COHERENT SUMMATION OF WAKE FIELDS EXCITED BY AN ELECTRON BUNCH SEQUENCE IN A PLANAR MULTIMODE DIELECTRIC RESONATOR

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The possibility of realization of resonator concept of dielectric wake field accelerator is studied. The requirements of implementation of this concept are obtained. Numerical simulations of wake field excitation in planar dielectric resonator by bunch sequence testify the obtained requirements are true.

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1. INTRODUCTION

Historically, the most of studies on the dielectric waveguides used both for Cherenkov radiation sources and for the dielectric wake field accelerators (DWFA) is carried out for cylindrical structures. Intensity of electric fields in DWFA can be considerably increased due to coherent summation of fields of many traversal harmonics of a field [1]. For this purpose it is necessary to provide the equal spacing of resonant eigen-frequencies of the structure. In rectangular structures [2] such mode to realize is essentially easier. The equal spacing of resonant frequencies is especially important at summation of fields from regular sequence of bunches.

In the dielectric waveguide excited wake field is removed from structure with group velocity. In finite length waveguide on building up of the wake field maximum the part of bunches of sequence works only [3]. The using of resonator eliminates effect of removing of wake field [4,5]. Under optimum conditions, excitation of dielectric resonator by a sequence of bunch is similar to the excitation of an optical resonator by a mode-locked laser equipped with an “optical switch”. At the moment when the short pulse of radiation reflected from the output of the resonator appears at the input, the optical switch injects the next impulse into the resonator.

Below we shall find optimum regimes of excitation of the dielectric resonator by sequence of bunches and we investigate the time dynamics of excited fields.

2. WAKE FIELD OF BUNCH TRAIN IN DIELECTRIC RESONATOR

Let the rectangular metal resonator have a transverse size a $(-a/2 \le x \le a/2)$ and b $(-b/2 \le y \le b/2)$, its length equal to L. The resonator is filled with a homogeneous dielectric with permittivity ε. Along the axis of the resonator there is a drift channel, the transverse dimensions which are small if compared with those of the resonator. We will suppose that monoenergetic, thin electron bunches are injected into the input of the resonator $z=0$ and then move with a constant velocity v_0 along the axis. The distribution of current density of a single bunch has the form of:

$$j_z = Q_b \delta(x-x_0)\delta(y-y_0)\delta(t-t_{0i}-z/v_0) \times [\theta(t-t_{0i}) - \theta(t-t_{0i}-L/v_0)]/v_0,$$

where Q_b is the charge of a bunch, t_{0i} is the time of the i-th bunch injection into the resonator, x_0, y_0 are transverse coordinates of a bunch, $\theta(t)$ is the Heaviside function, $\delta(t)$ is the Dirac function.

Having solved the wave equation and taking into account the vanishing of the tangential components of electric field on the metal walls of the resonator, we obtain the expression for the longitudinal electric field:

$$E_z = E_0 \sum_{i=1}^{N_b} \sum_{\substack{m,n=1\\ l=0}}^{\infty} G_{mn}(x,y,x_0,y_0)\cos(k_l z)\frac{\delta_l \omega_{100}}{\omega_{mnl}^2-\omega_l^2} \times \left\{\left[\frac{\omega_{mn0}^2}{\omega_{mnl}}\sin\omega_{mnl}(t-t_{0i}) - \omega_l\left(1-\frac{c^2}{v_0^2\varepsilon}\right)\sin\omega_l(t-t_{0i})\right]\right.$$
$$\times\theta(t-t_{0i}) - (-1)^l\left[\frac{\omega_{mn0}^2}{\omega_{mnl}}\sin\omega_{mnl}(t-t_{0i}-L/v_0)\right.$$
$$\left.\left. - \omega_l\left(1-\frac{c^2}{v_0^2\varepsilon}\right)\sin\omega_l(t-t_{0i}-L/v_0)\right]\theta(t-t_{0i}-L/v_0)\right\},$$

where: $E_0 = -16\pi Q_b v_0 / abL\varepsilon\omega_{100}$; $k_l = \pi l/L$, $\omega_l = k_l v_0$, $\omega_{mnl}^2 = c^2(\kappa_{xm}^2 + \kappa_{yn}^2 + k_l^2)/\varepsilon$, $\kappa_{xm} = \pi m/a$, $\kappa_{yn} = \pi n/b$, the function δ_l is equal 1 if $l=0$ and is equal 2 if $l \neq 0$; N_b is the number of bunches in a sequence; $G_m(x,x_0) = \sin[\kappa_{xm}(x+a/2)] \times \sin[\kappa_{xm}(x_0+a/2)]\sin[\kappa_{yn}(y+a/2)]\sin[\kappa_{yn}(y_0+a/2)]$.

When all particles left the resonator ($t > t_{0Nb} + L/v_0$), the field of the space charge (second expression in square brackets) disappears, and the expression for the longitudinal electric field is the sum of traveling forward and backward eigen-waves of the dielectric resonator:

$$E_z = E_0 \sum_{i=1}^{N_b} \sum_{\substack{m,n=1\\ l=0}}^{\infty} G_{mn}(x,y,x_0,y_0)\cos(k_l z)\frac{\delta_l \omega_{100}}{\omega_{mnl}^2-\omega_l^2} \times \frac{\omega_{mn0}^2}{\omega_{mnl}}\left[\sin\omega_{mnl}(t-t_{0i}) - (-1)^l \sin\omega_{mnl}(t-t_{0i}-L/v_0)\right].$$

We note, that for the condition

$$\omega_{mnl} = \omega_l$$

the relevant items in the sum (2),(3) become dominant. This condition is nothing but the condition of Cherenkov radiation in a slowing medium. This is why these resonant items can be treated as a Cherenkov field accumulated in the dielectric resonator. The rest of the field is the field of the transition radiation from both boundaries.

Because of the discreteness of the longitudinal wave numbers of the oscillation spectrum, the resonant condition for the optiomum sizes of the resonator, permittivity, and energy of the bunch can be fulfilled only approximately and only for a finite number of harmonics. Taking into account that we want to implement a multimode condition of DWFA, it makes sense to find the relation between the above quantities from the resonant requirement.

We now find the relation for a 2-d dielectric resonator (size in the y-direction is greater than the size in x-direction). Let the resonant condition be fulfilled for harmonic $m = 1$, $l = N$. Then for coherent summation of fields of all harmonics $m = k$, $l = kN$ from expression (3) we obtain

$$L = Na\sqrt{\beta_0^2\varepsilon - 1}, \quad \beta_0 = v_0 / c .$$

Conditions (5) and (6) are the basis of the resonator concept of the dielectric wake field accelerator. In such a resonator there will occur a multimode regime of field excitation and a coherent summation of fields from the injected bunches.

3. NUMERICAL SIMULATIONS OF WAKE FIELDS BUILD UP

The wakefield from a sequence of bunches of finite size is obtained by integration of expression (2) over the time of injection t_{0i} and the locations x_0 of the point bunches.

We choose the following parameters for numerical calculations: dielectric permittivity $\varepsilon = 2.1$; frequency of bunch repetition $f = 2.85$ GHz; energy of the bunches is 4MeV; transverse size chosen according to condition (6) $a = 5{,}045$ cm and the length of the resonator, chosen according to requirement (5), $L = 15.677$ cm (tenth longitudinal harmonic is chosen as resonant, $N = 10$); the length of bunch $L_b = 1.7$ cm; the height of bunch $a_b = 0.1a$; $2Q_b / b = 0.32$ nC/cm.

In Fig.1 the time dynamics of the wakefield ($x = 0$) at the input of the dielectric resonator after injection of a single bunch is presented, the bunch having left the resonator at time $t \approx 1.75$ ns. The wake field at the input of the resonator (as well as at any spatial point inside the resonator) has a nonregular behavior in time (see the left graph), even though the peaks of the wake field follow with a period approximately equal to the period of the basic resonant mode.

The longitudinal distribution of the wakefield also has a nonregular character. The envelope of the field is nonuniform along the length of the resonator. Its structure varies with time; the maximum is periodically displaced from one wall of the resonator to another. The irregularity of the wake field is related to the excitation by a single bunch of set of nonresonant longitudinal harmonics. These nonresonant harmonics can be attributed to the transition radiation [3], which has a wide spectrum. This is verified by the spectral density of the longitudinal electric field given in right part of Fig.1. In this graph the peaks of spectral density corresponding to the excited longitudinal harmonics of the field are clearly visible.

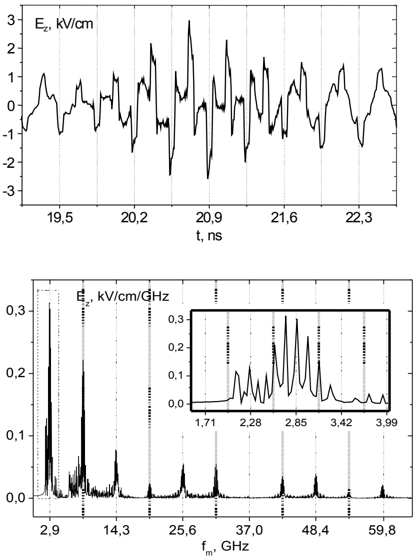

Fig.1. Time dependence of longitudinal electric field at the input of the dielectric resonator and its spectral density after injection of a single bunch

In Fig.2 the axial distributions of wakefield build up by injecting a sequence of 101 bunches into resonator is presented: the top figure corresponds to the time $t = 1.738$ ns (the first 5 bunches are injected into the resonator) and the down figure corresponds to the time $t = 35.639$ ns (when the last bunch of the sequence is injected into the resonator).

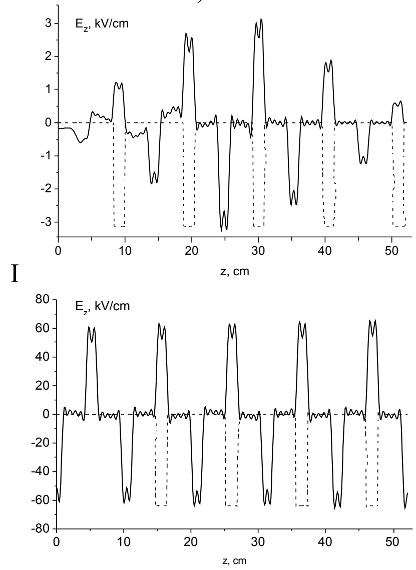

Fig.2. Axial distributions of wake field in the centre of resonator ($x = 0$) when injecting a train of bunches. Dashed lines show the location of bunches

At the initial stage, the amplitude of the field grows from the head of a sequence of the bunches to the location of the group wave front, excited by the first bunch, and then decreases backwards to the input of the resonator. The wakefield in the resonator before the first bunch leaves is qualitatively and quantitatively the same as the field in a semi-infinite waveguide. The shape of the wake field impulses and their duration approximately repeats the shape and duration of bunches. At later times, after all bunches have been injected into the resonator, a nearly homogeneous distribution of field amplitude is formed.

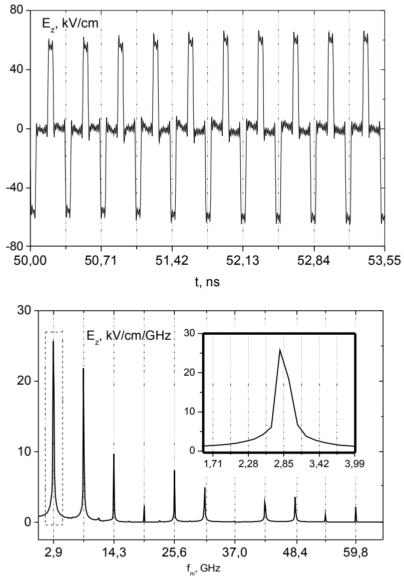

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Fig.3. Time dependence (left) of the longitudinal electric field and its spectral density (right) in the dielectric resonator after injection of 101 bunches

The case in the semi-infinite waveguide in contrast to the resonator with regard to the increasing input to output distribution of field amplitude [3] is formed. The number of bunches participating in the build up to maximum amplitude is much lower, compared with the resonator. For a dielectric waveguide having the same length, transverse size and permittivity as the resonator, this number of bunches is equal 6. By comparison of the top and down graphs of Fig.2 we conclude that all bunches of the sequence equally contribute to the formation of the longitudinal electric field amplitude in resonator: i.e., it is possible to excite a wakefield in the resonator with the amplitude considerably exceeding the amplitude of the field in the semi-infinite waveguide. The regularity of the oscillations is maintained.

We now show the mode-locking from the excitation of the dielectric resonator by a sequence of electron bunches. For this purpose we shall compare the temporal dynamics of the longitudinal electric field at a fixed point of the resonator when the field is excited by a single bunch and by a train of 101 bunches. In Fig.3 (top part), the time dependence of the longitudinal electric field at the input end of the resonator is presented. The last bunch has left the resonator at time $t \approx 38$ ns. The wakefield at the input of the resonator has a pattern of a sequence of rectangular pulses with period equal to the period of the bunch repetition rate. The amplitude of the pulses varies weakly with time. At the down side of Fig.3 the spectrum of the longitudinal electric field is presented. It is seen that wakefield contains only odd resonant frequencies. Non-resonant harmonics contribute very little to the amplitude of the wakefield.

We now compare this spectral density with the spectral density of the field, excited by a single bunch (see Fig.1). It is seen that a train of bunches regularizes the wake field, suppresses non-resonant longitudinal harmonics and strengthens the resonant harmonics of a field. In other words, the regular sequence of bunches realizes mode-locking of the dielectric resonator.

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КОГЕРЕНТНОЕ СЛОЖЕНИЕ КИЛЬВАТЕРНЫХ ПОЛЕЙ ПОСЛЕДОВАТЕЛЬНОСТИ ЭЛЕКТРОННЫХ СГУСТКОВ В ПЛОСКОМ МНОГОМОДОВОМ ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРЕ

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Исследована возможность реализации концепции ускорителя на кильватерных полях в диэлектрическом резонаторе. Аналитически определены условия когерентного сложения кильватерных полей последовательности электронных сгустков. Проведенное численное моделирование подтверждает условия когерентности.

КОГЕРЕНТНЕ ДОДАВАННЯ КІЛЬВАТЕРНИХ ПОЛІВ ПОСЛІДОВНОСТІ ЕЛЕКТРОННИХ ЗГУСТКІВ У ПЛОСКОМУ МНОГОМОДОВОМУ ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРІ

М.І. Оніщенко, Г.В. Сотніков

Досліджено можливість реалізації концепції прискорювача на кільватерних полях у діелектричному резонаторі. Аналітично визначені умови когерентного додавання кільватерних полів послідовності електронних згустків. Проведене чисельне моделювання підтверджує умови когерентності.