OPTIMIZATION OF AXIALLY SYMMETRIC DRIFT TUBE GEOMETRY

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The integral equation method had been used for calculation of the electric field strength between drift tubes as well as on their surfaces, power distribution of RF surface losses, and mutual tubes capacitance. The method gives an opportunity to find the critical surface points where the field strength and the linear power density have the maximum values.
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1 INTRODUCTION

For building high-current ion linear resonance accelerators it is necessary to utilize different types of accelerating structures depending on the particle energy [1]. For an initial part, from the injection energy W_o*=50÷100 keV* up to *W=2÷5 MeV/nucleon*, as a rule, the structures with the radio-frequency quadrupole focusing (RFQ) are used. For higher energies, down to *100 MeV/nucleon*, the structures on the basis of cavities with drift tubes are used. The lengths of the drift tubes, their apertures and gaps between them are determined by an ion beam dynamics, that is to ensure a radial and longitudinal stability of beam motion, an acceleration rate and other factors.

To construct the real geometry of drift tubes, i.e. to choice their outside radius, the radii of outside and inside curvatures of the end-walls, it is necessary to take into account the influence of these parameters on a mutual capacitance of the drift tubes, an increase factor of an electric field strength (overvoltage) on the electrode surface in comparison with the average value in the gap, ohmic losse distributions along the tube surfaces. It is especially important to know an overvoltage factor and loss power density in critical points of the electrode surfaces, where they have the maximal values, since this defines the electric sparking of the gaps and thermal stability of the accelerating structure.

Thus for optimization of the drift tubes construction, it is need to have a simple mathematical formalism for calculating the electric field strength between tubes, as well as on their surface, and distribution of ohmic power losse density, and mutual capacitance of the tubes.

2 METHOD OF INTEGRAL EQUATIONS

There are some possible approaches to the solution of this problem. One consists in the preliminary solution of the Laplace equation $\Delta U=0$ for finding of a potential distribution $U(r,z)$ in some closed area including electrodes. On a boundary G of the area it is supposed that a potential distribution $U(G)$ is known. As a rule, for the solution of this Dirichlet problem the grid methods are used, which allow to find the potential values $U(r_i,z_k)$ for axial geometry in nodes (r_i,z_k) of a computing grid. Further the electric field components may be found by numerical differentiation of the potential: $E_z(r_i,z_k)=-\partial U(r_i, z_k)/\partial z$ and $E_r(r_i, z_k)=-\partial U(r_i, z_k)/\partial r$ in the nodes of the grid. For definition of an electric field on the surface of metal electrodes it is necessary to extrapolate the field values from the nearest nodes of the computing grid, since the nodes of the grid, as a rule, do not coincide with the electrode boundaries. The surface charge density distribution $\sigma(r_s,z_s)$ and a current density $j(r_s,z_s)=d\sigma(r_s,z_s)/dt$ may be founded after definition of a normal component $E_n(r_s,z_s)$ of electric field on the electrode surface: $\sigma(r_s,z_s)=\varepsilon_0 E_n(r_s,z_s)$. The mutual capacitance of electrodes is $c=Q/(U_1-U_2)$, where $Q=\oint\!\!\oint_s \sigma(r_s,z_s)dS$ is the total charge on one of the electrodes; U_1-U_2 is a potential difference between the drift tubes.

Unfortunately in many cases, this algorithm is difficult to realize with a needed accuracy reasonable for designing. The caused is that, as a rule, the boundary potential $U(G)$ is known only on the metal electrodes. On sections of the boundary G, lying outside of them, it is necessary to use different physically reasonable approximations of the potential. Besides for determination of an electric field strength on the electrode surfaces (r_s,z_s), it is necessary to extrapolate the node functions $E_z(r_i,z_k)$ and $E_r(r_i,z_k)$, obtained by numerical differentiation of the .potential $U(r_i,z_k)$ on a computing grid.

To directly determine an electric charge density and electric field on the electrode surfaces the integral equations method had been used. The method is grounded on the usage of a source function, and for its application it is enough to know only the electrode geometry and their potentials [2].

It is known, if the surface charge density $\sigma(x_s,y_s,z_s)$ has been determined then in the absence of a space charge the potential $U(x,y,z)$ in any given space point (x,y,z) will be:

$$U(x,y,z)=\frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{n}\oint\!\!\oint_{S_k}\frac{\sigma(x_s,y_s,z_s)dS}{\sqrt{(x-x_s)^2+(y-y_s)^2+(z-z_s)^2}},$$

where (x_s,y_s,z_s) is a coordinate of a point on an electrode surface; and the integrals are taken over surfaces S_k of all n electrodes.

In case of two axial symmetric electrodes, in a cylindrical frame (r,z), this expression is:

$$U(r,z)=\frac{1}{\pi\varepsilon_0}\sum_{k=1}^{2}\oint_{L_k}\frac{\sigma(l)r_s K(t_s)dl}{\sqrt{(r+r_s)^2+(z-z_s)^2}},$$

where the integration is taken along closed loops L_k, which are the drift tube contours; $K(t) = \int_0^{\pi/2} \frac{d\beta}{\sqrt{1 - t^2 \sin^2 \beta}}$ is the complete elliptic integral of the 1-st kind [3]; $t_s = \frac{2\sqrt{r r_s}}{\sqrt{(r + r_s)^2 + (z - z_s)^2}}$. If $q(l)=2\pi r_s \sigma(r_s,z_s)$ is the line charge density along the electrode contours then the expression for the potential will be:

$$U(r,z) = \frac{1}{2\pi^2 \varepsilon_0} \sum_{k=1}^{2} \left(\oint_{Lk} \frac{q(l) K(t_s) dl}{\sqrt{(r + r_s)^2 + (z - z_s)^2}} \right) \quad (1).$$

To determine $q(l)$ the contour of each electrode is divided on n elements. Since a potential U_i of every i-element is known, i.e. U_i is either U_1 or U_2, then the expression (1) gives a system of $2n$ linear equations with $2n$ unknown linear charge densities q_i on the electrode contours:

$$\sum_{i=1}^{2n} a_{ij} q_j = 2\pi^2 \varepsilon_0 U_i \quad (2),$$

where U_i is equal to U_1 or U_2 depending on what electrodes the element is situated.

The coefficient matrix of the system of equations looks like:

$$a_{ij} = \frac{K(t_{ij}) \cdot h}{\sqrt{(r_i + r_j)^2 + (z_i - z_j)^2}},$$

$$t_{ij} = \frac{2\sqrt{r_i r_j}}{\sqrt{(r_i + r_j)^2 + (z_i - z_j)^2}},$$

where h is equal to either l_1/n or l_2/n, depending on along which of contours the summing is taken; l_1 and l_2 are contour lengths of electrodes, respectively.

The numerical solution of the system (2) allows to determine the line charge densities q_i along each contour and, accordingly, the total charge Q. For example, for the first drift tube we have: $Q_1 = \frac{l_1}{n} \sum_{i=1}^{n} q_i$. Accordingly, a mutual capacitance of drift tubes will be: $c=Q_1/(U_1-U_2)$. The distribution of a *RF* power loss, for example, along a contour of the first drift tube, i.e. on i-element will be:

$$W_i = \frac{\rho \omega^2 l_1}{2\pi \delta n} \cdot \frac{q_i^2}{r_i}.$$

The total ohmic loss power for this tube is:

$$W_1 = \frac{\rho \omega^2 l_1}{2\pi \delta n} \sum_{i=1}^{n} \frac{q_i^2}{r_i}.$$

Here ω is the cyclic frequency of the *RF* field; ρ is the specific resistance of an electrode material; $\delta=(2\rho/\omega\mu)^{1/2}$ is the skin layer width; μ is the magnetic permeability of vacuum. The electric field components $E_r(r,z)$ and $E_z(r,z)$, in an arbitrary space point, are the partial derivatives of analytical expression (1):

$$E_z(r,z) = \frac{1}{2\pi^2 \varepsilon_0} \sum_{k=1}^{2} \Big(\oint_{L_k} \frac{q(l)(z - z_s) K(t_s) dl}{[(r + r_s)^2 + (z - z_s)^2]^{3/2}} + 2\sqrt{r} \oint_{L_k} \frac{q(l)\sqrt{r_s}(z - z_s) K'(t_s) dl}{[(r + r_s)^2 + (z - z_s)^2]^2} \Big);$$

$$E_r(r,z) = \frac{1}{2\pi^2 \varepsilon_0} \sum_{k=1}^{2} \Big(\oint_{L_k} \frac{q(l)(r + r_s) K(t_s) dl}{[(r + r_s)^2 + (z - z_s)^2]^{3/2}} - \frac{1}{\sqrt{r}} \oint_{L_k} \frac{q(l)\sqrt{r_s}[(r + r_s)^2 + (z - z_s)^2 - 2r(r + r_s)] K'(t_s) dl}{[(r + r_s)^2 + (z - z_s)^2]^2} \Big).$$

For the numerical modeling the contour integrals for $E_r(r,z)$ and $E_z(r,z)$ are replaced by summing the integrands on i elements of the electrode contours. The precision of the calculations on the basis of this algorithm may be checked by comparison of the known potential values, accordingly U_1 and U_2, in the points of the electrode surfaces with the results of a numerical integration of expression (1).

The program LOZOVA for optimization of the geometry of axial symmetric drift tubes, grounded on usage of the integral equations method, had been developed in DELPHI32 environment for the WINDOWS 95/98 operational system.

3 RESULTS OF CALCULATION OF HELIUM ACCELERATOR DRIFT TUBES

The developed LOZOVA program has been used for constructing the drift tubes for an accelerating channel of a helium-3 ion linac with a working frequency *f=425 MHz* for the ion-induced low energy proton therapy [4].

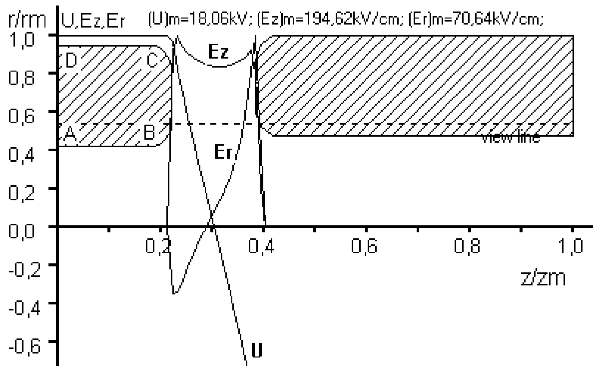

Fig. 1. Distribution of the potential U and the field components E_r and E_z along a dotted line, crossing the electrodes.

A radial and longitudinal particle dynamics was calculated for the usage of an alternating phase focusing and the π-wave accelerating structure. The ion dynamics had defined the drift tubes lengths and their apertures and the gaps between them.

For illustration in Fig. 1 the distributions of a potential U, a radial E_r and an axial E_z components of the electric field between a couple of drift tubes are given.

They are shown for one of accelerating periods, where the channel aperture changes.

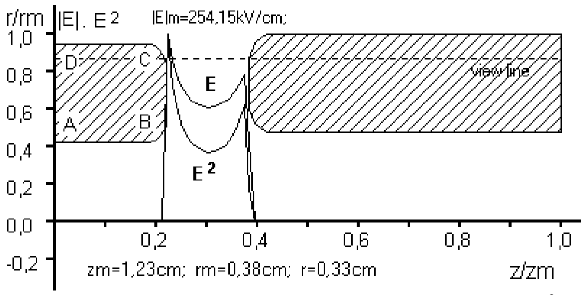

Fig. 2. Distribution of the field modulus E and E^2 along the dotted line.

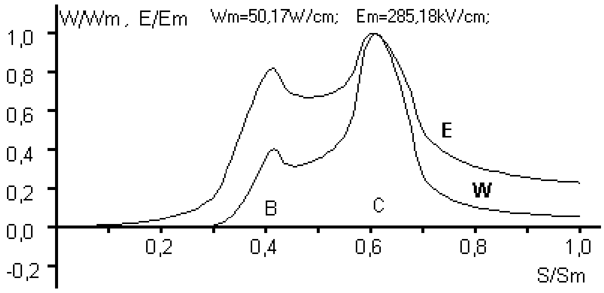

Fig. 3. Distribution of the field strength E and a linear power density W along the first electrode contour.

For this accelerating period, Fig. 1, the drift tubes potentials were: U_1 = *18 kV*, U_2=*-18 kV*; tubes lengths l_1 = *5.4 mm*, l_2 = *15 mm, respectively*; the gap was *g=2 mm*; the tubes aperture radiuses were *1.6 mm* and *1.8 mm*; and the tubes wall width was *2 mm*.

The distributions in Fig. 1 are given for the same radii of the interior and exterior curvature of electrode end-walls equal to *0.5 mm*, and are shown along a view line, marked by a dotted line. This line crosses a *B* point of the *ABCD* contour of the first electrode, where one of the surface maxima of the electric field strength takes place $E\approx195$ *kV/cm*. However the greatest value of the electric field module $E\approx254$ *kV/cm* is observed for the second maximum in a *C* point of the first electrode, Fig. 2. An average field along the gap is *180 kV/cm*. Here, Fig. 2, shows the exhibited distribution of $E^2(z)$ along the view line, parallel *z* axis. This quantity is measured in the experiment for examinations of a *RF* field topography by the disturbing body method. The mutual capacitance of the drift tubes makes *0.5 pF*.

The distributions of the electric field *E* and the linear density of a *RF* loss power *W* along the *ABCD* contour of the first half-tube are given in Fig. 3 (S_m is the length of a half of the contour). As one may see from Fig. 3, the critical electric strength is observed at the outer electrode rounding and reaches $E_c\approx285$ *kV/cm*. In this critical point the linear power density of the ohmic loss reaches W_c=*50 W/cm*; while the total power of the first half tube is *P=7.3 W*. The obtained critical value of a surface electric field E_c is still acceptable with regard to gap sparking at the frequency *f=425 MHz*, taking into account the empirical criterions [5, 6]. In particular, according to [6] at high frequencies, the limiting electric field E_{max} on a metal electrode surface can be taken according to the empirical relation: $E_{max}\approx3.1\cdot10^7/\lambda^{0.5}$ (E_{max} is in *V/m*, a wave length λ is in *m*). At *f=425 MHz* the maximum field strength E_{max} makes *369 kV/cm*.

The critical values of E_c and W_c are essentially decreasing with increasing the radius of the electrode outer rounding. When it varies from *0.5 mm* to *1 mm*, the field E_c decreases down to *248 kV/cm* and the power W_c falls down to *33 W/cm*, and mutual capacitance down to *0.48 pF*.

Naturally, these levels of the power and the field strength are acceptable only if the accelerator proposed [4] operates with the duty factor no more than 0.5 % and the time of the irradiation cycle is about several minutes.

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