NUMERICAL SIMULATION OF THE SUPERCRITICAL ELECTRON BEAM DYNAMICS IN MAGNETIC FIELD OF FINITE SIZE IN THE PRESENCE OF PLASMA

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The first results of development of a numerical electromagnetic 2.5-dimensional code «SOM 2.5» (3 dimensional by the velocities and 2 dimensional by the coordinates) for researching of a virtual cathode in the presence of plasma are submitted.

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1. INTRODUCTION

Basis of a method of collective acceleration of ions is the slow wave of a spatial charge formed by high-current electron beam as a result of its spatial and time modulation. Methods of spatial [1, 2] and time [1, 3] modulations are well-known. The use of beams with current higher than limiting vacuum one opens the opportunity of time modulation by the virtual cathode field in the presence of plasma [4, 5].

The virtual cathode is a strongly nonlinear formation for complete description of which the numerical methods are used. In the present report the results of development of the numerical electromagnetic code describing the self-consistent dynamics of a virtual cathode in the cylindrical resonator are adduced.

2. THE NUMERICAL ALGORITHM

The theoretical analysis of dynamics of electron-ion formation is based on a PIC method [6]. A statement of the problem is the following.

The relativistic electronic beam having a ring section, is injected in the cylindrical resonator. Beam thickness is $\Delta = r_2 - r_1$, where r_1 and r_2 are the inner and external radii of E-beam. Beam current is I_b. At the resonator input (at $z = 0$) injected beam is monoenergetic. The transversal components of electron velocities are equal to zero. In the drift space there is an external magnetic field of intensity H_0 that is directed along the longitudinal axis z of the resonator.

The system is axially symmetrical. It allows being restricted to the solution of set of Maxwell equations for the E-type wave at numerical simulation of dynamics of electromagnetic fields:

$$\partial E_r/\partial t = -c\,\partial H_\varphi/\partial z - 4\pi j_r;$$

$$\partial E_z/\partial t = c\left[\partial\left(rH_\varphi\right)/\partial r\right]/r - 4\pi j_z;$$

$$\partial H_\varphi/\partial t = c\left(\partial E_z/\partial r - \partial E_r/\partial z\right),$$

where j_r, j_z are the radial and azimuthal components of macroparticles current density which are contained in the resonator. They are calculated by means of the mechanism of weighing of currents in nodes of a two-dimensional spatial grid [6]. Thus it is necessary to know a position and velocity of each macroparticle. They are determined from the solution of motion equations which have been written down in cylindrical coordinates:

$$\frac{d^2 r_p}{dt^2} = \frac{q}{m\gamma}\left[E_r - \frac{dr_p}{dt}\delta + \frac{1}{c}\left(r_p\frac{d\varphi_p}{dt}H_0 - \frac{dz_p}{dt}H_\varphi\right) + r_p\left(\frac{d\varphi_p}{dt}\right)^2\right];$$

$$\frac{d^2\varphi_p}{dt^2} = \frac{q}{m\gamma}\left(\frac{1}{c r_p}\frac{dr_p}{dt}H_0 - \frac{d\varphi_p}{dt}\delta\right) - 2\frac{1}{r_p}\frac{dr_p}{dt}\frac{d\varphi_p}{dt};$$

$$\frac{d^2 z_p}{dt^2} = \frac{q}{m\gamma}\left(E_z - \frac{dz_p}{dt}\delta + \frac{1}{c}\frac{dr_p}{dt}H_0\right),$$

where $\gamma = \left(1 - v_p^2/c^2\right)^{-1/2}$,

$v_p^2 = \left(dr_p/dt\right)^2 + \left(r_p\,d\varphi_p/dt\right)^2 + \left(dz_p/dt\right)^2$,

$\delta = \left(E_r\,dr_p/dt + E_z\,dz_p/dt\right)/c^2$, q and m are the charge and weight of the macroparticle.

The numerical solution of Maxwell equations for a E-type wave and weighing of charges were carried out on shifted spatial and time grids one from other. The spatial grid for evaluation of these quantities is shown on Fig.1. For a time discretization of motion equations the predictor-corrector method was used [7]. Values of the macroparticles velocities are calculated in half-integer time steps $t^{n+1/2} = (n + 1/2)\tau$, and coordinates $\left(z_p, r_p\right)$ - in the integer time steps $t^n = n\tau$ (n is integer, τ is a time step). Thus the values of the fields components, contained in the motion equations , are calculated by the linear interpolation from nodes of a grid. Owing to the selected scheme, the solution of Maxwell equations is necessary to carry out twice more often, than solution of motion equations. Function H_φ is calculated in time steps $t^{n\pm 1/4}$, and E_z and E_r in time steps t^n and $t^{n+1/2}$ respectively.

Boundary condition for fields consists in vanishing of tangential components of an electromagnetic field on walls of the drift chamber. At an initial time step the value of electromagnetic fields components are equal to zero; particles in the resonator are absent.

In the issue the operations flowchart on one time step $\tau = t^{n+1} - t^n$ looks like this:

1. Finding the value of a magnetic field H_φ at time step $t^{n+1/4}$

$$H_{\varphi}^{n+1/4} = F_{H_{\varphi}}\left(H_{\varphi}^{n-1/4}, E_r^n, E_z^n\right).$$

2. Calculation the values of components of electric field E_z and E_r at time step $t^{n+1/2}$

$$E_r^{n+1/2} = F_{E_r}\left(E_r^n, H_{\varphi}^{n+1/4}, j_r^{n-1/2}\right);$$

$$E_z^{n+1/2} = F_{E_z}\left(E_z^n, H_{\varphi}^{n+1/4}, j_z^{n-1/2}\right).$$

3. Finding the value of a magnetic field H_{φ} at time step $t^{n+3/4}$

$$H_{\varphi}^{n+3/4} = F_{H_{\varphi}}\left(H_{\varphi}^{n+1/4}, E_r^{n+1/2}, E_z^{n+1/2}\right).$$

   Finding the value of a magnetic field $H_{\varphi}^{n+1/2}$ by averaging.
4. Solving of motion equations by the predictor-corrector method:
   a. calculation a preliminary value of an angular velocity of macroparticle $\omega^{n+1/2}$;
   b. the values of r^{n+1}, z^{n+1}, v_r^{n+1}, v_z^{n+1};
   c. finding the values of the same quantities at time step $t^{n+1/2}$ by averaging;
   d. calculation a final value of an angular velocity of macroparticle $\omega^{n+1/2}$.
5. Injecting new macroparticles in the resonator and with taking into account of newly injected particles calculating values j_z and j_r in grid nodes for time step $t^{n+1/2}$.
6. Calculation the values a components of an electric field E_z and E_r at time step t^{n+1}

$$E_r^{n+1} = F_{E_r}\left(E_r^{n+1/2}, H_{\varphi}^{n+3/4}, j_r^{n+1/2}\right);$$

$$E_z^{n+1} = F_{E_z}\left(E_z^{n+1/2}, H_{\varphi}^{n+3/4}, j_z^{n+1/2}\right).$$

3. RESULTS OF THE NUMERICAL SIMULATION

The mentioned algorithm has been implemented as a complex of programs in C++ language. For numerical calculations the following parameters of the experimental installation "Agat" [4, 5] has been chosen: $R = 2{,}5\,cm$, $r_1 = 1{,}5\,cm$, $r_2 = 1{,}7\,cm$, $L = 15\,cm$, energy of beam electrons $280\,keV$, intensity of external driving magnetic field $H_0 = 15\,kOe$.

We carried out the calculations of the electron beam dynamics at various values of input current of the beam. In Fig.2 the results of simulation for beam current $I_b = 0{,}5\,kA$, and in Fig.3 - for current $I_b = 4{,}0\,kA$ are given.

For chosen sizes of the drift chamber and electron beam the limiting vacuum current I_{cr} is equal to $I_{cr} = 3{,}76\,kA$.

The calculations which have been carried out by the 2.5-dimensional electromagnetic code "SOM 2.5" have shown, that on taken times, which are equal approximately to one pass of the beam through the resonator, when currents less than critical one the virtual cathode is not forming. The motion of macroparticles along the drift chamber is laminar, and the beam keeps the tubular shape. During propagation along the resonator beam thickness varies insignificantly.

At currents higher than critical one the forward front of the beam moves as laminar one. The Coulomb field of the forepart macroparticles decelerates the following macroparticles and scatters them in a transverse direction. As a result the virtual cathode reflecting newly injected electrons is forming. Eventually laminar motion of beam particles is completely broken. Though for the value of a magnetic intensity used in calculations the beam remains tubular, its thickness varies essentially.

4. CONCLUSIONS

1. The numerical 2.5-dimensional electromagnetic code "SOM 2.5", describing dynamics of a high-current beam and fields in the cylindrical resonator, based on a PIC method is created.
2. The numerical code adequately to physical insights describes the process of creation of a virtual cathode in the finite magnetic field. The limiting values of currents received in 2.5-dimensional model, approximately coincide with calculated earlier in one-dimensional model.
3. Nevertheless, on large considered times at big values of currents of injected beams the accumulation of calculating errors takes place. In result there are reflected macroparticles even at $I_b < I_{cr}$. Thus, there is necessary to optimize the created numerical code with the purpose of minimization of calculating errors.

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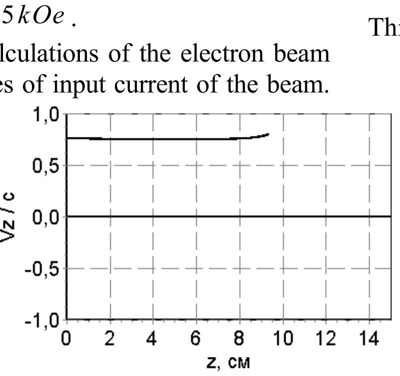
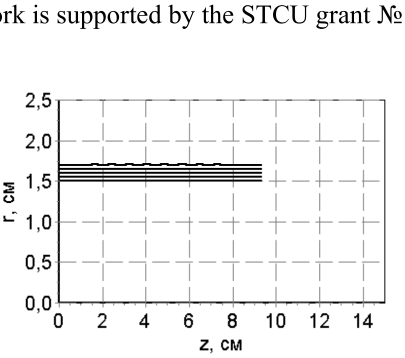
$t = 0{,}4\,ns$





$t = 1,0\,ns$

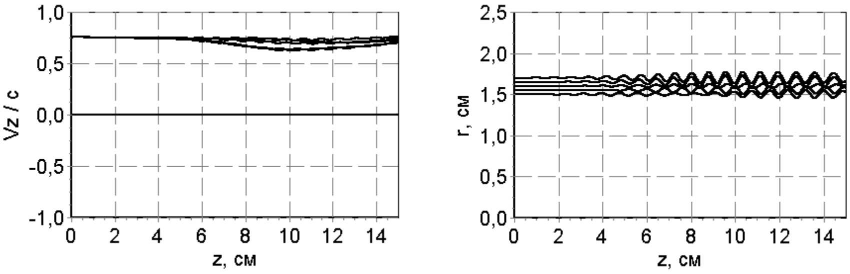


Fig.2. Results of simulation for the beam current $I_b = 0,5\,kA$

$t = 0,4\,ns$

$t = 1,0\,ns$

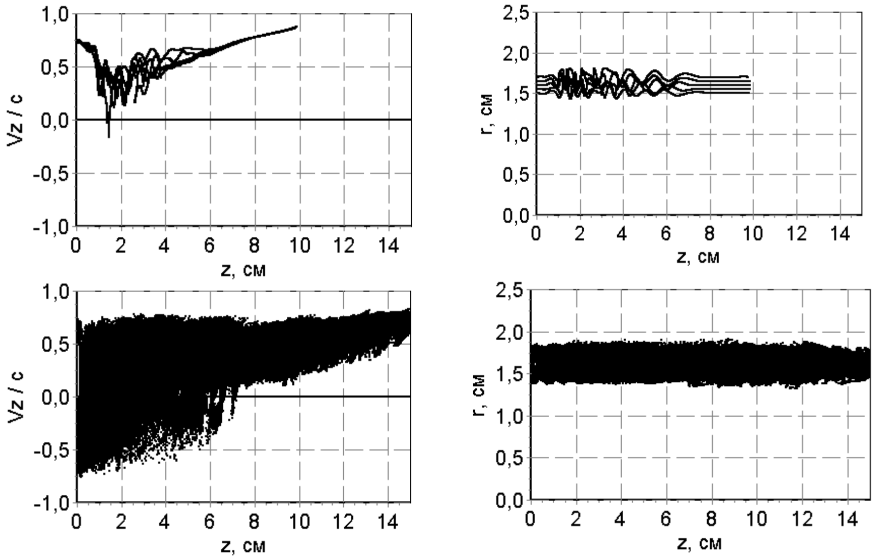


Fig.3. Results of simulation for beam $I_b = 4,0\,kA$

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ДИНАМИКИ СВЕРХКРИТИЧЕСКОГО ЭЛЕКТРОННОГО ПУЧКА В МАГНИТНОМ ПОЛЕ КОНЕЧНОЙ ВЕЛИЧИНЫ В ПРИСУТСТВИИ ПЛАЗМЫ

П.И.Марков, И.Н.Онищенко, Г.В.Сотников

Представлены первые результаты создания численного электромагнитного 2,5-мерного (3-х мерного по скоростям и 2-х мерного по координатам) кода для исследования виртуального катода в присутствии плазмы.

ЧИСЕЛЬНЕ МОДЕЛЮВАННЯ ДИНАМІКИ НАДКРИТИЧНОГО ЕЛЕКТРОННОГО ПУЧКА В МАГНІТНОМУ ПОЛІ СКІНЧЕННОЇ ВЕЛИЧИНИ В ПРИСУТНОСТІ ПЛАЗМИ

П.І.Марков, І.М.Онищенко, Г.В.Сотників

Представлено перші результати створення чисельного електромагнітного 2,5-мірного (3-х мірного по швидкостях і 2-х мірного по координатах) коду для дослідження віртуального катоду в присутності плазми.