ANALYTICAL FORMULAS FOR ALTERNATING WAKE FORCE OF CORRUGATED WAVEGUIDES

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Using the method of field decomposition on the eigen modes in partly intersecting regions, the analytical formula of alternating wake force induced by a bunch of relativistic charged particles in a periodic rf structure is derived.

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1. INTRODUCTION

As is known, wakefields excited by a bunch of relativistic charged particles in a periodic corrugated waveguide and corresponding wake forces can be expressed as a spatial harmonic Floquet's series

$$\vec{F}(\vec{r},t)=\sum_{p=-\infty}^{\infty}\vec{F}^{(p)}(\vec{r}_\perp,t-z/\mathrm{v}_z)\,e^{i\frac{2\pi p}{D}z},\quad(1)$$

where $F^{(p)}$ is the p^{th} space harmonic of the wake force, v_z is the bunch velocity, D is the waveguide period.

Usually, the wake force harmonic synchronous with the bunch (p=0) is of interest. Action of this harmonic on the bunch results in the well known beam loading and beam break up effects in accelerating rf structures. However, the alternating transverse wake force which consists of the nonsynchronous spatial harmonics ($p\neq0$) can give rise to undulating particles with the alternating transverse velocity

$$\vec{\mathrm{v}}_\perp=\frac{ic}{2\gamma}\sum_{p\neq0}\vec{K}^{(p)}e^{i\frac{2\pi p}{D}z},\quad(2)$$

where $K^{(p)}$ is the WF undulator parameter

$$\vec{K}^{(p)}=-\frac{\vec{F}_\perp^{(p)}(r_\perp,t-z/\mathrm{v}_z)\,D}{p\pi mc\mathrm{v}_z},\quad(3)$$

γ is the Lorentz factor (γ>>1), c is the velocity of light, m is the electron mass of rest.

The undulation of charged particles with the alternating velocity Eq.(2) causes emitting wakefield undulator-type (WFU) radiation, properties of which have been studied recently in [1-5]. In this connection there exists an interest in developing methods of calculation of the alternating wakefields induced by a relativistic charged bunch in corrugated waveguides.

The goal of the present paper is to derive analytical relations for nonsynchronous wakefield harmonics in the lowest pass-band of a dick loaded waveguide (DLW) (TM01-type mode) in order to be able to estimate magnitudes of WFU radiation fluxes quantitatively. For this we will base on the approaches represented in [6, 7].

2. WAKE FORCES IN DWL

Let the DLW geometry has a form shown in Fig.1. The components of the TM_{01}-type mode in the space $0\le r\le a$ can be written in the form [8].

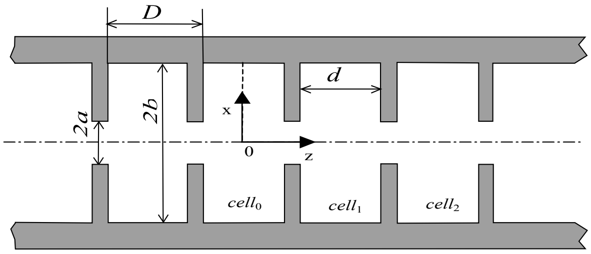

Fig. 1. Schematic representation of the DLW

$$E_z=A\sum_{p=-\infty}^{+\infty}E_p\frac{I_0(\chi_p r)}{I_0(\chi_p a)}\exp\left[i(h_p z-\omega t)\right],\quad(4)$$

$$E_r=-iA\sum_{p=-\infty}^{+\infty}E_p\frac{h_p}{\chi_p}\frac{I_1(\chi_p r)}{I_0(\chi_p a)}\exp\left[i(h_p z-\omega t)\right],\quad(5)$$

$$H_\varphi=-iA\sum_{p=-\infty}^{+\infty}E_p\frac{\omega}{c\chi_p}\frac{I_1(\chi_p r)}{I_0(\chi_p a)}\exp\left[i(h_p z-\omega t)\right],\quad(6)$$

where ω is the resonant frequency, $h_p=\psi/D+2\pi p/D$ and $\chi_p=\sqrt{h_p^2-(\omega/c)^2}$ are the longitudinal and radial wavevectors, respectively; ψ is the phase shift per the period D for resonant frequency, E_p is the spectral amplitude of the p^{th} spatial harmonic, I_0 and I_1 are the modified Bessel functions.

On the other hand, a longitudinal electric field in the n^{th} cell in the space ($0\le r\le b$, $|z-nD|\le d/2$) can be given by

$$E_{z,cell_n}=Be^{i(n\psi-\omega t)}G_z(r,z-nD).\quad(7)$$

Here, $Be^{in\psi}$ is the complex amplitude of the lowest axisymmetrical coupled mode, $G_z(r,z-nD)$ is the longitudinal component of electric field of coupled mode in the n^{th} cell.

Further, by matching the longitudinal electric fields Eqs.(4) and (7) on the common surface, $r=a$, $nD-D/2\le z\le nD+D/2$, it can be expressed AE_p via B

$$AE_pD=B\int_{-d/2}^{d/2}G_z(a,z)\,e^{-ih_pz}dz.\quad(8)$$

Furthermore we suppose that the small coupler factor between cells at the given frequency ω results in the big perturbation of the cavity lower eigenmode only near the cavity aperture. Thus, the integral in Eq.(8) may be replaced by the integral of the zeroth

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approximation mode $G_z(a,z)\approx J_0(\mu_{01}a/b)$, ($\mu_{01}$ are the zeros of Bessel functions).

Taking into account said above we can derive some important relations. The ratio of spectral harmonics of the longitudinal electrical field on the axis can be obtained in the following form:

$$\frac{E_p}{E_q}=\frac{h_q}{h_p}\frac{\sin(h_p d/2)}{\sin(h_q d/2)}. \tag{9}$$

Let the q^{th} spatial harmonic of the fundamental mode pass-band at a frequency ω be synchronous with a relativistic bunch (i.e. $\omega/h_q=v_z\approx c$). Besides, the phase shift per period structure is $\theta_q=h_q D=\omega D/v_z$, $\chi_q=\omega/(\gamma v_z)$. In this case, the loss factor for the fundamental mode pass-band can be found in the form (SI units):

$$k_q=\frac{|AE_q|^2}{2\varepsilon_0\int_{V_{cell}}|E_{z,cell}|^2\,dV}=\frac{cZ_0}{2\pi b^2}\frac{d}{D}\left[\frac{J_0(\mu_{0,1}a/b)}{J_1(\mu_{0,1})}\frac{\sin\left(\frac{\theta_q d}{2D}\right)}{\frac{\theta_q d}{2D}}\right]^2 \tag{10}$$

where ε_0 is the dielectric constant, Z_0=120π Ohm is the vacuum impedance.

Because an amplitude of the synchronous spatial harmonic of longitudinal wakefield excited by a single ultrarelativistic particle with a charge e is given by [9]

$$E_z=AE_q=-e2k_q, \tag{11}$$

then, one induced by a bunch with the longitudinal time distribution function, $f_b(\tau)=\iint_{S_\perp}d^2r_\perp f_b(r_\perp,\tau)$, can be expressed as

$$\mathcal{E}_z(\tau)=-eN2k_q\int_{-\infty}^{\tau}f_b(\tau')\,e^{-i\omega(\tau-\tau')}\,d\tau', \tag{12}$$

where N is the number of particles in the bunch.

Using Eqs.(5), (6), we can define the alternating wake force induced by a single ultrarelativistic particle of charge e as

$$\mathrm{F}_r=e\,\mathrm{Re}\left\{E_r-\frac{v_z}{c}H_\varphi\right\}\approx -ie\frac{AE_q}{2}e^{-i\omega\tau}\sum_{p=-\infty}^{\infty}\frac{E_{q+p}}{E_q}\frac{I_1(\chi_{q+p}r)}{I_0(\chi_{q+p}a)}\frac{\pi p}{\chi_{q+p}D}e^{i\frac{2\pi p}{D}z}+c.c. \tag{13}$$

Taking into account Eqs.(11) and (12), from Eq.(13) we can derive a formula for the radial alternating wake force excited by a bunch in the DWL

$$\vec{F}_r(r,\tau)=-ie\sum_{p=-\infty}^{\infty}\frac{\pi p}{D}\left[\frac{\mathcal{E}_z(\tau)}{\chi_{q+p}}\frac{I_1(\chi_{q+p}r)}{I_0(\chi_{q+p}a)}\frac{E_{q+p}}{E_q}+\frac{\mathcal{E}_z(\tau)^*}{\chi_{q-p}}\frac{I_1(\chi_{q-p}r)}{I_0(\chi_{q-p}a)}\frac{E_{q-p}}{E_q}\right]e^{i\frac{2\pi p}{D}z} \tag{14}$$

Inserting the p^{th} harmonic of radial wake force Eq.(14) in the definition Eq.(3), and taking into account relation Eq.(9), we can found the WFU parameter in the following form:

$$K_r^{(p)}(r,\tau)=\frac{ie}{mc^2}\frac{\frac{\theta_q d}{2D}}{\sin\left(\frac{\theta_q d}{2D}\right)}\left[\frac{\mathcal{E}_z(\tau)}{\chi_{q+p}}\frac{I_1(\chi_{q+p}r)}{I_0(\chi_{q+p}a)}\frac{\sin\left((\theta_q+2\pi p)\frac{d}{2D}\right)}{(\theta_q+2\pi p)\frac{d}{2D}}+\frac{\mathcal{E}_z(\tau)^*}{\chi_{q-p}}\frac{I_1(\chi_{q-p}r)}{I_0(\chi_{q-p}a)}\frac{\sin\left((\theta_q-2\pi p)\frac{d}{2D}\right)}{(\theta_q-2\pi p)\frac{d}{2D}}\right] \tag{15}$$

3. WF UNDULATOR OPTIMIZATION

To perform under-estimations of photon fluxes WFU radiation from the DLW called as the WF undulator [4,5], it is frequently required to find DLW dimensions for which a WFU parameter achieves the maximum magnitude. The optimization is simple if it is naturally supposed that bunches enough short and uniform charged. So, due to $\omega\Delta\tau<<2\pi$, ($\Delta\tau$ is the bunch duration), the amplitude of synchronous spatial harmonic of the longitudinal wakefield Eq.(12) has the simplest form:

$$\mathcal{E}_z(\tau)=-eN2k_q\frac{\tau}{\Delta\tau}, \tag{16}$$

where τ varies in the limits $0\le\tau\le\Delta\tau$. In this case the Eq.(15) can be rewritten as:

$$K_r^{(p)}(r,\tau)=-\frac{ie^2N2k_q}{mc^2}\frac{\tau}{\Delta\tau}\frac{\frac{\theta_q d}{2D}}{\sin\left(\frac{\theta_q d}{2D}\right)}\left[\frac{I_1(\chi_{q+p}r)}{I_0(\chi_{q+p}a)}\frac{\sin\left((\theta_q+2\pi p)\frac{d}{2D}\right)}{\chi_{q+p}(\theta_q+2\pi p)\frac{d}{2D}}\pm\frac{I_1(\chi_{q-p}r)}{I_0(\chi_{q-p}a)}\frac{\sin\left((\theta_q-2\pi p)\frac{d}{2D}\right)}{\chi_{q-p}(\theta_q-2\pi p)\frac{d}{2D}}\right] \tag{17}$$

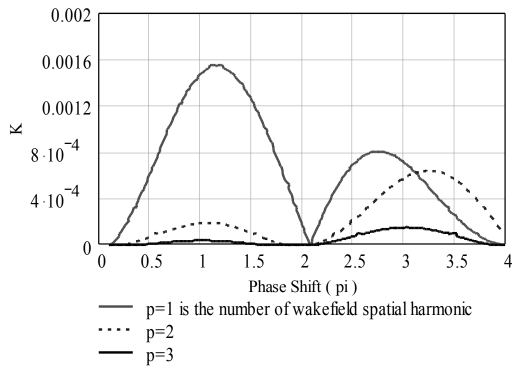

Fig.2. The mean-square WF undulator parameters vs a phase shift per period of the synchronous spatial harmonic

Fig.2 shows the dependence of the mean-square WFU parameter Eq.(17) on the phase shift per period of the resonant spatial harmonic with the phase velocity equal c. The dependences are calculated for the bunch with charge 12 nC, which moves at the distance r_b=0.75a from a DWL axis. As follows from this figure the optimal phase shift equal to 9π/8 is much close to the phase shift 4π/3 operating for the accelerating section STRUM90 produced and tested at KIPT [10]. Therefore, as prototype we just choose this rf structure.

Modifying the other DLW dimensions (the aperture radius a, dick thickness D-d, and cell radius b) it is found the optimal dimensions which are given in Table.

The optimal dimensions of a S-band DLW

θ_q	D (cm)	d (cm)	a (cm)	b (cm)
$4\pi/3$	7.1	3.9	2	4.1

4. CONCLUSION

The method developed above allows analytical relations for an alternating wake forces in the lowest pass-band of an axis symmetric DLW to be obtained. That results in the possibility to perform under-estimations of photon fluxes of WFU radiation from DLW, and to design the DLW with maximal magnitudes of WFU parameters.

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АНАЛИТИЧЕСКИЕ ФОРМУЛЫ ДЛЯ ЗНАКОПЕРЕМЕННОЙ КИЛЬВАТЕРНОЙ СИЛЫ ДИАФРАГМИРОВАННОГО ВОЛНОВОДА

А.Н.Опанасенко

Используя метод разложения полей по собственным модам в частично пересекающихся областях, выведена формула для знакопеременной кильватерной силы, возбуждаемой сгустком релятивистских заряженных частиц в периодической резонансной структуре.

АНАЛІТИЧНІ ФОРМУЛИ ДЛЯ ЗНАКОЗМІННОЇ КІЛЬВАТЕРНОЇ СИЛИ ДІАФРАГМОВАНОГО ХВИЛЕВОДУ

А.М.Опанасенко

Використовуючи метод розкладу полів по власних модах в областях, які частково перетинаються, виведено формулу для знакозмінної кільватерної сили, що збуджується згустком релятивістських заряджених частинок у періодичній резонансній структурі.