CHARGED PARTICLES DYNAMICS OPTIMIZATION IN A DRIFT-TUBE LINEAR ACCELERATOR

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A problem of charged particle dynamics optimization in a drift-tube linear accelerator is considered. Discrete optimization model based on the Solovjev equations is suggested. Functional is considered for dynamics estimation to allow conducting optimization and taking into account a density distribution of charged particles. Analytical expression for functional variation that helps constructing of various directed methods of optimization is suggested.

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1. INTRODUCTION

We consider this problem as a non-standard problem of the theory of optimal control in discrete systems. This problem of control of particular trajectory and ensembles of trajectories (beams of trajectories) had been considered under various criteria of quality. Along trajectories of the system we consider functional characterizing the dynamics of programmed motion (motion of synchronous particle) as well as dynamics of perturbed motion. The mathematical model suggested in this work also allows accounting for a density of particle distribution in phase space. The method of solving of this problem is proposed.

2. MATHEMATICAL MODEL

We consider the problem of optimizing of the beam parameters for the drift-tube linear accelerator. To characterize the motion in this accelerator we have applied the discrete Solovjev equations [1].

In common case, Solovjev equations can be written in the following form:

$$x(k+1) = f(k, x(k), u(k)), \tag{1}$$

$$y(k+1) = F(k, x(k), y(k), u(k)), \tag{2}$$

for $k = 0,..., N-1$, where $x(k)$ is the $n-$ dimensional phase vector defining programmed motion, $y(k)$ is the $m-$ dimensional phase vector defining perturbed motion, $u(k)$ is $r-$ dimensional control vector, and $f(k,x(k),u(k))$ is the $n-$ dimensional vector function defining the process dynamics at each step. For all $k \in \{0,1,\dots,N\}$ the vector function $f(k,x(k),u(k))$ is assumed to be definite and continuous on $\Omega_x \times U(k)$ at all its arguments $(x(k),u(k))$, along with partial derivatives with respect to these variables. $F(k,x(k),y(k),u(k))$ is the $m-$ dimensional vector function; for all $k \in \{0,1,\dots,N\}$ it is assumed to be definite and continuous on $\Omega_x \times \Omega_y \times U(k)$ at all its arguments $(x(k),y(k),u(k))$, along with the partial derivatives with respect to these variables and second partial derivatives. Ω_x – is the domain in R^n, Ω_y is the domain in R^m, and $U(k)$, $k = 0,1,\dots,N-1$ is a compact set in R^r. In this case we consider the Jacobian function

$$J_k = J(k, x(k), y(k), u(k)) = \left|\frac{\partial F(k)}{\partial y(k)}\right|$$

to be non-zero for all changes of $k, x(k), y(k)$ and $u(k)$. Eq. (1) describes the dynamics of programmed motion. Eq. (2) describes the dynamics of perturbed motions.

We assume that $x(0) = x_0$ is fixed and the initial state of system (2) is described by set M_0 – a compact set of non-zero measures in R^m. We call the sequence of vectors $\{u(0),u(1),\dots,u(N-1)\}$ the control of the system described by Eqs. (1) and (2) and denote it by u for brevity. We call the corresponding sequence of vectors $\{x(0), x(1),\dots, x(N)\}$ the trajectory of programmed motion and denote it by $x = x(x_0,u)$. We denote the phase state of a programmed trajectory for the $k-$ th step by $x(k) = x(k,x_0,u(k))$. Similarly, we call the sequence of vectors $\{y(0), y(1),\dots, y(N)\}$ the trajectory of perturbed motion and denote it by $y = y(x, y_0, u)$. We denote the phase state of the particle for the $k-$ th step by $y(k) = y(k,x,y_0,u)$. The set of trajectories $y(x, y_0, u)$ corresponding to the initial state x_0, the control u and different states $y_0 \in M_0$ are referred to as an ensemble of trajectories, or the beam of trajectories. The phase state of the beam at the $k-$ th step is also called the cross-section of the beam of trajectories and is denoted by $M_{k,u}$ i.e.:

$$M_{k,u} = \{y(k) : y(k) = y(k, y_0, x(k), u(k)), y_0 \in M_0\},$$

and the controls satisfying conditions $u(k) \in U(k)$, $k = 0,1..., N-1$ are admissible.

Let x_0 be the initial state of a synchronous particle and M_0 be the set of initial phase states of charged particles with density distribution $\rho_0(y_0) = \rho(0, y_0)$. We would like to determine how the distribution density function along the beam trajectories is transformed. Let us fix an instant $k+1$ and a point $\bar{y}_{k+1} \in M_{k+1,u}$. Let $\bar{y}(k+1)$ be an image of the point $\bar{y}(k)$ in view of Eq. (2). We denote by $G(y(k))$ the set of points $y^i(k) \in M_{k,u}$, such that the trajectories of the system emanating at the instant k from $y^i(k)$ at the step $k+1$

fall within a certain r- neighborhood $S_r(\bar{y}(k+1))$ of the point $\bar{y}(k+1)$.

By the distribution density of trajectories from Eq. (2) at the point $\bar{y}(k+1)$ on the $k+1$- th step, we mean the limit:

$$\rho(k+1,\bar{y}_{k+1}) = \lim_{r\to 0}\left\{\frac{1}{mes(S_r(\bar{y}(k+1)))}\int_{G(\bar{y}(k))}\rho(k,y_k)\,dy_k\right\}, \quad (3)$$

where

$$mes(S_r(\bar{y}(k+1))) = \int_{S_r(\bar{y}(k+1))} dy(k+1).$$

From the one-to-one correspondence of the sets $G(y(k))$ and $S_r(\bar{y}(k+1))$, the integral in Eq. (3) transforms to the form:

$$\int_{S_r(\bar{y}(k+1))} \rho(k,y_k)J_k^{-1}dy(k+1), \quad (4)$$

where $y_k = y(k)$ and

$$J_k^{-1} = J^{-1}(k,x(k),y(k),u(k)) = \det\left(\frac{\partial y(k)}{\partial y(k+1)}\right) = \det{}^{-1}\left(\frac{\partial F(k,x(k),y(k),u(k))}{\partial y(k)}\right).$$

In view of this and the form $mes(S_r(\bar{y}(k+1)))$, we obtain the following equation for $\rho(k,y_k)$:

$$\rho(k+1,y(k+1)) = J_k^{-1}\rho(k,y_k),\ \rho(0,y_0) = \rho_0(y_0). \quad (5)$$

The function $\rho(k) = \rho(k,y_k)$ denotes the distribution density function for the k -th step.

We introduce the following functional:

$$I(u) = \sum_{k=1}^{N-1}\int_{M_{k,u}}\varphi_k(x_k,y_k,\rho(k,y_k),u_k)dy_k + \int_{M_{N,u}} g(y_N,\rho(N,y_N))dy_N, \quad (6)$$

where φ_k and g are continuously differentiable functions, $x_k = x(k)$.

The functional (6) allows the simultaneous estimation of programmed and perturbed motions, as well as their simultaneous optimization and taking into account the density of particle distribution in the phase space.

3. VARIATION OF FUNCTIONAL

Let us rewrite Eqs. (1) and (2) and an equation for the distribution density along trajectories of the system (2):

$$x(k+1) = f(k,x(k),u(k)),$$
$$y(k+1) = F(k,x(k),y(k),u(k)), \quad (7)$$
$$\rho(k+1) = J^{-1}(k,x(k),y(k),u(k))\cdot\rho(k),$$

for $k = 0,\dots,N-1$.

Let us denote variations of trajectories of system (7) as $\delta x(k),\delta y(k)$, and $\delta\rho(k)$, with admissible variation of control Δu and a given u.

Now we define the corresponding equations for variations:

$$\delta x(k+1) = \frac{\partial f(k)}{\partial x(k)}\delta x(k) + \frac{\partial f(k)}{\partial u(k)}\Delta u(k), \quad (8)$$

$$\delta y(k+1) = \frac{\partial F(k)}{\partial x(k)}\delta x(k) + \frac{\partial F(k)}{\partial y(k)}\delta y(k) + \frac{\partial F(k)}{\partial u(k)}\Delta u(k), \quad (9)$$

$$\delta\rho(k+1) = \rho(k)\frac{\partial J^{-1}(k)}{\partial x(k)}\delta x(k) + \rho(k)\frac{\partial J^{-1}(k)}{\partial y(k)}\delta y(k) + J^{-1}(k)\delta\rho(k) + \rho(k)\frac{\partial J^{-1}(k)}{\partial u(k)}\Delta u(k). \quad (10)$$

We also have the following Eqs. [2-3]:

$$div_y\delta y(k+1) = div_y\delta y(k) + J^{-1}(k)\left(\frac{\partial J(k)}{\partial y(k)}\delta y(k) + \frac{\partial J(k)}{\partial x(k)}\delta x(k) + \frac{\partial J(k)}{\partial u(k)}\Delta u(k)\right), \quad (11)$$

where

$$div_y\delta y(k) = \sum_{i=1}^{m}\frac{\partial\delta y_i(k)}{\partial y_i(k)}.$$

Taking into account Eqs. (8)–(11), the initial values of variations $\delta x(0) = 0, \delta y(0) = 0, \delta\rho(0) = 0$, $div_y\delta y(0) = 0,$ and using methods of investigation for functions of type (6) [2-3], variation of functional (6) (for admissible variation of control Δu) can be represented in the following form:

$$\delta I = \sum_{k=0}^{N-1}\int_{M_{k,u}}\left(J(k)p^T(k+1)\frac{\partial F(k)}{\partial u(k)} + J(k)\gamma^T(k+1)\frac{\partial f(k)}{\partial u(k)} + J_k\xi(k+1)\rho(k)\frac{\partial J^{-1}(k)}{\partial u(k)} + q(k+1)\frac{\partial J(k)}{\partial u(k)} + \frac{\partial\varphi(k)}{\partial u(k)}\right)dy_k\Delta u(k), \quad (12)$$

where $p(k),\gamma(k),\xi(k)$ and $q(k)$ are the following auxiliary functions:

$$p^T(N) = \left(\frac{\partial g(y_N,\rho_N)}{\partial y(N)}\right),\ \xi(N) = \left(\frac{\partial g(y_N,\rho_N))}{\partial\rho(N)}\right),$$

$$q(N) = g(y_N,\rho_N),\quad \gamma(N) = 0,$$

$$q(k) = J(k)q(k+1) + \varphi(k),\ \xi(k) = \xi(k+1) + \frac{\partial\varphi(k)}{\partial\rho(k)},$$

$$p^T(k) = J(k)p^T(k+1)\frac{\partial F(k)}{\partial y(k)} + J(k)\xi(k+1)\rho(k)\frac{\partial J^{-1}(k)}{\partial y(k)} + q(k+1)\frac{\partial J(k)}{\partial y(k)} + \frac{\partial\varphi(k)}{\partial y(k)},$$

$$\gamma^T(k) = J(k)p^T(k+1)\frac{\partial F(k)}{\partial x(k)} + J(k)\gamma^T(k+1)\frac{\partial f(k)}{\partial x(k)} + J(k)\xi(k+1)\rho(k)\frac{\partial J^{-1}(k)}{\partial x(k)} + q(k+1)\frac{\partial J(k)}{\partial x(k)} + \frac{\partial\varphi(k)}{\partial x(k)},$$

for $k = 1,\dots,N-1$.

Eq. (12) for functional variation allows the construction of various methods of optimization of the functional in Eq. (6).

4. CONCLUSION

Simultaneous optimization of programmed and perturbed motions under various quality criteria was considered in previous works [3–5] and the results were applied to the optimization of beam dynamics in linear-tube accelerators [4-5]. The analytical representation obtained in this paper for variation of the functional examined allows taking into account the density distribution of charged particles as well.

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ОПТИМИЗАЦИЯ ДИНАМИКИ ЗАРЯЖЕННЫХ ЧАСТИЦ В УСКОРИТЕЛЕ С ТРУБКАМИ ДРЕЙФА

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Рассматривается проблема оптимизации динамики заряженных частиц в линейном ускорителе с трубками дрейфа. Предлагается дискретная модель оптимизации, основанная на уравнениях Соловьева. Для оценки динамики вводится функционал, позволяющий проводить совместную оптимизацию программного и возмущенных движений с учетом плотности распределения частиц в фазовом пространстве. Выписывается аналитическое представление вариации предложенного функционала, дающее возможность построения направленных методов оптимизации.

ОПТИМІЗАЦІЯ ДИНАМІКИ ЗАРЯДЖЕНИХ ЧАСТОК У ПРИСКОРЮВАЧІ З ТРУБКАМИ ДРЕЙФУ

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Розглядається проблема оптимізації динаміки заряджених часток у лінійному прискорювачі з трубками дрейфу. Пропонується дискретна модель оптимізації, заснована на рівняннях Соловйова. Для оцінки динаміки вводиться функціонал, що дозволяє проводити спільну оптимізацію програмного і збуреного рухів з урахуванням густини розподілу часток у фазовому просторі. Виписується аналітичне зображення варіації запропонованого функціонала, що дає можливість побудови спрямованих методів оптимізації.