BIPERIODICAL BUNCHING SYSTEM BASED ON THE EVANESCENT WAVES

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To improve the beam bunching at the initial stage of acceleration it is necessary to create an increasing field distribution. Such distribution can be created in a biperiodic disk-loaded waveguide. It is well known that in periodic structures there are two different types of electromagnetic oscillations. In the passbands they exist in the form of travelling waves. In the stopbands electromagnetic oscillations exist in the form of evanescent waves and have the decreasing (increasing) dependence on the coordinate. The properties of electromagnetic oscillations in the stopband that exists in the biperiodic structure due to its biperiodicity are investigated. The results of the simulation of bunching process in the system based on the evanescent wave are presented.
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1 INTRODUCTION

Electron beam characteristics at the exit of the accelerator are substantially defined by parameters of a bunching system. Using standing waves in the bunching systems improves characteristics of the accelerated beams. In bunching systems based on the standing wave the klystron bunching mechanism can be combined with the accelerating process. For this purpose it is necessary to create the field distribution, at which the field amplitude increases and intervals with zero field exist. It was shown, that in the biperiodic structure there is an eigen oscillation appropriate to such a non-trivial field distribution [1]. The increasing field distribution cannot be obtained in the smooth waveguide. As a result of the boundary conditions, the amplitude of the increasing solution in the smooth waveguide is always less than the amplitude of the decreasing one. In this paper the properties of evanescent oscillations in the biperiodic structure are investigated. The results of simulation of the bunching system based on an evanescent oscillation are presented.

2 MATHEMATICAL MODEL

Consider the boundless structure consisting of the cavity chain, when geometrical sizes of cavities and radii of apertures change periodically. Let us designate: the amplitude of E_{010} - mode in the cavity with length D_1 and radius b_1 as A_n (A - cavity); the amplitude of E_{010} - mode in the resonator with length D_2 and radius b_2 as B_n (B - cavity); radii of coupling apertures as a_1 and a_2. A – cavity has aperture of the radius a_1 on the right. In comparison with a plane cavity chain ($D_1 = D_2$, $b_1 = b_2$, $a_1 = a_2$) the field amplitude in the biperiodical cavity chain is described by two difference homogenous equations with constant coefficients [2, 3]:

$$A_n\left(\omega^2-\omega_1^2(1+\alpha_1)-\frac{i\varpi_1\varpi}{Q_1}\right)+\omega_1^2\varepsilon_1 B_n+\omega_1^2\varepsilon_2 B_{n-1}=0, \quad (1.1)$$

$$B_n\left(\omega^2-\omega_2^2(1+\alpha_2)-\frac{i\varpi_2\varpi}{Q_2}\right)+\omega_2^2\varepsilon_1 A_n+\omega_2^2\varepsilon_2 A_{n+1}=0, \quad (1.2)$$

where

$$\varepsilon_{1,2}=\frac{2}{3\pi J_1^2(\lambda_{01})}\frac{a_{1,2}^3}{b_1 b_2\sqrt{D_1 D_2}},\quad \alpha_{1,2}=\frac{2}{3\pi J_1^2(\lambda_{01})}\frac{a_1^3+a_2^3}{b_{1,2}^2 D_{1,2}},$$

$\omega_{1,2}=c\lambda_{01}/b_{1,2}$, λ_{01} is the first root of the zero order Bessel function; ω_1 (ω_2) is the resonant frequency of E_{010} – mode of the A (B) – cavity; Q_1 (Q_2) is the quality factor of the A (B) - cavity.

Consider the ideal structure ($Q=\infty$). As the structure is periodic with the period $D = D_1 + D_2$, then the field amplitude in the *(n+1)* resonator can be expressed through the field amplitude in the *n* resonator:

$$A_{n+1}=\rho A_n,\ B_{n+1}=\rho B_n. \quad (2)$$

By using condition (2), from equations (1.1, 1.2) we get the set of equations to define amplitudes A_n and B_n:

$$A_n\left(\omega^2-\omega_1^2(1+\alpha_1)\right)+\omega_1^2\varepsilon_1 B_n\left(1+\Delta\rho^{-1}\right)=0, \quad (3.1)$$

$$B_n\left(\omega^2-\omega_2^2(1+\alpha_2)\right)+\omega_2^2\varepsilon_1 A_n\left(1+\Delta\rho\right)=0, \quad (3.2)$$

where the parameter Δ is equal to the ratio a_2^3/a_1^3. The set of homogenous equations (3.1, 3.2) has the non-trivial solution if its determinant is equal to zero:

$$\rho^2-2\theta\rho+1=0, \quad (4)$$

where $\theta=\dfrac{\left(\omega_1^2(1+\alpha_1)-\omega^2\right)\left(\omega_2^2(1+\alpha_2)-\omega^2\right)}{2\omega_1^2\omega_2^2\varepsilon_1^2\Delta}-\dfrac{1+\Delta^2}{2\Delta}$.

Equation (4) is called characteristic. There are two roots of the characteristic equation $\rho_{1,2}=\theta\pm\sqrt{\theta^2-1}$. The existence of two roots of the characteristic equation shows that there are two eigen oscillations in the boundless biperiodic structure: $A_n = C_1\rho_1^n$, $B_n = F_1\rho_1^n$ and $A_n = C_2\rho_2^n$, $B_n = F_2\rho_2^n$. The ratio C_1/F_1 (C_2/F_2) depends on ω:

$$\frac{C_{1(2)}}{F_{1(2)}}=-\frac{\omega_1^2\varepsilon_1\left(1+\Delta\rho_{1(2)}^{-1}\right)}{\omega^2-\omega_1^2(1+\alpha_1)}=-\frac{\omega^2-\omega_2^2(1+\alpha_2)}{\omega_2^2\varepsilon_1\left(1+\Delta\rho_{1(2)}\right)}. \quad (5)$$

Analysis of the dependence of θ versus ω shows, that in comparison with plane periodic structure there are two passbands in the biperiodic one. Passbands are separated by the stopband. The minimal value θ corresponds to the middle of the forbidden zone:

$$\omega_*^2 = \frac{1}{2}\left[\omega_1^2(1+\alpha_1)+\omega_2^2(1+\alpha_2)\right], \qquad (6)$$

$$\theta(\omega_*) = \frac{\left(\omega_1^2(1+\alpha_1)-\omega_2^2(1+\alpha_2)\right)^2}{2\omega_1^2\omega_2^2\varepsilon_1^2\Delta} - \frac{1+\Delta^2}{2\Delta}. \qquad (7)$$

If $a_1 = a_2$ and eigen frequencies of resonators (including frequency shifts caused by coupling apertures) tend to ω_*:

$$\omega_1^2(1+a_1)=\omega_2^2(1+a_2)=\omega_*^2, \qquad (8)$$

than the inner stopband disappears and so-called compensation takes place [2,3].

Consider the case when condition (8) is satisfied and $a_1 \neq a_2$. In this case

$$\theta(\omega_*) = -\frac{1+\Delta^2}{2\Delta},\ \rho_1(\omega_*) = -\Delta,\ \rho_2(\omega_*) = -\frac{1}{\Delta}. \qquad (9)$$

When $\omega \to \omega_*$, than $C_1 / F_1 \to 0$, $C_2 / F_2 \to \infty$, as it follows from equation (5). Hence, at this frequency $C_1 = 0$ and $F_2 = 0$.

Consider the bounded structure consisting of *2N-1* cavities. Assume that the structure is bounded by A -cavities. In this case boundary conditions takes the form:

$$B_1\left(\omega^2-\omega_2^2(1+\alpha_2)-\frac{\tilde\omega_1^2(\gamma_1)^2\omega_2^2}{\omega^2-\tilde\omega_1^2(1+\tilde\varepsilon_1)}\right)+\omega_2^2\varepsilon_2A_2=0, \qquad (10.1)$$

$$B_{N-1}\left(\omega^2-\omega_2^2(1+\alpha_2)-\frac{\tilde\omega_N^2(\gamma_N)^2\omega_2^2}{\omega^2-\tilde\omega_N^2(1+\tilde\varepsilon_N)}\right)+\omega_2^2\varepsilon_1A_{N-1}=0, \qquad (10.2)$$

where $\tilde\omega_1$ is the resonant frequency of E_{010} – mode of the first resonator, $\tilde\omega_N$ is the resonant frequency of E_{010} – mode of the last resonator, $\tilde\varepsilon_1 = \frac{2}{3\pi J_1^2(\lambda_{01})}\frac{a_1^3}{\tilde b_1^2\tilde D_1}$,

$\tilde\varepsilon_N = \frac{2}{3\pi J_1^2(\lambda_{01})}\frac{a_2^3}{\tilde b_N^2\tilde D_N}$, $\quad \gamma_1 = \frac{2}{3\pi J_1^2(\lambda_{01})}\frac{a_1^3}{\tilde b_1 b_2\sqrt{\tilde D_1 D_2}}$,

$\gamma_N = \frac{2}{3\pi J_1^2(\lambda_{01})}\frac{a_2^3}{\tilde b_N b_2\sqrt{\tilde D_N D_2}}$. The structure consisting of *2N-1* cavities has *2N-1* resonant frequencies. The amplitude distribution of the eigen oscillation in the bounded structure is equal to the sum of amplitude distributions of eigen oscillations in the boundless structure:

$$A_n = C_1\rho_1^n + C_2\rho_2^n,\ B_n = F_1\rho_1^n + F_2\rho_2^n. \qquad (11)$$

Designate: $g_1 = \left(\omega^2-\omega_2^2(1+\alpha_2)-\frac{\tilde\omega_1^2(\gamma_1)^2\omega_2^2}{\omega^2-\tilde\omega_1^2(1+\tilde\varepsilon_1)}\right)$,

$g_N = \left(\omega^2-\omega_2^2(1+\alpha_2)-\frac{\tilde\omega_N^2(\gamma_N)^2\omega_2^2}{\omega^2-\tilde\omega_N^2(1+\tilde\varepsilon_N)}\right)$. Taking into account (11), we can rewrite equations (10.1, 10.2) in such a form:

$$F_1\rho_1\left(g_1+\frac{C_1}{F_1}\omega_2^2\varepsilon_2\rho_1\right)+F_2\rho_2\left(g_1+\frac{C_2}{F_2}\omega_2^2\varepsilon_2\rho_2\right)=0, \qquad (12.1)$$

$$F_1\rho_1^{N-1}\left(g_N+\frac{C_1}{F_1}\omega_2^2\varepsilon_1\right)+F_2\rho_2^{N-1}\left(g_N+\frac{C_2}{F_2}\omega_2^2\varepsilon_1\right)=0, \qquad (12.2)$$

The ratio C_1/F_1 (C_2/F_2) is determined by the expression (5). Solution of the set of equations (12.1)-(12.2) exists in the case, when determinant is equal to zero:

$$\rho_1\left(g_1+\frac{C_1}{F_1}\omega_2^2\varepsilon_2\rho_1\right)\rho_2^{N-1}\left(g_N+\frac{C_2}{F_2}\omega_2^2\varepsilon_1\right)- \rho_2\left(g_1+\frac{C_2}{F_2}\omega_2^2\varepsilon_2\rho_2\right)\rho_1^{N-1}\left(g_N+\frac{C_1}{F_1}\omega_2^2\varepsilon_1\right)=0 \qquad (13)$$

Equation (13) determines the resonant frequencies of the bounded structure.

At the frequency $\omega=\omega_*$

$$\frac{F_1}{C_2}=\rho_2^3\frac{\varepsilon_2}{\tilde\omega_1^2(\gamma_1)^2}\left(\omega_*^2-\tilde\omega_1^2(1+\tilde\varepsilon_1)\right), \qquad (14.1)$$

$$\frac{F_1}{C_2}=\rho_2^{2(N-1)}\frac{\varepsilon_1}{\tilde\omega_N^2(\gamma_N)^2}\left(\omega_*^2-\tilde\omega_N^2(1+\tilde\varepsilon_N)\right). \qquad (14.2)$$

If parameters of boundary resonators are chosen so, that $\tilde\omega_1^2(1+\tilde\varepsilon_1)=\tilde\omega_N^2(1+\tilde\varepsilon_N)=\omega_*^2$, then ω_* will be the resonant frequency of the bounded structure. The amplitude distribution of this resonant oscillation is such:

$$A_n = C_2\rho_2^n,\ B_n = 0. \qquad (15)$$

This resonant oscillation is based on the one eigen oscillation of the boundless structure. Let us suppose, that $\Delta<1$ ($a_2<a_1$). In this case the field amplitude distribution is increasing as $\rho_2 = -1/\Delta$. Thus, if we want to obtain the increasing amplitude distribution, we should bound the biperiodic waveguide by cavities of A - type.

It can similarly be shown that in order to obtain the resonant oscillation based on the decreasing eigen oscillation of the boundless structure, the structure should be bounded by cavities of B - type.

Eigen oscillations have the following feature: in the cavity chain the field amplitude turns into zero in each cavity with even number.

Thus, choosing in a certain way boundary conditions, it is possible to create the amplitude distribution that is based on only one fundamental solution. It should be noted that it is possible only in the stopband. Inside the passband such a condition cannot be fulfilled.

3 SIMULATION

Proceeding from the theory presented above, the injector system based on evanescent oscillation was simulated using SUPERFISH [4] and PARMELA [5] codes. The simulation was held under the electron beam initial energy W_0=25 keV and current 50 mA with space charge forces taken into account. The peak value of on-axis electric field is 30 MV/m.

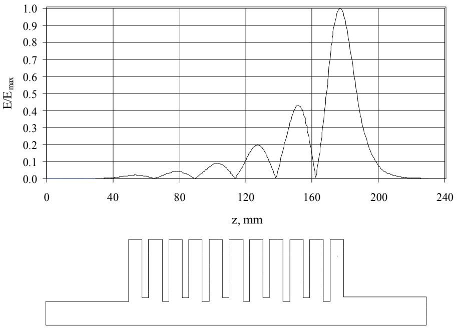

Fig. 1. Geometry of a bunching system based on the ordinary biperiodical waveguide and corresponding

on-axis electric field distribution.

The waveguide section composed of eleven accelerating cells was taken for simulations of the bunching system based on the biperiodic waveguide. Biperiodic waveguide having all corresponding sizes to be equal except iris radii was considered. The time-transit angle for relativistic particle was chosen equal to 0.3π per cell. In considered case $\rho < 0$, than the phase shift per period is π. The required on-axis increasing field distribution was obtained by periodical variations of iris radii and by boundary cells tuning. On-axis field distribution and geometry of a resonance system are shown in Fig 1.

Electrodynamics performances of the simulated system are as follows: the quality factor Q=5446 and shunt impedance R_{sh}=10.8 MOhm/m. Simulation of particle dynamics in the system has shown that the maximum energy is 0.7 MeV, average energy is 0.57 MeV, energy spectrum is 27% (70% of particle), phase length is 20°, normalized emittance is 18 mm·mrad and capture is 96%.

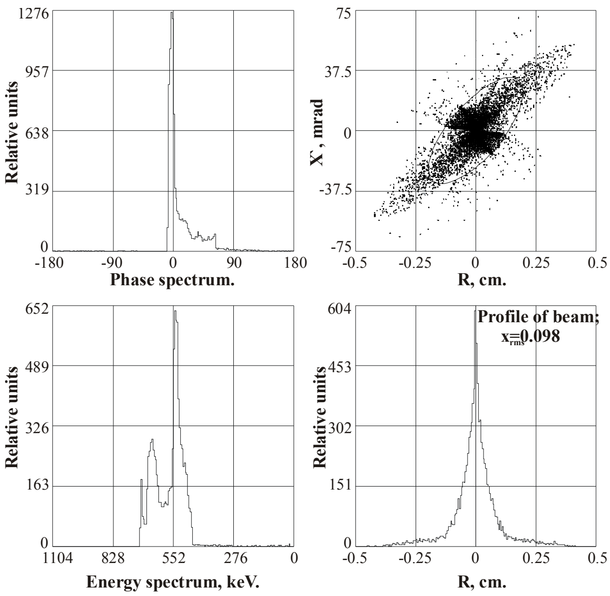

Fig. 2. Characteristics of the electron beam at the injector exit.

Parameters of the electron beam at the injector exit (phase and energy spectrum, emittance and transverse beam profile) are shown in Fig. 2.

4 CONCLUSION

Our investigations have been shown the possibility of electron beam bunching and accelerating in the biperiodic structure. The appropriate field distribution is based on the one eigen oscillation of the boundless biperiodic structure in the stopband (evanescent oscillation).

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