PARAMETRIC BEAM INSTABILITY IN A CRYSTAL

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Analysis of the optimal conditions for realization of X-ray free electron laser is considered for the case when an electron beam radiates in a crystal. It is shown that the use of the parametric X-ray radiation under the condition of multi-wave diffraction allows essential decreasing of the value of threshold beam density for the coherent generation. Estimation for the critical current density for the coherent X-ray generation in some crystals is also obtained.

PACS: 42.55.V

1. INTRODUCTION

Different mechanisms of induced X-ray radiation are thoroughly studied, which may serve as a basis for construction of an X-ray laser, operating within the 10…100 keV range. Various types of free electron lasers, using the Compton scattering of a light wave by an electron beam, resonance transition radiation, channeling particle radiation have been considered. The analysis of the obstacles revealed that large current densities j > 10^{13} A/cm^2 of relativistic electrons is necessary to realize the induced radiation mode. This fact stands the question mark on the possibility of the X-ray coherent generation within the mentioned range. The emission from charged particles in a crystal to the large angles to the particle velocity has been shown [1]–[4] to provide a radical change in a gain of the radiation and reduce the required current density for generation launch to the values j ~ 10^8 A/cm^2, which makes possible the development of PXR FEL). This possibility follows from the recently discovered law of parametric beam instability [1]–[4], which occurs when an electron (positron) beam passes through a crystal. The instability leads to a drastic increase of the coefficient of induced radiation in the crystal and reduces the threshold current for the generation start. The more detailed theory of this phenomenon is considered in [5].

2. EQUATIONS FOR FIELD AND DENSITY WAVES

The classic approach to the electron movement assumes the Maxwell's equations for the description of the interaction between charged particle and wave fields of radiation, and crystal. Keeping in mind the X-ray diffraction of emitted photons in the framework of two-wave approximation, these equations are:

$$\left(k^2 - \frac{\omega^2}{c^2}\varepsilon_0\right)\mathbf{E}_{\mathbf{k},\omega} - \mathbf{k}(\mathbf{k}\mathbf{E}_{\mathbf{k},\omega}) - \frac{\omega^2}{c^2}\chi_{-g}\mathbf{E}_{g,\omega} = \frac{4\pi i\omega}{c^2}\mathbf{j}(\mathbf{k},\omega),$$

$$\left(k_g^2 - \frac{\omega^2}{c^2}\varepsilon_0\right)\mathbf{E}_{g,\omega} - \mathbf{k}_g(\mathbf{k}_g\mathbf{E}_{g,\omega}) - \frac{\omega^2}{c^2}\chi_g\mathbf{E}_{\mathbf{k},\omega} = \frac{4\pi i\omega}{c^2}\mathbf{j}(\mathbf{k}_g,\omega),$$

$$\mathrm{k}(\varepsilon_0\mathrm{E}_{\mathrm{k},\omega} + \chi_{-g}\mathrm{E}_{g,\omega}) = 4\pi i n(\mathrm{k},\omega),$$

$$\mathrm{k}_g(\varepsilon_0\mathrm{E}_{g,\omega} + \chi_g\mathrm{E}_{\mathrm{k},\omega}) = 4\pi i n(\mathrm{k}_g,\omega), \qquad (1)$$

$$\varepsilon_0 = 1 + \chi_{0,\omega}\, n(\mathrm{k},\omega) + \mathrm{kj}(\mathrm{k},\omega) = 0.$$

Here $\mathbf{E}_{\mathbf{k},\omega}$ is the amplitude of direct wave and $\mathbf{E}_{g,\omega}$ – of diffracted wave with vector $\mathbf{k}_g = \mathbf{k} + \mathbf{g}$, $n(\mathbf{k},\omega)$ and $\mathbf{j}(\mathbf{k},\omega)$ are Fourier-components of charge density and current of beam, respectively:

$$n(\mathbf{k},\omega) = \int d\mathbf{r}dt\, e^{i(\omega t - \mathbf{kr})} n(\mathbf{r},t),$$
$$n(\mathbf{r},t) = e\sum_j \delta[\mathbf{r} - \mathbf{r}_j(t)],$$
$$\mathbf{j}(\mathbf{r},t) = e\sum_j \mathbf{v}_j(t)\delta[\mathbf{r} - \mathbf{r}_j(t)], \qquad (2)$$

for electrons on the trajectory $\mathbf{r}_j(t)$.

To analyze the influence of the diffraction on the instability, we separate the amplitudes of transverse $E_t^{(s)}$ (polarization e_s, $s = \sigma, \pi$) and longitudinal $E_l^{(s)}$ electromagnetic waves.

The longitudinal component creates also a wave with charge density:

$$4\pi\, n(\mathbf{k},\omega) = -kE_l^{(s)}(\mathbf{k},\omega).$$

The Maxwell's equations (1) have to be supplemented by motion equations for electron in electromagnetic field $E(\mathbf{r}_j(t),t)$ which should be solved in linear approximation ($\gamma^2 = 1 - u^2/c^2$) and for a primary homogeneous beam:

$$\sum_j e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_{j0}} = n_b(2\pi)^3\delta(\mathbf{k}-\mathbf{k}'),\ n_b = \frac{J}{ecS}, \qquad (3)$$

where n_b is an electron density of the beam of current J and cross-section S. Then the following closed linear equation system for transverse and longitudinal wave fields is found for two-wave diffraction:

$$\{k^2c^2 - \omega^2[\varepsilon_0 + \chi_{\mathrm{tt}}^{(s)}(\mathbf{k})]\}E_{\mathbf{k},t}^{(s)} - \omega^2\chi_{-g}C_s\mathbf{E}_{\mathrm{g},t}^{(s)} =$$
$$= \omega^2\chi_{\mathrm{tl}}^{(s)}(\mathbf{k})E_{\mathbf{k},l}^{(s)},$$
$$\{k_g^2c^2 - \omega^2[\varepsilon_0 + \chi_{\mathrm{tt}}^{(s)}(\mathbf{k}_g)]\}E_{\mathrm{g},t}^{(s)} - \omega^2\chi_g C_s\mathbf{E}_{\mathrm{g},t}^{(s)} =$$
$$= \omega^2\chi_{\mathrm{tl}}^{(s)}(\mathbf{k}_g)E_{\mathbf{k}_g,l}^{(s)},$$
$$\{(\omega - \mathbf{ku})^2 - \omega^2\chi_{\mathrm{ll}}^{(s)}\}E_{\mathbf{k},l}^{(s)} = \omega^2\chi_{\mathrm{lt}}^{(s)}(\mathbf{k})E_{\mathbf{k},l}^{(s)}, \qquad (4)$$

$\chi_{\mathrm{tl}}^{(s)}$ is a tensor of beam polarizability, which is physically analogous to crystal polarizability:

$$\chi_{\mathrm{tt}}^{(s)}(\mathbf{k}) = -\frac{\omega_b^2}{\gamma\omega^2}\left[1 - \frac{\omega(\mathbf{e}_s\mathbf{u})^2}{c^2(\omega - \mathbf{ku})}\right], \quad \chi_{\mathrm{tl}}^{(s)}(\mathbf{k}) = -\frac{\omega_b^2}{\gamma\omega^3}k(\mathbf{e}_s\mathbf{u}),$$

$$\chi_{\parallel}^{(s)}(\mathbf{k}) = \frac{\omega_b^2}{\gamma\omega^2}\left[1 - \frac{(\mathbf{ku})^2}{k^2c^2}\right],$$

$$\chi_{\parallel t}^{(s)}(\mathbf{k}) = \frac{\omega_b^2}{\gamma\omega^2}(\mathbf{e}_s\mathbf{u})\left[\frac{k}{\omega} - \frac{(\mathbf{ku})}{kc^2}\right],$$

$$\omega_b^2 = \frac{4\pi e^2 n_b}{m}. \qquad (5)$$

3. CALCULATION OF THE INCREMENT

The determinant of (5) delivers a dispersion equation for k and ω. For relativistic particles, the terms with $(\omega - \mathbf{ku})$ in denominator are essential, whereas the terms with $(\omega - \mathbf{k}_g\mathbf{u})^{-1}$ are negligible. Thus, the dispersion equation is:

$$\left\{\left[k^2c^2 - \omega^2\varepsilon_0 + \frac{\omega_b^2}{\gamma}\right]\left[k_g^2c^2 - \omega^2\varepsilon_0 + \frac{\omega_b^2}{\gamma}\right] - \omega^4 C_s^2\chi_g\chi_{-g}\right\} =$$

$$-\frac{\omega_b^2u^2}{\gamma(\omega - \mathbf{ku})^2}\theta^2 k\left(k - \frac{\omega u\cos\theta}{c^2}\right)\left[k_g^2c^2 - \omega^2\varepsilon_0 + \frac{\omega_b^2}{\gamma}\right] \equiv$$

$$\equiv \frac{A(k_z,\omega)}{(\omega - \mathbf{ku})^2} \qquad (6)$$

Here $\theta << 1$ is an angle between $\mathbf{k}$ and beam velocity $\mathbf{u}$, the latter corresponds to axis Z. The most essential influence on the dispersion equation is exerted by variation of current due to the work of longitudinal component of radiation field performed under the beam particles (the right part of Eq.(6)). The solution of non-linear dispersion equation (6) is localized in the intersection of dispersion equations, corresponding to the wave fields of radiation in the crystal and radiation of the spatial charged beam. The coordinates k_{z0} and ω_0 of this intersection are found from:

$$\left\{[k^2c^2 - \omega^2\varepsilon_0][k_g^2c^2 - \omega^2\varepsilon_0] - \omega^4C_s^2\chi_g\chi_{-g}\right\} \equiv D(k_z,\omega),$$

$$(\omega - \mathbf{ku})^2 = 0. \qquad (7)$$

The frequency and wave vector of radiation have to satisfy the diffraction conditions at the moment of PXR emission, and therefore:

$$k_{z0} = k_B(1+\delta), \quad \omega_0 = uk_B(1+\delta), \quad |\delta| << 1, \qquad (8)$$

with k_B determined from the Bragg condition for the excited reciprocal lattice vector

$$k_B = -\frac{2\mathbf{k}_\perp\mathbf{g}_\perp + g^2}{g\sin\theta_B}, \quad \sin\theta_B = \frac{(\mathbf{gu})}{gu}, \qquad (9)$$

where deviation δ is:

$$\delta = \frac{\chi_g\chi_{-g} - (\eta + \xi)^2}{2\nu(\eta + \xi)}, \qquad (10)$$

$$\nu = -\frac{g\sin\theta_B}{k_B}, \quad \eta = \frac{k_\perp^2}{k_B^2} \approx \theta^2, \quad \xi = \gamma^{-2} - \chi_0.$$

Accordingly to general method for instability analysis, the solution of dispersion equation (6) may be found as an expansion series over the detuning of ω' and k_z' near the point (8).

For arbitrary crystal orientation and radiation angle like for homogeneous medium Eq.(6) leads to an amplification coefficient on the unity length, i.e. the gain:

$$G[\mathrm{cm}^{-1}] = \mathrm{Im}\,(k_z),$$

which is the principal characteristics of FEL. The following expression G for parametric beam instability can be found:

$$G = \frac{\sqrt{3}}{2}\left(\frac{Q\omega_0\chi_g\chi_{-g}}{u^2c^2k_{z0}[\chi_g\chi_{-g} + (1+\nu)(\eta+\xi)^2]}\right)^{1/3},$$

$$Q = -\frac{\omega_b^2u^2\theta^2k_0(k_0 - \omega_0\cos\theta\, u/c^2)}{2\omega_0\gamma}. \qquad (11)$$

Contrary to the Cherenkov instability in homogeneous medium, the amplification coefficient in 3D periodic medium depends essentially on crystal material and orientation that is a direct way to gain an interaction between radiation and electron beam. The geometry of emission, when:

$$\theta = \left[\frac{|\chi_g|}{\sqrt{|\cos 2\theta_B|}} - \frac{1}{\gamma^2} - |\chi_0|\right]^{1/2},$$

$$1 < 2\sin^2\theta_B < 1 + \frac{\chi_g\chi_{-g}}{(\gamma^{-2} + |\chi_0|)^2}, \qquad (12)$$

the diffracted wave in this case is traveling at the angle $\sim 90^\circ$, to the beam velocity:

$$2\theta_B = \frac{\pi}{2} + \psi,$$

$$\theta\sin 2\theta_B < \psi < \theta\sin 2\theta_B + \frac{\chi_g\chi_{-g}}{(\gamma^{-2} + |\chi_0|)^2}. \qquad (13)$$

Additionally to the longitudinal component, the transverse component of a diffracted wave becomes parallel to electron velocity in this case and performs a supplemental work on beam modulation (bunching).

For the optimal case of small detuning $\omega' \to 0$, the gain of parametric instability is:

$$G = \left(\frac{Q\omega_0|\chi_g|}{|\cos 2\theta_B|^{1/2}}\right)^{1/4}. \qquad (14)$$

Thus, at certain choice of 3D crystal parameters, the dependence of the amplification coefficient on the particle density in beam can qualitatively be changed, in comparison to the homogeneous medium: $n_b^{1/3} \to n_b^{1/4}$. Taking into consideration the inequality $\omega^3 Q << 1$, which is fulfilled for the current in real devices, the change in power degree influences significantly on G. Later on [4], the influence of quantum effects on instability increment has been studied, and N-wave diffraction is shown to increase the increment due to the dependence:

$$G \sim (n_b)^{1/(N+2)}.$$

Under the conditions of multiwave diffraction, the generation threshold is proved to be reduced due to the phenomenon of the parametric beam instability. This fact makes it possible to observe an induced PXR radiation as well as induced channelling radiation in LiH crystal at electron beam current density j ~ 10^8 A/cm^2, and energy from tens to hundreds MeV.

Basing on these investigations, a new type of FEL has been proposed, named a volume free electron laser (VFEL) [6]. The first lasing of VFEL in millimeter wavelength range was observed in 2001 [7].

This work is supported by International Scientific and Technical Center (project B-626).

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ПАРАМЕТРИЧЕСКАЯ ПУЧКОВАЯ НЕУСТОЙЧИВОСТЬ В КРИСТАЛЛЕ

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Анализ оптимальных условий для реализации рентгеновского лазера на свободных электронах проведен в случае, когда электроны излучают в кристалле. Показано, что использование параметрического рентгеновского излучения в условиях многоволновой дифракции позволяет существенно уменьшить величину пороговой плотности пучка для получения когерентного излучения. Получена также оценка для критической плотности для когерентной генерации рентгеновского излучения в некоторых кристаллах.

ПАРАМЕТРИЧНА ПУЧКОВА НЕСТІЙКІСТЬ У КРИСТАЛІ

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Аналіз оптимальних умов для реалізації рентгенівського лазера на вільних електронах проведений у випадку, коли електрони випромінюють у кристалі. Показано, що використання параметричного рентгенівського випромінювання в умовах багатохвильової дифракції дозволяє істотно зменшити величину порогової густини пучка для одержання когерентного випромінювання. Отримана також оцінка критичної густини для когерентної генерації рентгенівського випромінювання в деяких кристалах.