PROPERTIES AND EXCITATION OF SOLITARY PERTURBATIONS BY ELECTRON BEAM IN ACCELERATOR

V.I. Lapshin, V.I. Maslov, I.N. Onishchenko, V.L. Stomin
NSC Kharkov Institute of Physics & Technology, 61108 Kharkov, Ukraine
E-mail: vmaslov@kipt.kharkov.ua

The excitation of a solitary wave perturbation of electric potential hump type with a large amplitude of an electron beam at the accelerator has been considered. Its properties and dependencies of properties on the amplitude have been investigated. This perturbation propagates in the rest frame of the beam with a velocity, approximately equal to the thermal velocity of beam electrons. The perturbation forms hole and vortex in the electron phase space. The solitary perturbation is excited due to nonlocal interaction of the beam with a metallic wall of final conductivity. This hump of electric potential is the BGK perturbation.
PACS numbers: 29.17.+w

1 INTRODUCTION

If the wall of the accelerator has a finite conductivity, then the penetration of electron beam field, propagating along the wall, into the wall is possible and the excitation of volume charge perturbation, δq, is also possible in the wall. It means that the interaction of the electron beam with the wall is possible. We consider the possibility of non-linear perturbation excitation in the electron beam due to this interaction with the wall.

2 PROPERTIES OF A SOLITARY HUMP-FIELD

The perturbation of volume charge of the wall is described by the following equation

$$\partial_t \delta q = \sigma \partial_z E. \tag{1}$$

Here σ is the conductivity of the wall. The Poisson equation for the longitudinal component of electric field in a long- wave approximation is as follows

$$\partial_z E = -4\pi(e\delta n_b + R\delta q). \tag{2}$$

Here δn_b is the perturbation of the beam density, R is the small parameter or geometrical factor, taking into account the skin depth and radial spatial distance between the beam and conducting wall. From equations of a continuity and motion for beam electrons (1), (2) one can obtain a dispersion relation for high-frequency perturbations with the frequency ω and wave vector k:

$$1 - \omega_b^2/(\omega - kV_b)^2 - i4\pi\sigma R/\omega = 0. \tag{3}$$

Here ω_b is the plasma frequency of the electron beam, and V_b is its velocity. From (3) it follows that the maximum growth rate of amplitude $\gamma_L \approx (\pi\sigma R\omega_b)^{1/2}$ has perturbations with the wave vector $k \approx \omega_b/V_b$.

With amplitude growth excited perturbations become non-linear. We consider a limiting case, when perturbations are transformed in chain of short perturbations. We investigate the excitation of such short solitary perturbation at a non-adiabatic stage of its evolution. In other words, the approach is considered, when during the time of perturbation excitation the beam electrons are shifted relatively to perturbation on a distance, not exceeding its width.

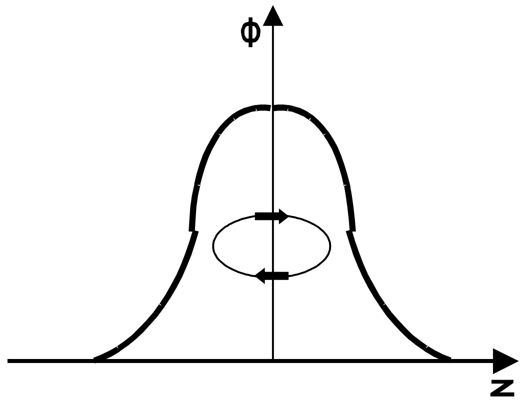

Fig. 1.

We consider the solitary electric potential hump, propagating with velocity V_s along z in conductive waveguide with a strong magnetic field and with high-energy electron beam. At first we describe the solitary perturbation of electric potential φ of a small amplitude, φ_o. From Maxwell equations one can derive the equation for the electric field E

$$\Delta E - (V_s\nabla)^2 E/c^2 + 4\pi e[\nabla n - (V_s\nabla) n v/c^2] = 0. \tag{4}$$

Here $E = -\partial_z\varphi$; n, v are the electron density and velocity. For latter determining we use the kinetic equation for the electron distribution function f_e

$$\partial f_e/\partial t + (v\nabla) f_e - (e/m_e)(E + [v,B]/c)\partial f_e/\partial v = 0. \tag{5}$$

As we consider a strong magnetic field $H_o \rightarrow \infty$, directed along z-axis, then the electrons propagate along this axis. Because, at first, we derive the solution in the form of a stationary soliton, propagating with a velocity V_s, we use dependence of f_e on the coordinate and time $\xi = z - V_s t$. In this case from the Vlasov equation for high-energy electrons (5) one can obtain expressions for a high-energy electron density perturbation $\delta n = n - n_o$

$$\delta n_h = n_{oh}[e\varphi/T_h - (4/3\sqrt{\pi})(e\varphi/T_h)^{3/2} + (e\varphi/T_h)^2/2]. \tag{6}$$

Here T_h, is the temperature of high-energy electrons.

Using (6), from (4) we derive the nonlinear equation for small amplitudes

$$(\varphi')^2=\varphi^2(k_\perp^2-\omega_p^2/V_s^2+r^{-2}_d)-(16/15)r^{-2}_d(e/\pi T_h)^{1/2}\varphi^{5/2}, \quad (7)$$

$r^2_d=T_h/4\pi e^2 n_{oh}$.

Using $\varphi'|_{\varphi=\varphi_o}=0$, one can derive the equation for the velocity of solitary perturbation of electric potential hump type

$$k_\perp^2\gamma^2_o-\omega_p^2/V_s^2\gamma^3_o+r^{-2}_d=(16/15)r^{-2}_d(e/\pi T_h)^{1/2}\varphi^{1/2}_o. \quad (8)$$

Using (8), one can solve the equation (7) as follows

$$\varphi=\varphi_o/ch^4[\xi/\Delta\xi],\ \Delta\xi=4r_d(T_h/e\varphi_o)^{1/4}, \quad (9)$$

where ξ is the coordinate along the direction of perturbation propagation.

At large amplitudes the shape of the solitary electric potential hump is determined by

$$(\varphi')^2=k_\perp^2\gamma_o^2\varphi^2+ \quad (10)$$
$$+8\pi e n_o\{-\varphi+(mV_s c/e)[[(\gamma_o+e\varphi/mc^2)^2-1]^{1/2}-(\gamma_o^2-1)^{1/2}]\}+$$
$$+8\pi e n_{oh}\{-T/e+2(T\varphi/e\pi)^{1/2}(1+T_{tr}/T)+$$
$$+\exp(e\varphi/T)[1-(2/\sqrt{\pi})\int_o^{\sqrt{(e\varphi/T)}} dx\, \exp(-x^2)]T/e-$$
$$-(2T_{tr}/e\sqrt{\pi})\exp(-e\varphi/T_{tr})\int_o^{\sqrt{(e\varphi/T)}} dx\, \exp(x^2T/T_{tr})\}.$$

3 EXCITATION OF NONLINEAR PERTURBATIONS IN AN ELECTRON BEAM DUE TO DISSIPATIVE INSTABILITY DEVELOPMENT AT INTERACTION OF BEAM AND WALL WITH A FINITE CONDUCTIVITY

Taking into account the nonstationary terms in a kinetic equation for beam electrons one can obtain, for correction to density of beam electrons, δn_{bt}, proportional to $\partial_t\phi_o$, the expression

$$\partial_z\delta n_{bt} \approx \partial_t\phi[1/z - z + (2z^2 - 1)\phi_o/6z]V_{th}\sqrt{2}. \quad (11)$$

Here $\phi=e\varphi/T_h$, T_h is the electron beam temperature, ϕ_o is the amplitude of perturbation; V_{th} is the thermal velocity of beam electrons, $z=(V_s-V_b)/V_{th}\sqrt{2}$. Similarly [1] one can found that the velocity of the solitary perturbation is approximately equal to $V_s \approx V_b - 1.32V_{th}$.

Taking into account the electron beam interaction with perturbation of the charge density in the wall from (2) one can derive

$$e\,\delta n_{bt} \approx -R\delta q. \quad (12)$$

From (11), (12) the equation follows, describing the excitation of perturbation. In case of not so small amplitudes it has a form

$$\partial_\tau\phi_o\partial_\tau\phi \approx 8\sqrt{2}\partial^3_y\phi. \quad (13)$$

Here $y = z/r_b$, $r_b = (T/4\pi n_b e^2)^{1/2}$, $\tau = t\gamma_L$. The solution of equation (13) we search as

$$\phi(y, \tau) = \phi_o(\tau)\mu[y - \int^{\tau}_{-\infty} d\tau_o\delta V_s(\phi_o(\tau_o))]. \quad (14)$$

δV_s is the change of a soliton velocity, appeared as a result of its interaction with environment, $\mu(y)$ is the Spatial distribution of a soliton potential, described by equation (7). From (7), (13), (14) one can obtain the following expression for the growth rate of excitation of the solitary perturbation

$$\gamma \approx (\gamma_L/3)\phi_o^{1/4}\,[2/(\sqrt{3}-1)]^{1/2}. \quad (15)$$

Thus, the possibility of non-linear perturbation formation in an electron beam as a result of its interaction with a wall of a final conductivity is under consideration. The formation of perturbations in the electron beam was observed in [2].

REFERENCES

1. H.Schamel. Electron holes, ion holes and double layers // *Phys. Rep.* 1986. V. 140. N 3. P.163-191.
2. L.K.Spentzouris, P.L.Colestock, F.Ostiguy // *Proc. of the PAC'95*. 1995.