). Let's note the identities

$$\chi \equiv ig - h + S; \quad \bar{\chi} \equiv -ig - h + S.$$

The brackets of the constraints h and g with all first class constraints are equal to zero, and with each other it is

$$\{h, g\} = \zeta \hat{p} \bar{\zeta} = h + j.$$

If the column of constraints h and g is multiplied on an arbitrary matrix from the group $SL(2, \mathfrak{R})$, the real constraints forming a new column, will be equivalent initial and have the same brackets with each other and with the first class constraints as initial ones. In particular such transformation allows changing by places h and g , having replaced them, for example, on $-g$ and h respectively. Below, if not opposite is told, the constraints h and g are determined up to an arbitrary $SL(2, \mathfrak{R})$ -transformation.

Let's note, that the constraints h and g differ from the constraints $(\chi - S)/\sqrt{2}$ and $i(\bar{\chi} - S)/\sqrt{2}$ by a complex unimodular transformation.

3. TRANSITION AMPLITUDE AS THE BFV-BRST PATH INTEGRAL

The most profound method for calculation of transition amplitude for constrained systems is the BFV--BRST formalism [1]. In this approach, for each first-class constraint F_a the set of coordinates of the initial phase space is supplemented by 'dynamical' Lagrange multipliers λ^a with the same Grassmannian parity, their canonically conjugate momenta π_a , $\{\lambda^a, \pi_b\} = \delta^a_b$, and the ghost variables of the opposite parity. The ghost sector contains Grassmannian odd ghosts C^a , antighosts \bar{C}_a and their canonically conjugate quantities \tilde{P}^a and P_a , $\{C^a, \tilde{P}_b\} = \delta^a_b = \{P^a, \bar{C}_b\}$. The ghost numbers of the ghost variables are

$$gh(C) = gh(P) = -gh(\tilde{P}) = -gh(\bar{C}) = 1.$$

The variables λ , π , C , P are real, whereas \tilde{P} , \bar{C} are pure imaginary.

The variables of original phase space are subjected to the second-class constraints, but the algebra of the

first-class constraints F_a remains the same even after introducing the DBs. Thus, the BRST charge has a rank one and is a linear combination of the first-class constraints, F_a and π_a , of the extended phase space

$$\Omega = F_a C^a + \pi_a P^a + \frac{1}{2} \tilde{P}_a U_{bc}^a C^c C^b;$$

$$\{\Omega, \Omega\}_{PB} = \{\Omega, \Omega\}_{DB} = 0.$$

The BRST charge is real, $\bar{\Omega} = \Omega$, Grassmannian odd, $\varepsilon(\Omega) = 1$, and has the ghost number one, $gh(\Omega) = 1$.

The path integral for the transition amplitude,

$$Z_\psi = \int D[Z, p_Z] \prod_{i,\tau} \delta(G_i) \times \prod_{\tau} 2\pi |\det\{G_i, G_j\}|^{1/2} \exp(iA_{eff}), \quad (9)$$

includes the usual Liouville measure $D[Z, p_Z]$ in the phase space of BFV-BRST approach parameterized by the coordinates $Z = (z, C, P)$ and canonically conjugate variables $p_Z = (p_z, \tilde{P}, \bar{C})$. This means that in the standard finite-dimensional approximations of the path integral, the product of differentials of each pair of the canonically conjugate real bosonic variables in the measure is divided by 2π . The differential of each variable that remains without its pair, in accordance with the boundary conditions under consideration, is also divided by 2π . Similar multipliers are absent for the Grassmannian quantities. In the Hamiltonian approach, the multipliers corresponding to the realification Jacobian of the using complex variables do not appear in the measure.

Fulfillment of the second-class constraints $(G_i) = (h, g)$, which commute with the first class constraints, is provided by the functional δ -functions in expression (9). The multipliers corresponding to the realification Jacobian do not arise in the product $\prod_{i,\tau} \delta(G_i)$ of δ -functions of the complex second-class constraints. The measure is normalized by the determinant of Poisson brackets matrix for the second-class constraints, $\det\{G_i, G_j\} = (\zeta \hat{p} \bar{\zeta})^2$, which is equal j^2 on the surface of the second-class constraints. In addition, for every 'moment of time' τ the factor $(2\pi)^{-1}$ should be introduced into the measure on each pair of the real bosonic second-class constraints.

The effective Hamiltonian action is

$$A_{eff} = \int_{\tau_i}^{\tau_f} dt (p_Z \dot{Z} - H_\psi) + A_{b.t.}$$

This expression can contain, and in our case it indeed contains, uncertainty, which should be eliminated during the calculation of amplitude. The question is in the ordering constants connected with the possible presence of products of canonically conjugate (noncommuting) variables in the BRST Hamiltonian H_ψ .

For the theory with reparametrization invariance, the BRST Hamiltonian H_Ψ is the ‘BRST derivative’ of the gauge fermion $\Psi : H_\Psi = \{ \Omega, \Psi \}$. In the amplitude (9), one can use on equal footing both Poisson and Dirac brackets because, in our case, the Poisson brackets of the first class constraints (entering into Ω) and the arbitrary function of phase space variables differ from the Dirac brackets by addends which are proportional to the second class constraints only. Thus these terms vanish on the second-class constraint surface. The gauge fermion Ψ is Grassmannian odd, $\varepsilon(\Psi) = 1$, pure imaginary, $\bar{\Psi} = -\Psi$, and has a negative ghost number, $gh(\Psi) = -1$. The relativistic gauge with derivatives for the Lagrange multipliers ($\dot{\lambda} = 0$) corresponds to $\Psi = \tilde{P}_a \dot{\lambda}^a$, then

$$H_\Psi = F_a \dot{\lambda}^a + \tilde{P}_a P^a.$$

As it is known [1], the transition amplitude does not depend on a choice of the gauge fermion if the path integral is taken over the paths, which belong to the one class of equivalence with respect to the BRST transformation. Such class is extracted by choosing the appropriate gauge and boundary conditions.

For standard formulated theory with the first class constraints the canonical gauges, connecting coordinates and momenta, and ‘relativistic’ gauges, fixing derivatives of Lagrange multipliers in term of the phase variables, are physically equivalent. The proof of the equivalence involves a permutation of some limit transition and path integration.

The boundary conditions for the ghosts and antighosts as well as for the momenta of Lagrange multipliers consist in vanishing of these quantities in initial and final moments. For real basic coordinates the boundary conditions fix both initial and final values. For complex basic coordinates, in general, due to noncommutativity of mutually conjugate quantities, the boundary conditions fix eigenvalues of corresponding operators either in bra or in ket vectors, i.e. either in initial or in final states only. Therefore, unification of conditions on considered states and canonical conjugacy of complex conjugate quantities lead to situation when boundary conditions fix initial values of a half of complex coordinates and final values of the rest of complex conjugate for them coordinates. In considered model, because of presence of the second-class constraints and occurrence of some gauge and physical variables in uniform object (index spinor), the choice of correct boundary conditions is not quite trivial.

Let us transform the expression (9) to standard form of transition amplitude for systems without second-class constraints. For this purpose we introduce auxiliary variables $\lambda^h, \pi_h, P^h, C^h, \tilde{P}_h, \tilde{C}_h$ with natural Grassmannian parities and use obvious equalities

$$\int D[P^h, \tilde{P}_h] \exp(-i \int_{\tau_i}^{\tau_f} d\tau \tilde{P}_h P^h) = 1;$$

$$\int D[\lambda^h, \pi_h] \exp(-i \int_{\tau_i}^{\tau_f} d\tau (\lambda^h \dot{h} + \pi_h \dot{g})) = \prod_{\tau} 2\pi \delta(h) \delta(g);$$

$$\int D[C^h, \tilde{C}_h] \exp(-i \int_{\tau_i}^{\tau_f} d\tau C^h \{h, g\} \tilde{C}_h) = \prod_{\tau} \{h, g\} = \prod_{\tau} (\det \{G_i, G_j\})^{1/2},$$

where the integrand variables do not satisfied any boundary conditions. Then, after permutation of the limit transition and path integration, we have

$$Z_\Psi = \lim_{\varepsilon \rightarrow 0} \int D[Z', p_Z] \exp \left(i \int_{\tau_i}^{\tau_f} d\tau [p_Z \dot{Z}' + \varepsilon^2 \pi_h \dot{\lambda}^h + \varepsilon \tilde{P}_h \dot{C}^h + \varepsilon \tilde{C}_h \dot{P}^h - H_\Psi - \tilde{P}_h P^h - \lambda^h \dot{h} - \pi_h \dot{g} - C^h \{h, g\} \tilde{C}_h] + i A_{b.t.} \right) \quad (10)$$

where $Z' \equiv (Z, \lambda^h, C^h, P^h), p_Z \equiv (p_Z, \pi_h, \tilde{P}_h, \tilde{C}_h)$. After change of variables $\varepsilon C^h \rightarrow C^h, \varepsilon \tilde{C}_h \rightarrow \tilde{C}_h, \varepsilon^2 \pi_h \rightarrow \pi_h$ with unit super-Jacobian we obtain

$$Z_\Psi = \lim_{\varepsilon \rightarrow 0} \int D[Z', p_Z] \times \exp \left\{ i \int_{\tau_i}^{\tau_f} d\tau [p_Z \dot{Z}' - \{\Omega', \Psi_1\}] + i A_{b.t.} \right\}, \quad (11)$$

where

$$\Omega' = \Omega + g C^h + \pi_h P^h,$$

$$\Psi_1 = \Psi + \tilde{P}_h \dot{\lambda}^h + \frac{1}{\varepsilon^2} \tilde{C}_h \dot{g}.$$

It is easy to verify $\{\Omega', \Omega'\} = 0$. Since Ω' is real, odd and has ghost number one, so that it may be interpreted as BRST charge for only first-class constraints $T, S, \phi, \bar{\phi}, h$. Simultaneously Ψ_1 is interpreted as gauge fermion, since it is odd and have negative ghost number. Underline that ‘new’ BRST-charge Ω' includes one second-class constraint h along with first-class constraints. The second-class constraint g of initial model enters into gauge fermion Ψ_1 and plays a role of ‘nonrelativistic’ canonical gauge condition.

Expression (11) depends on the parameter ε through the gauge fermion Ψ_1 only and therefore this path integral does not depend on ε . It makes possible in (11) at first to omit passage to the limit $\varepsilon \rightarrow 0$ and then to do passage to the limit $\varepsilon \rightarrow \infty$. Then in limit $\varepsilon \rightarrow \infty$ (permutations of the limit transitions with the path integration are made within the framework of the usual assumptions of properties last) we obtain

$$Z_\Psi = \int D[Z', p_Z] \exp \left\{ i \int_{\tau_i}^{\tau_f} d\tau [p_Z \dot{Z}' - \{\Omega', \Psi'\}] + i A'_{b.t.} \right\}$$

where $\Psi' = \tilde{P}'_A \lambda'^A$, $\lambda'^A = (\lambda^a, \lambda^h)$, $\tilde{P}'_A = (\tilde{P}_a, \tilde{P}_h)$ is gauge fermion for system with BFV-BRST phase space Z' , $p_{Z'}$ extended by auxiliary variables. This gauge fermion corresponds 'relativistic' gauge with usual boundary conditions for initial and auxiliary variables. Origin of boundary conditions for auxiliary variables C^h, \tilde{C}_h, π_h is a result of the mentioned limit transitions. The expression (11) has standard form for the transition amplitude in BFV-BRST approach for reparametric-invariant system without second-class constraints. Thus the calculation of propagator for initial system with first- and second-class constraints is reduced to the calculation of propagator for a system with only first-class constraints.

We carry out the calculation of transition amplitude in the coordinate representation for the variables z^A and in the mixed representation for the ghosts, i.e. we choose the boundary conditions

$$x^\mu(\tau_i) = x_i^\mu, \quad x^\mu(\tau_f) = x_f^\mu;$$

$$\zeta^a(\tau_1) = \zeta_1^a, \quad \bar{\zeta}^{\dot{a}}(\tau_2) = \bar{\zeta}_2^{\dot{a}};$$

$$\pi_a(\tau_f) = \pi_a(\tau_i) = 0;$$

$$C^a(\tau_i) = C^a(\tau_f) = 0, \quad \tilde{C}_a(\tau_i) = \tilde{C}_a(\tau_f) = 0,$$

where the marks (1,2) of spinors must be understood as (f, i) for the holomorphic choice and as (i, f) for the antiholomorphic one. The boundary values are not fixed for the rest of variables. The boundary conditions imposed are BRST-invariant and ensure vanishing of the BRST charge on the boundaries. This provides the form-invariance of amplitude.

The choice of boundary conditions for the index spinor is covariant. Such choice is not unique. Using combinations of the index spinor and its conjugate momentum with other variables of the phase space, one can propose a variety of covariant boundary conditions on index variables. All they are in essence equivalent and reflect a concrete choice of the quantum description of a spin (i.e., realization of the Hilbert space of quantum states). We restrict ourselves to the consideration of two basic variants. As the simplest ones, they are described in the literature now.

The boundary conditions imposed are BRST invariant and ensure vanishing of the BRST charge on the boundaries (for that it is sufficiently the conditions on π and C). One can understand the vanishing of the boundary values of the BRST charge as a classical manifestation of the quantum condition $\hat{\Omega} \left| \Psi_{phys} \right\rangle = 0$.

With boundary conditions, the correctness of the variational principle, i.e. independence of any variation of the action from the boundary values of the variation for variables, which are not fixed at the boundary, needs introducing the boundary term

$$A_{b.t.} = \varepsilon_\zeta (\zeta_2 p_{\zeta 2} - \bar{p}_{\zeta 1} \bar{\zeta}_1).$$

Here $\varepsilon_\zeta = +1$ corresponds to the holomorphic choice of the boundary conditions and $\varepsilon_\zeta = -1$ corresponds to the antiholomorphic one.

4. COMPLEXIFICATION OF PHASE VARIABLES

Complexification of some phase variables leads to the Gupta-Bleuler formalism [6] as to the result of calculation of the path integral by the saddle-point method. The Gupta-Bleuler formalism simplifies physical interpretation and mathematical calculations however it 'violates' simple formulations of some fundamental physical principles. In operator quantization this obstacle appears as necessity to use an indefinite metric in the state space and so demands certain freedom of formulations and exceptional caution in their application outside of the formally justified region. Of course, in the BFV-BRST approach, where indefiniteness of the metric is an element of basic formulation, it does not form any obstacle but attempts of 'direct' application of the Gupta-Bleuler procedure here collide with the problem of reality (Hermiticity) for BRST-charge, gauge fermion and Hamiltonian, i.e. ultimately with the unitary problem. In general case these difficulties are not overcome now [8]. Here we do not pursue purpose of solution for general problem restricting consideration with particular model. Therefore our consideration is justified in the framework of usual assumption on the properties of path integral.

Complex linear unimodular transformation of the second-class constraints and corresponding auxiliary variables

$$\begin{bmatrix} d \\ \bar{d} \end{bmatrix} = A \begin{bmatrix} g \\ h \end{bmatrix}, \quad \begin{bmatrix} \bar{\pi}_d \\ \pi_d \end{bmatrix} = \begin{bmatrix} \lambda^d \\ \bar{\lambda}^{\dot{d}} \end{bmatrix} = A^{-1T} \begin{bmatrix} \lambda^h \\ \pi_h \end{bmatrix},$$

$$\begin{bmatrix} C^d \\ \tilde{C}_d \end{bmatrix} = A^{-1T} \begin{bmatrix} C^h \\ \tilde{C}_h \end{bmatrix}, \quad \begin{bmatrix} P^d \\ \tilde{P}_d \end{bmatrix} = A \begin{bmatrix} P^h \\ \tilde{P}_h \end{bmatrix},$$

where

$$A = \frac{1}{\sqrt{-2i}} \begin{bmatrix} i & -1 \\ -i & -1 \end{bmatrix}, \quad A^{-1} = \frac{1}{\sqrt{-2i}} \begin{bmatrix} -1 & 1 \\ i & i \end{bmatrix},$$

allows writing the equalities (9) in terms of complex auxiliary variables. Then instead of (10) we obtain

$$\begin{aligned} Z_\Psi = \int D[Z_c, p_{z_c}] \exp \left\{ i \int_{\tau_i}^{\tau_f} d\tau [p_{z_c} \dot{Z}_c - H_\Psi - \right. \\ \left. - \tilde{P}_d P^d - \lambda^d (h + ig) - \frac{i}{2\varepsilon} \pi_d (h - ig) - \right. \\ \left. - \frac{1}{\varepsilon} C^d \{h, g\} \tilde{C} + iA_{b.t.} \right\}. \end{aligned}$$

As result of such replacement a complex Lagrange multiplier λ^d and its canonically conjugated momentum π_d appear. Here one should take into account corresponding modification of boundary conditions and boundary term. Return to prelimit form

(9) of the amplitude is necessary because of the constraints h and \mathcal{G} still enter in it on equal footing.

As has been shown above the propagator Z_Ψ does not depend on ε and so in the limit $\varepsilon \rightarrow \infty$ the amplitude Z_Ψ takes the form

$$Z_\Psi = \int D[Z_c, p_{Z_c}] \exp\left\{i \int_{\tau_i}^{\tau_f} dt [p_{Z_c} \dot{Z}_c + \pi_d \dot{\lambda}^d + \tilde{P}_d \dot{C}^d + \tilde{C}^d \dot{P}^d - H_\Psi - \tilde{P}_d P^d - \lambda^d (g + ih)] + iA_{b,l}\right\}.$$

Equality

$$H_\Psi = \tilde{P}_d P^d + \lambda^d (g + ih) = \{\Omega + (g + ih)C^d + \pi_d P^d, \Psi + \tilde{P}_d \lambda^d\}$$

allows us to write the amplitude in the form

$$Z_\Psi = \int D[Z_c, p_{Z_c}] \exp\left\{i \int_{\tau_i}^{\tau_f} dt [p_{Z_c} \dot{Z}_c - \{\Omega_c, \Psi_c\}] + iA_{b,l}\right\}$$

where $\Omega_c = \Omega + (g + ih)C^d + \pi_d P^d$ is the new ‘‘BRST charge’’ and $\Psi_c = \Psi + \tilde{P}_d \lambda^d$ is ‘‘gauge fermion’’.

The new ‘‘BRST charge’’ Ω_c satisfies to the conventional nilpotency condition $\{\Omega_c, \Omega_c\} = 0$ but includes, along with the first class constraints, the complex second-class constraint $d \equiv g + ih$ and is not real. The ‘‘gauge fermion’’ Ψ_c is complex as well due to nonreality of the Lagrange multiplier λ^d and antighost \tilde{P}_d . All that is not unessential for developing consideration because of can be regard as a formal method.

Relativistic gauge with derivatives for Lagrange multipliers corresponds to the choice $\Psi_c = \tilde{P}_a \lambda^a$ then

the BRST Hamiltonian $H_{\Psi_c} = \{\Omega_c, \Psi_c\}$ is equal to

$$H_{\Psi_c} = F_a \lambda^a + \tilde{P}_a P^a - i \tilde{P}_\phi C^{\bar{\phi}} \lambda^S + i \tilde{P}_\phi C^S \lambda^{\bar{\phi}} + i \tilde{P}_\phi C^{\bar{\phi}} \lambda^S - i \tilde{P}_\phi C^S \lambda^{\bar{\phi}} + 4i(S+J) \tilde{P}_T C^{\bar{\phi}} \lambda^{\bar{\phi}} - 4i(S+J) \tilde{P}_T C^{\bar{\phi}} \lambda^{\bar{\phi}},$$

where $a = T, S, \phi, \bar{\phi}, d$.

5. THE SECOND CLASS CONSTRAINTS AND COMPLEX STRUCTURE ON THE CONSTRAINED PHASE SPACE

It should be instructive to give a short review of the main ideas, which are concern to the splitting of the second-class constraints into holomorphic, and antiholomorphic sets. We start from a phase manifold M with a symplectic form ω_M , which is restricted by a set of real constraints φ_A , $A = 1, \dots, k$. The pullback of ω_M on the constraint surface N via its identical imbedding into the initial phase space M defines a presymplectic form ω_N there. The Poisson brackets of

the constraints do not vanish on the constraint surface in general, i. e.

$$\{\varphi_A, \varphi_B\}_{PB} = C_{AB}^C \varphi_C + Z_{AB}$$

where the structure functions should satisfy the Jacobi identity. The Hamiltonian vector field space falls into a subset being tangent to the constraint surface and a subset, which is skew orthogonal to it. The tangent fields correspond to the first class constraints φ_a , $a = 1, \dots, l$, and another ones φ_α , $\alpha = 1, \dots, k - l$, to the second-class constraints.

In quantization procedure, if any anomalies are absent, to the first class constraints one can apply the Dirac prescription, i. e. impose such constraints on the physical states $\hat{\varphi}_a |\Psi\rangle = 0$. The BFV-BRST quantization is far-reaching generalization of this prescription.

However, if the second-class constraints are present the BFV-BRST quantization, with its excellent covariance properties, has very limited applicability. The majority of the developed methods need in covariant separation of constraints in classes and either transform the second class constraints into the first class one due to extending of the phase space and at the price of essential losing of transparent physical picture or turn to the Gupta-Bleuler quantization. It should be noted that covariant anomaly-free solution of the second-class constraints with complete exclusion of corresponding variables is impossible in the nontrivial cases by definition. The Gupta-Bleuler procedure inserts minimal deformation of the physics, however in general also needs in covariant separation of constraints and existence of complex structure which leads to involutive (anti)holomorphic set of constraints.

For the case of even variables the existence of such complex structure was proved in neighborhood of constraint surface with using of Darboux’s theorem [8]. In our case such structure is inseparable property of the model and does not need in any proof.

A (pseudo) Hilbert-space, which is constructed on the phase space M enlarged for first-class constraints by ghost and antighost coordinates in the BFV-BRST quantization procedure and does not have a positive definite metric in general, is too large. It contains the physical Hilbert space as a subspace. The physical states are extracted as BRST-closed states which are annihilated by a Hermitian nilpotent BRST operator $\hat{\Omega}$, $\hat{\Omega} |\Psi_{phys}\rangle = 0$. The physical states are defined up to BRST-exact states, i. e. states $|\Psi_{phys}\rangle$ and $|\Psi_{phys}\rangle + \hat{\Omega} |\Lambda\rangle$, with arbitrary $|\Lambda\rangle$, are physically equivalent. The space of physical states is defined by the cohomologies of the BRST operator, i. e. by the elements of the coset space $\text{Ker} \hat{\Omega} / \text{Im} \hat{\Omega}$.

If the second-class constraints are present, it seems natural to regard a part of the splitted second-class constraints as the first-class ones and the other part as gauge-fixing condition. Actually it is BFV-BRST modification Gupta-Bleuler prescription. The problems

in such approach first of all lie in covariant separation of the constraints in the classes and in the Hermiticity conditions. Covariant separation is not a problem for our model of particle with spin as one can see from the consideration above.

A standard way to ensure uniform conditions on ket and bra vectors in the Gupta-Bleuler approach consists in modification of scalar product. This modification takes transparent form if one accounting its aim realizes the imaginary unit i (in the mixed real-complex subspace of the phase space for the complex second-

class constraints) as a matrix $i = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and

introduces a metric operator $G = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ so that

$Gi = -iG$. Now redefining the scalar product as $\langle \Phi, \Psi \rangle \equiv (\Phi, G\Psi)$ we have

$$\langle \Phi, \hat{\Omega} \Psi \rangle = \langle \hat{\Omega} \Phi, \Psi \rangle$$

due to the identity $G\hat{\Omega} = \hat{\Omega}^*G$. So we introduce an indefinite metric in the state space.

One should realize the metric operator in terms of the variables of the model. Such a problem has well known solution in holomorphic representation for harmonic oscillator.

We imply an evident equality

$$e^{-a^*a} a = -\frac{d}{da^*} e^{-a^*a}$$

where the operator $-\frac{d}{da^*} = \hat{a}$ is, by definition,

Hermitian conjugated to the creation operator a^+ in the holomorphic representation with diagonal creation operator $\hat{a}^+ = a^*$. Such a problem does not appear for the first class constraints written in terms of complex variables because of the complex structure is fictive because it introduced for convenient and does not change the basic principle of the theory. In our formulation a role of a plays that or other of the complex second-class constraints. Let's notice, that one can add linear combination of the first class constraints to a understood in this way in the exponent, as working on physical states they should give zero. It allows appreciably simplify expression in the exponent of our model. In a result we come to expression for scalar product already used in work [6].

6. CONCLUSION

In this paper the propagator of free massless particle with arbitrary spin is represented as BFV-BRST path integral within the index spinor formalism. Classical formulation of the theory is given. Its Hamiltonization is carried out. The constraints are investigated and the structure functions are obtained. The BRST-charge is found and it is shown that its rank is 1. Expression for transition amplitude as path integral is transformed to the form of transition amplitude for system with the first class constraints only. Complexification of some phase variables is carried out that allows one simplifies

calculations and, that is more important, physical interpretation.

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