CHARACTERISTICS OF UNDULATOR-TYPE RADIATION EMITTED BY BUNCH OF CHARGED PARTICLES IN WAKEFIELD

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Characteristics of the undulator radiation, emitted by a bunch of relativistic charged particles interacting with the nonsynchronous harmonics of the transverse wakefields in a periodic structure, are presented.

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1. INTRODUCTION

The undulator type radiation, UR, emitted by a beam of relativistic charged particles, undulating in the alternating wakefields induced by this beam when it is moving in the periodic structure, is interesting for practical areas of applications of accelerator and FEL physics which recently has received development. Two new radiation mechanisms have been proposed for generating ultra-short wavelength light. In papers [1,2] the image charge wakefields produced by the sheet electron beam passing between two periodic grating surfaces were considered as wiggler type fields, and a new device, an image charge undulator was proposed. The image charge wakefields are quasi-static fields which dominate when the bunch length is much longer than the maximum wavelength λ_{max} of the Cherenkov-type radiation, CR, emitted by an electron in the periodic structure. (The term CR assimilates different, used in literature, names of the radiation of a charged particle moving with a constant velocity in periodic structures.)

When the bunch length is shorter than λ_{max}, the fields of coherent Cherenkov-type radiation become prevalent wakefields, WF. In this case, as it has been shown in [3-5], the transverse components of the nonsynchronous spatial wakefield harmonics acting on the particles can give rise to their undulating motion and, consequently, to generating the undulator-type radiation. A theory of the fundamental mechanism of the radiation produced by the ultrarelativistic charge single-particle in the self-wakefield induced in an infinitely long periodic structure was given in [4,5]. As follows from the theory, in the relatively long-wave spectral region, where diffraction of generated waves is essential, the radiation manifests itself in the coherent interference of WF and UR. A pure UR takes place only in the relatively ultra-short wave range where the wave diffraction can be neglected. The power of coherent UR emitted by a bunch of N particles is proportional to N^4 [4]. In the case of incoherent emission it is proportional to N^3 [5].

In the present paper, we will consider UR spectral–angular characteristics for relativistic charged particles interacting with transverse components of the nonsynchronous spatial harmonics of the self-wakefields excited in a periodic structure. Also, we will analyze the conditions when the incoherent UR power can exceed the loss power caused by exciting wakefields.

2. RADIATION BY SINGLE-PARTICLE

As a periodic structure, we will consider a vacuum corrugated waveguide with a metallic surface. Let a particle with the ultrarelativistic longitudinal velocity v_0, the charge e, and the mass m moves through the structure with the period, D. The UR power emitted by the particle in the spectral region where the wave diffraction can be neglected ($\omega_{pe}<\omega$) is given for dipole approximation in the form [4]

$$P_{UR} = \frac{e^2}{16\pi c}\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \,\sin\theta \int_{\omega_{pe}}^{\gamma c/r_0} \omega^2 d\omega \sum_{p\neq 0}\left\{\frac{\left|\alpha_x^{(p)}\right|^2}{p^2}\left[1-\sin^2\theta\,\cos^2\varphi\, R(\omega,\theta,p)\right]\right.$$
$$\left.+\frac{\left|\alpha_y^{(p)}\right|^2}{p^2}\left[1-\sin^2\theta\,\sin^2\varphi\, R(\omega,\theta,p)\right]-\mathrm{Re}\left(\frac{\alpha_x^{(p)}\alpha_y^{(p)}}{p^2}\right)\sin 2\varphi\,\sin^2\theta\, R(\omega,\theta,p)\right\}$$
$$\times\left\{\delta\left[\omega\left(\beta_0\cos\theta-1\right)-p\Omega\right]+\delta\left[\omega\left(\beta_0\cos\theta+1\right)-p\Omega\right]\right\} \tag{1}$$

Here $R(\omega,\theta,p)\equiv\left(1-\frac{\omega}{\Omega p}\beta_0\cos\theta\right)^2-\beta_0^2\left(\frac{\omega}{\Omega p}\right)^2$, $\Omega=2\pi v_0/D$, θ is the angle between the wave vector $\boldsymbol{k}$ and the longitudinal axis OZ, ω *is* the frequency, ω_{pe} is the electron plasma frequency of the waveguide metal, φ is the angle between the axis OX and the XOY is the plane projection of $\boldsymbol{k}$, $\alpha^{(p)}\equiv\frac{2e^2\boldsymbol{u}_\perp^{(p)}}{mc\gamma\Omega}$ is small parameters $|\alpha^{(p)}| <<1$, $\boldsymbol{u}_\perp^{(p)}$ is a transverse component of the p^{th} harmonic of the wake function defined as $\boldsymbol{u}_\perp^{(p)}=\boldsymbol{w}_\perp^{(p)}+\boldsymbol{w}_\perp^{(-p)*}$, where $\boldsymbol{w}^{(p)}$ is

$$\boldsymbol{w}^{(p)}\equiv\frac{Dv_0}{4c^2V_{cell}}\sum_{n=0}^{\infty}\sum_{\lambda'_j}\frac{g_{z,\lambda_j}^{(n)*}}{\left|v_0-\frac{d\omega_\lambda}{dh}\right|_{\lambda=\lambda_j}}\left[\boldsymbol{g}_{z,\lambda'_j}^{(n+p)}-i\frac{v_0}{\omega_{\lambda_j}}\nabla_\perp g_{z,\lambda_j}^{(n+p)}-\frac{\Omega p}{\omega_{\lambda_j}}\boldsymbol{g}_{\perp,\lambda_j}^{(n+p)}\right]$$

$\boldsymbol{g}_\lambda^{(n)}\equiv\boldsymbol{g}_\lambda^{(n)}(\mathbf{r}_{0,\perp})$ is the n^{th} spatial harmonics of the vector potential eigenfunction, h is the propagation constant, c is the velocity of light, γ is the Lorentz's factor, $\omega_{\lambda j}$ is the set of eigenfrequencies satisfying resonant conditions $\omega_\lambda-h(\omega_\lambda)v_0=n\Omega$ of radiation in periodic structures.

Integrating Eq.(1) over frequency ω and angle φ, we can find the resonant UR frequencies

$$\omega^{(p)}=\frac{|p|\Omega}{1-\beta_0\cos\theta}, \tag{2}$$

and the angle distribution of UR power

$$P_{UR}=\frac{e^2\Omega^2}{4c}\left\{\sum_{p=p_{mim}}^{p=p_0}\left|\alpha^{(p)}\right|^2\int_0^{\theta_p}d\theta\,\frac{\sin\theta}{\left|1-\beta_0\cos\theta\right|^3}\left(1-\frac{\left(1-\beta_0^2\right)\sin^2\theta}{2\left(1-\beta_0\cos\theta\right)^2}\right)\right.$$
$$\left.+\sum_{p=p_0}^{p<<p_{\lim}}\left|\alpha^{(p)}\right|^2\int_0^{\pi}d\theta\,\frac{\sin\theta}{\left|1-\beta_0\cos\theta\right|^3}\left(1-\frac{\left(1-\beta_0^2\right)\sin^2\theta}{2\left(1-\beta_0\cos\theta\right)^2}\right)\right\} \quad (3)$$

Here,
$$\theta_p=\arccos\left[\frac{1}{\beta_0}\left(1-\frac{p\Omega}{\omega_{pe}}\right)\right],\ p_0 \text{ is integer of}\left[\frac{\omega_{pe}}{\Omega}\left(1+\beta_0\right)\right],$$
$$p_{min}=\begin{cases}1, & if\ \left(1-\beta_0\right)\omega_{pe}/\Omega\le 1\\ \text{integer of}\left[\left(1-\beta_0\right)\omega_{pe}/\Omega\right], & if\ \left(1-\beta_0\right)\omega_{pe}/\Omega>1\end{cases}$$

The number of harmonics in the sum is defined by the dipole limit resulting in $p<<p_{\lim}=2\pi\gamma/\max\{\alpha^{(p)}\}$.

Integrating Eq.(1) over θ and φ, the spectrum distribution of the UR power is obtained in form

$$P_{UR}=\frac{e^2}{4c}\left\{\sum_{p=p_{mim}}^{p=p_0-1}\left|\frac{\alpha^{(p)}}{p}\right|^2\int_{\omega_{pe}}^{\frac{p\Omega}{1-\beta}}d\omega\,\frac{\omega}{\beta_0}\left\{1-\frac{1}{2}\left(\frac{\omega}{\gamma p\Omega}\right)^2\left[1-\frac{1}{\beta_0^2}\left(1-\frac{p\Omega}{\omega}\right)^2\right]\right\}\right.$$
$$\left.+\sum_{p=p_0+1}^{p<<p_{\lim}}\left|\frac{\alpha^{(p)}}{p}\right|^2\int_{\frac{p\Omega}{1+\beta}}^{\frac{p\Omega}{1-\beta}}d\omega\,\frac{\omega}{\beta_0}\left\{1-\frac{1}{2}\left(\frac{\omega}{\gamma p\Omega}\right)^2\left[1-\frac{1}{\beta_0^2}\left(1-\frac{p\Omega}{\omega}\right)^2\right]\right\}\right\} \quad (4)$$

3. RADIATION BY BUNCH

Dimensions σ of bunches accelerated in the high-energy rf linacs satisfy the relation $D/\gamma^2<<\sigma$ as well as $\omega_{pe}<<2\Omega\gamma^2$. So, it is of interest to consider characteristics of incoherent hard radiation of real beams. In this case, the UR power may be written as [5]

$$P_{UR}=\frac{4e^6}{3m^2c^3}N^3\sum_{p=1}^{p<<p_{\lim}}\int_{-\infty}^{\infty}d\tau\iint_{S_\perp}d^2\boldsymbol{r}_\perp\, f_b(\boldsymbol{r}_\perp,\tau)\gamma(\boldsymbol{r}_\perp,\tau)^2\left|\boldsymbol{u}_\perp^{(p)}(\boldsymbol{r}_\perp,\tau)\right|^2, \quad (5)$$

where $f_b(\boldsymbol{r}_\perp,\tau)/\mathrm{v}_0$ is the normalized function of charge-density distribution, $S_\perp$ is the cross-section of the periodic structure, $\tau=t-z/\mathrm{v}_0$,

$$\boldsymbol{u}^{(p)}(\boldsymbol{r}_\perp,\tau)\equiv-\frac{1}{e^2N}\int_0^{\infty}d\tau'\iint_{S_\perp}d^2\boldsymbol{r}'_{0\perp}\, f_b(\boldsymbol{r}'_{0\perp},\tau-\tau')\boldsymbol{F}^{(p)}(\boldsymbol{r}_\perp,\boldsymbol{r}'_{0\perp},\tau') \quad (6)$$

$\boldsymbol{F}^{(p)}(\boldsymbol{r}_\perp,\boldsymbol{r}_{0\perp},\tau)$ is the *pth* spatial harmonics of the force, produced by the point charge *eN* (with the transverse coordinate $\boldsymbol{r}_{0\perp}$) acting on the charge *e* (with the transverse coordinate $\boldsymbol{r}_\perp$) moving at the distance $\mathrm{v}_0\tau$ after the point charge.

For analytical calculations it is convenient to consider a monochromatic filamentary uniform bunch of the length l_b moving in a weakly corrugated circular waveguide of the radius

$$b(z)=b_0\left[1+\varepsilon(z)\right]=b_0\left[1+\sum_{p=-\infty}^{\infty}\varepsilon_p\exp\left(i\frac{2\pi p}{D}z\right)\right]. \quad (7)$$

Here $\varepsilon(z)<<1$ is the relative depth of the corrugation, b_0 is the average radius of the waveguide.

The UR power lost by the bunch moving at the distance r_b from the waveguide axis is given by [5]

$$P_{UR}=\frac{4e^6}{3m^2c^3}\gamma^2N^3\left(\frac{4\pi}{b_0D}\right)^2\sum_{p=1}^{\infty}p^2\left|2\varepsilon_p\right|^2\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\left(r_b/b_0\right)^{m+n}}{\left(1+\delta_{0,m}\right)\left(1+\delta_{0,n}\right)}$$
$$\times\sum_{s=1}^{\infty}\sum_{q=1}^{\infty}\left\{A_{m,s}A_{n,q}I\left(\omega_{m,s,p},\omega_{n,q,p}\right)+B_{m,s}B_{n,q}I\left(\omega'_{m,s,p},\omega'_{n,q,p}\right)\right.$$
$$\left.-A_{m,s}B_{n,q}I\left(\omega_{m,s,p},\omega'_{n,q,p}\right)-B_{m,s}A_{n,q}I\left(\omega'_{m,s,p},\omega_{n,q,p}\right)\right\} \quad (8)$$

where $\delta_{0,m}$ is the Chronicler's symbol,

$$I(\omega_1,\omega_2)\equiv\frac{c^2}{\omega_1\omega_2l_b^2}\left[1-\frac{\sin(\omega_1l_b/c)}{\omega_1l_b/c}-\frac{\sin(\omega_2l_b/c)}{\omega_2l_b/c}+\frac{\sin((\omega_1-\omega_2)l_b/c)}{(\omega_1-\omega_2)l_b/c}\right],$$

$$A_{m,s}(r/b_0)\equiv\frac{J'_m(\mu_{m,s}r/b_0)}{J'_m(\mu_{m,s})},\qquad B_{m,s}(r/b_0)\equiv\frac{b_0}{r}\frac{m^2}{m^2-\mu'^2_{m,s}}\frac{J_m(\mu'_{m,s}r/b_0)}{J_m(\mu'_{m,s})},$$

$\mu_{m,s}$ and $\mu'_{m,s}$ are the zeros of the Bessel functions $J_m(\mu_{m,s})=0$ and $J'_m(\mu'_{m,s})=0$, respectively,

$$\omega_{m,s,p}=\frac{\pi pc}{D}\left[1+\left(\frac{D\mu_{m,s}}{2\pi pb_0}\right)^2\right]\qquad \omega'_{m,s,p}=\frac{\pi pc}{D}\left[1+\left(\frac{D\mu'_{m,s}}{2\pi pb_0}\right)^2\right]$$

are the frequencies of resonant WF modes. Therewith, the WF power loss of the bunch is [5]

$$P_{WF}=\frac{2\pi(eN)^2}{D}\sum_{p=1}^{\infty}p\left|2\varepsilon_p\right|^2\sum_{m=0}^{\infty}\left(\frac{r_b}{b_0}\right)^{2m}\frac{1}{\left(1+\delta_{0,m}\right)}$$
$$\times\sum_{s=1}^{\infty}\left\{\omega_{m,s,p}\left(\frac{\sin(\omega_{m,s,p}l_b/2c)}{\omega_{m,s,p}l_b/2c}\right)^2-\frac{m^2\omega'_{m,s,p}}{m^2-\mu'^2_{m,s}}\left(\frac{\sin(\omega'_{m,s,p}l_b/2c)}{\omega'_{m,s,p}l_b/2c}\right)^2\right\} \quad (9)$$

Let us consider the sinus-type corrugated waveguide of a radius $b(z)=b_0\left[1+2\varepsilon_1\sin(2\pi z/D)\right]$, with $\varepsilon_1=0.05$. Let an electron bunch, with the typical for SLAC Linac parameters, $l_b=500$ μm, $N=4\times10^{10}$, and $\gamma=10^5$ moves in a sub-millimeter structure with $b_0=D=0.3$ mm. The bunch distance from the axis is chosen equal to $r_b=0.9b_0$ to estimate the maximal values of the UR power.

The distribution of the synchronous harmonic of the longitudinal wake function $u_z^{(0)}$ and the $(+1)^{st}$ harmonic transverse wake function $u_r^{(+1)}$ along the bunch are represented in Figs.1 and 2, respectively. In calculations 120 resonant WF modes are taken intoaccount.

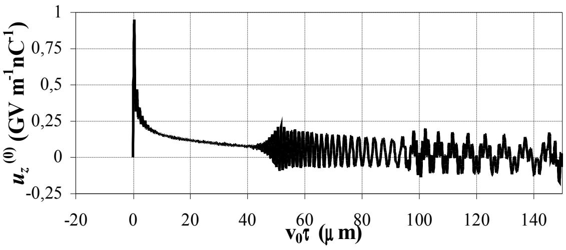

Fig. 1. *The synchronous harmonic of longitudinal wake function*

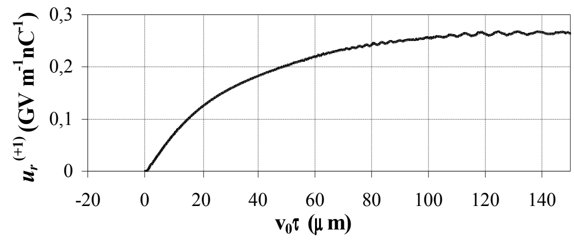

Fig. 2. *The nonsynchronous harmonic of the transverse wake function*

As shown in Fig.1, the bunch head, up to 50 μm, basically excites WF, while UR is predominantly emitted by the next part after head of the bunch (see Fig.2).

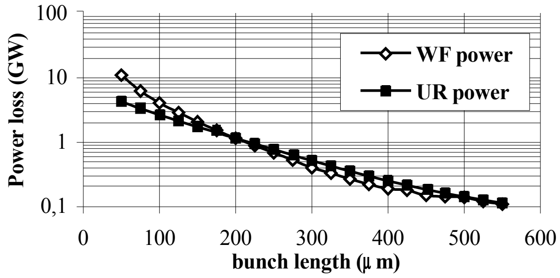

***Fig.3**. The UR and WF power versus the bunch length*

Eq.(5) shows that in the range of ultra-sort wavelength light the UR power grows as a square of particle energy. Therefore, we can expect that for very high-energy particles the UR power can exceed the WF power which, as well-known, is independent on the particle energy. Figs.1,2 indicate to increasing the UR power fraction with lengthening the bunch. This is confirmed by Fig.3. The incoherent UR power reaches the WF power at the bunch length 200 μm.

The incoherent UR power and WF power for the bunch of length 500 μm versus electron energy are represented in Fig.4. As seen from this figure the UR predominates at energies above 50 GeV.

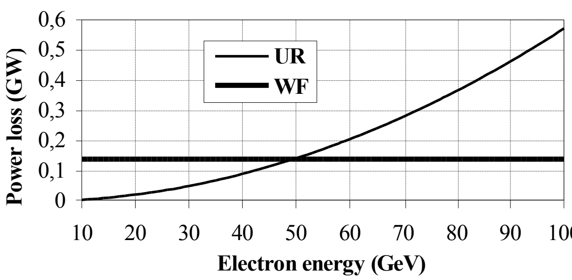

***Fig.4**. The incoherent UR and WF power*

Fig. 5 represents the dependence of a number of electrons in the bunch of length 500 μm on their energy when the incoherent UR power equals to the WF power. This dependence determinates the limit above which the incoherent UR power exceeds the WF power.

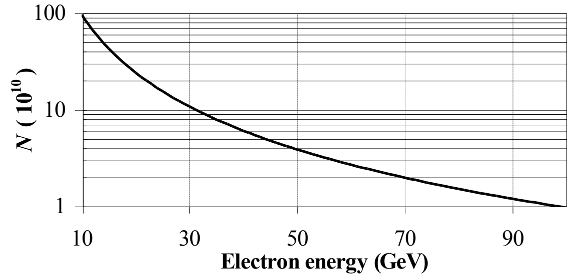

***Fig. 5**. The number of bunch electrons versus the electron energy when UR power equals the WF power*

4. CONCLUSIONS

The spectral–angular characteristics of undulator-type radiation of the ultrarelativistic charged particle undulating in nonsynchronous spatial harmonics of the self-wakefields in the periodic structure are obtained. It is shown numerically that whereas the beam power losses fall with lengthening the bunch, the relative fraction of the UR power increases. The conditions when the incoherent UR power becomes comparable with the WF power are defined.

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ХАРАКТЕРИСТИКИ ОНДУЛЯТОРНОГО ИЗЛУЧЕНИЯ СГУСТКА ЗАРЯЖЕННЫХ ЧАСТИЦ В КИЛЬВАТЕРНОМ ПОЛЕ

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Получены характеристики ондуляторного излучения сгустком релятивистских заряженных частиц, взаимодействующих с несинхронными гармониками кильватерного поля в периодической структуре.

ХАРАКТЕРИСТИКИ ОНДУЛЯТОРНОГО ВИПРОМІНЮВАННЯ ЗГУСТКОМ ЗАРЯДЖЕНИХ ЧАСТИНОК В КІЛЬВАТЕРНОМУ ПОЛІ

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Одержано характеристики ондуляторного випромінювання згустком релятивістських заряджених частинок, які взаємодіють з несинхронними гармоніками кільватерного поля в періодичній структурі.