ON TRANSVERSAL INSTABILITY OF BEAM IN POWERFUL S-BAND LINEAR ELECTRON ACCELERATORS

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High pulse current of electrons in a linear resonance accelerator assists the increase of overall efficiency of the facility. At the same time, there is a danger of development of transversal instability of a beam (BBU), especially in linacs consisting of a few accelerating sections. In the paper the method of calculation of starting current of regenerative BBU instability in the tapered disc loaded waveguides is presented as well as simulation results of BBU build-up in the linac consisting of a few accelerating sections.

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1. INTRODUCTION

Phenomena of beam pulse shortening occurring in power linear electron accelerators (linacs) were discovered in the middle of the past century [1]. Some late analogous phenomena came out during commissioning of multi-sectional high-energy linacs [1,2]. Extensive researches on the problem have shown that such effect is caused by beam-excited waves (mostly E_{11} like wave) propagated in the higher pass bands of the disk-loaded waveguide (DLW). At some condition interaction between beam and waves causes exponential growth of transversal oscillations of the particles until they hit into the linac walls. Occurrence of such phenomenon in linacs consisting of just one accelerating section is called as regenerative BBU in contradistinction to cumulative BBU that is characteristic for multisectional linacs. Although BBU has rather long history of study there is no cardinal solution for its complete suppression till now.

Usage of the tapered DLWs is the one method for diminishing of starting current of regenerative BBU instability. In such waveguides besides providing the synchronism between the particles and wave propagating in the fundamental pass band, a region where particles can interact effectively with waves in the higher pass bands is well shorter than waveguide length. Owing this fact, for example, the starting current of regenerative BBU instability in the piece-wise homogeneous section "Kharkov-85" is more than tenfold higher than that in the homogeneous section "Kharkov-65" [2]. Accelerating sections of the KUT type [3,4] are steeply tapered 1.24 m long DLWs with 2π/3 mode of oscillation and relative phase velocity $\beta_{ph} = 1$. They have high efficiency (more than 80% at current accelerated of 1 A) During operation of linacs consisting one or two such sections at current accelerated more than 1 A there was no beam pulse shortening observed. We plan to upgrade the power linac for industrial purposes [4] to provide higher beam energy at beam current about 1 A by installing the third accelerating section. Because absence of pulse shortening does not mean that the BBU instability does not develop, we anticipate appreciable influence of possible BBU instability developing in the three-section linac on beam transversal characteristics. It forced us to start developing a new computer code to simulate the process of E_{11} wave excitation in DLW during acceleration of electron beam with pulse length τ of several microseconds.

There are the two main questions needed to be answered at BBU instability studding. Firstly, it is necessary to estimate the starting current value of regenerative BBU instability[1]. Secondly, conditions of cumulative BBU instability arising in the multisectional linac need to be studied.

This work is dedicated to description of the mathematical model that the code developed is based. Results of preliminary study of BBU instability development in the three-sectional linac with the pilot version of the code also are presented.

2. MATHEMATICAL MODEL

The code under development is based on the mathematical model that represents a DLW as a chain of coupled resonators. The model has the following substantially new feature. Motion of particles in the self-consistent field of the chain in the fundamental pass-band that is excited both by particles and external RF source taking into account input and output couplers and their transversal motion in the self-consistent field in the higher pass-band that corresponds to the E_{110} oscillations of the single cell of the chain are simulated simultaneously. We do know about existent of similar developed codes that realized such possibilities.

The mathematical model of such combined excitation is based on the two sets of coupled equations. The sets have the similar structure but their physical meaning is substantially different. Because DLW is presented as the chain of coupled resonators, electromagnetic field in each cell of the chain can be written in the form of

$$E_n = \mathsf{E}_{1,n}(t)E_{n,010}(r) + \mathsf{E}_{2,n}(t)E_{n,110}(r),$$

$$H_n = \mathsf{H}_{1,n}(t)H_{n,010}(r) + \mathsf{H}_{2,n}(t)H_{n,110}(r).$$

The time dependent functions obey the following systems of coupled equations:

[1] In general, operating current of a linac should be less than the BBU instability starting current.

$$\frac{d^2 E_{1,n}}{dt^2} + \frac{\omega_{1,n,0}}{Q_{1,n}} \frac{dE_{1,n}}{dt} + \omega_{1,n}^2 E_{1,n} =$$
$$= \omega_{1,n,0}^2 \left(\mu_{n,n+1}^{(1)} E_{1,n+1} + \mu_{n,n-1}^{(1)} E_{1,n-1} \right) -$$
$$- \frac{1}{N_{1,n}} \frac{d}{dt} \int_{V_n} j_z E_{z,n,010} \, dV$$

$$\frac{d^2 H_{2,n}}{dt^2} + \frac{\omega_{2,n,0}}{Q_{2,n}} \frac{dH_{2,n}}{dt} + \omega_{2,n}^2 H_{2,n} =$$
$$= \omega_{2,n,0}^2 \left(\mu_{n,n+1}^{(2)} H_{2,n+1} + \mu_{n,n-1}^{(2)} H_{2,n-1} \right) +$$
$$+ \frac{\omega_{2,n}}{N_{2,n}} \int_{V_n} j_z E_{z,n,110} \, dV$$

where $\omega_{i,n,0}$ and $\omega_{i,n}$ are the eigen frequencies of separated cell without and with taking into account influence of coupling holes, correspondently. These systems have to be combined with the equations that connect the current j_z with the excited fields.

During interaction of a beam consisting of a long bunch train with DLW, the dominant role will be played by storage processes of field excitation by numbers of particles at time range that is long as compare with oscillation periods. Therefore, it is possible to assume that maximal spectral components of excited electromagnetic oscillations will be localized mostly around the linac operating frequency and in the higher pass-bands of DLW. The spectral components of heterodyne frequencies will be negligible excluding particular cases, which are rarely realized in practice. Then, time dependence of excited fields can be presented as the product of the slow amplitudes and the fast oscillating exponents. However, a methodological problem concerned with the fact that frequencies of E_{010} and E_{110} oscillations are not multiple each other is aroused during the simultaneous simulation both beam acceleration described by the first set of equations (0.1) and BBU instability excitation described by the second set of equations (0.2). This problem even at slow amplitude varying of both oscillations does not allow us to use efficient method of fast oscillation averaging both for substantial simplification of equations and for computational resource decrease. We use the method of fast oscillation averaging to find solution of (0.1). To obtain solution of the second set of equations (0.2) we used another method based on more detailed time consideration. The last one uses the two assumptions. First one is about slow varying amplitudes of E_{110} oscillations during particles pass through the interaction region and second one is about independence of longitudinal motion from transversal motion while detailed time dependent characteristics of longitudinal particle motions (energy and position) are used at simulation of transversal dynamics, which is defined by E_{110} oscillations.

3. SIMULATION RESULTS

On a basis of the above-mentioned mathematical model we have developed the computer code that allows us to carry out numerical simulations simultaneously both self-consisted transient process of particle acceleration and self-consisted transient beam dynamics in the transversal plane that defined by the E_{110} oscillations. The code developed takes into account the fine beam structure, particularly: bunch shape, energy distribution of particles and their change with time, etcetera. Simulation of beam dynamics can be carried out for arbitrary number of accelerating sections.

We also developed the auxiliary technique to analyze the higher eigen modes of oscillations in the KUT type accelerating sections using the characteristics of the E_{110} oscillation of each cell. Because these sections are strong tapered DLWs, the most dangerous modes are located near the section entrance as calculations indicate. Fig.1 shows comparison of the measured distribution of the longitudinal component of the lowest non-axisymmetrical mode of the KUT type accelerating section with simulated one.

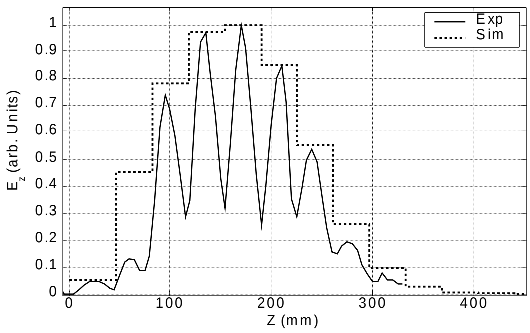

Fig. 1. Distribution of the longitudinal component of the lowest non-axisymmetrical mode along the initial part of the section

The field distribution was measured by bead-pull method. Because of technical reasons the bead did not have enough selectivity to measure the transversal and longitudinal components of the electric field separately, the experimental distribution showed in Fig.1 was obtained with following procedure. There were two measurements of field distribution with the same bead. The first measurement was carried out on the section axis, while the second one was carried out at 5 mm off the axis. Using the fact that the longitudinal component of the electrical field is zero on the axis while its transversal component is weakly dependent on the transversal coordinate, the experimental curve in Fig.2 was obtained by subtraction of the on-axis field distribution from the off-axis one. A good agreement of experimental and simulated data is evident.

During the next stage of our research we studied processes of BBU instability build-up in the first accelerating section of the linac described in reference [4] (piece-wise homogeneous section) and the starting current of regenerative BBU instability was derived.

The time dependent field amplitudes of the E_{110} oscillations in the fourth cell of the DLW are shown in Fig.2 for the set of accelerated current.

It follows from Fig.2 that the starting current of regenerative BBU instability is about 0.65 A. Time dependent transversal size of bunches at the section exits for the three-sectional linac are shown in Fig.3. The first section was piece-wise homogeneous section, the second one was a smooth tapered section and the third one was the same as the first section. Accelerated current corresponded to the starting current in the first section (I=0.65 A). Root mean square transversal size of a bunch

$$X_{s,q} = \sqrt{\left\langle (x - \langle x \rangle)^2 \right\rangle}$$

was chosen as the analyzed characteristic.

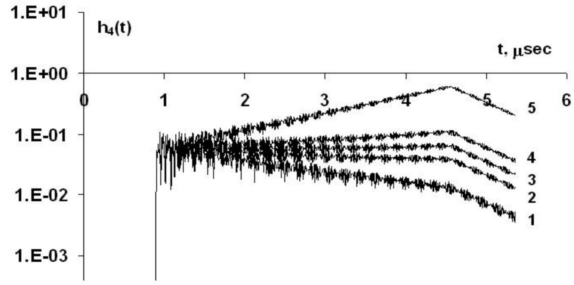

Fig. 2. Time dependent field amplitudes of the E_{110} *oscillations in the fourth cell for the set of accelerated current (1 – I=0.54 A, 2 - I=0.65 A, 3 - I=0.72 A, 4 - I=0.78 A, 4- I=0.98 A)*

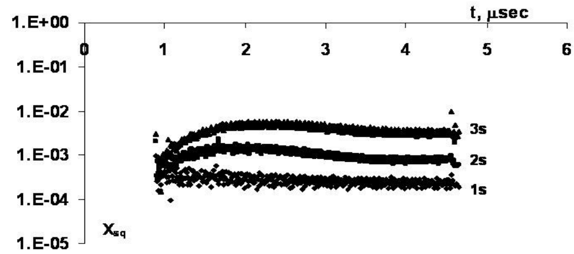

Fig. 3. Time dependent transversal size of bunches at the section exits at accelerated current of I=0.65 A

Analysis of these dependences shows that transversal size of bunches in the second and third section has tendency to increasing however it does not growth exponential with time. Simulation shows that acceleration of a beam with a current that is higher than starting one, causes exponential growth of transversal size of bunches with time.

4. CONCLUSION

Results of transversal particle dynamic simulation outlined in this work have shown that BBU instability caused by excitation of E_{110} oscillations in high current linacs consisting of few accelerating sections does not yet have cumulative character with exponential growth of beam size up to the starting current of the regenerative BBU instability in the first accelerating section.

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МОДЕЛИРОВАНИЕ ПОПЕРЕЧНОЙ НЕУСТОЙЧИВОСТИ ПУЧКА В МОЩНЫХ ЛИНЕЙНЫХ УСКОРИТЕЛЯХ ЭЛЕКТРОНОВ

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Приведены методика расчета порогового тока регенеративной поперечной неустойчивости в неоднородных цилиндрических диафрагмированных волноводах и результаты моделирования развития поперечной неустойчивости в ускорителе, состоящем из нескольких ускоряющих секций.

МОДЕЛЮВАННЯ ПОПЕРЕЧНОЇ НЕСТІЙКОСТІ ПУЧКА В ПОТУЖНИХ ЛІНІЙНИХ ПРИСКОРЮВАЧАХ ЕЛЕКТРОНІВ

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Приведено методику розрахунку стартового струму регенеративної поперечної нестійкості в неоднорідних циліндричних діафрагмованих хвилеводах та результати моделювання розвитку поперечної нестійкості в прискорювачі, що складається з декількох прискорювальних секцій.