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THE WEAK TURBULENCE METHODS IN THE PROBLEM OF GALAXY MASS DISTRIBUTION FUNCTION

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The analogy between the turbulent Kolmogorov type spectra and the distribution of galaxies on masses is discussed. Due to nonlocality [1] of galaxy mass spectrum formed by merging the additional approximate conservation law affects its intermediate asymptotics power index, which proves to be intervening between the constant mass flux and this of the number of massive galaxies. Analytical description of this asymptotics, which essentially uses the Smoluchowsky kinetic equation symmetry transformation, is presented. The result is compared with the recently discovered steepness of galaxy luminosity function on its faint edge. The problem of the main part of barionic mass in the Universe is shortly discussed in this context.

In the expanding Universe the galaxies as we know run away from each other. In the ideal scheme of such Hubble's expanding there is no converging of galaxies: merely the distances between the galaxies grow. Under such conditions the galaxies evolve independently and their masses do not essentially change.

In reality the galaxies have not only the Hubble's recession velocities. They also have some "peculiar" components which are the result of fluctuations. The latter are also relevant for formation of galaxies themselves - the gravitationally coupled systems. Immediate confirmation of existing of such initial fluctuations has been received by investigation of the relic radiation (on the microkelvin's level).

With such fluctuation the Jeans gravitation instability evolves. In the expanding Universe it transforms from exponential to power type, but is not removed under expansion.

In the issue the galaxies gather in groups. Our Galaxy - the Milky Way - belongs to the Local Group containing one dozen odd members. The big groups - clusters - contain themselves groups as a rule. Many clusters are parts of super clusters, and so on, thus forming hierarchical structure of the Universe.

In the vicinity of the Milky Way three colossal super clusters have been detected. The most famous is the Great Attractor (GA) on which our Local Group falls. This falling is reflected even in the dipole anisotropy of relic radiation on the millikelvin's level corresponding to the velocity of about 600 km/s. The Hubble low (as straight observation show) fails in the direction of GA: near GA (remote from us at about 60 Mpc) galaxies accelerate or decelerate due to their falling on GA. In similar cases of different scales the galaxy-number density grows, the peculiar velocity components change and the collisions and mergers of galaxies take place (see the review lectures in [2] and the numerous references there).

As a result of mergers the mass function (MF), i.e. distribution of galaxies on masses is formed. The answer to the question: "Where is the main part of barionic stellar mass of the Universe is concentrated?" depends very essentially on some of the MF-details.

On the other hand the activity of galaxies and their nuclei arises owing to merging and such objects as radio galaxies and quasars appear. The correlation between the merger process and activity proves this. Probably even the Black Holes which are responsible for the nuclear activity may appear in the merging processes. But this side of the problem goes beyond the scope of this article (see, for example, [3] and the references there).

We will be interested only in the galaxy MF and their analogy with the Kolmogorov spectrum of turbulence in liquid and with the similar weak turbulent spectra in plasma.

As we will see the methods being developed in plasma physics prove to be useful for the analysis of galaxy mass distribution. The difference is that we have in the MF case the mass flux on the spectrum instead of energy flux in the Kolmogorov turbulence case. And the former is found to be the nonlocal.

The main part of the computations of this paper will be published in PysicaD, 2000, with more details.

1. So, we will be interested in the mass spectrum formed while merging, the main attention being paid to the index of power intermediate asymptotics (IA), which in the merging model assumes a clear physical sense and can be obtained by analytical methods. This MF part is juxtaposed with the recently discovered greater steepness of the galaxy luminosity function at its faint edge (see as example [4]). This also may serve as an argument in favor of the galaxy evolution due to merging.

Below we will realize what could give us the pure analytical methods analogous to those in the weak turbulence theory [5,6]. The idea of nonlocality of the weak turbulent spectra which was discussed by Balk, Zakharov and Nazarenko in the plasma physics problems context [1] and symmetry transformations of kinetic equations [5,7] will play the most essential role in this description.

2. Before examining the real galaxy interaction we must recall that SE for MF,

$$\frac{\partial f(m,t)}{\partial m} = \int dm_1\, dm_2\ [U_{12}\, \delta_m\ f_1\ f_2 - $$
$$- \text{cycle} - \text{bicycle}\,] \qquad (1)$$

where $f \equiv f(m,t)$, $f_1 \equiv f(m_1,t)$, etc, are MFs, $\delta_m \equiv \delta(m - m_1 - m_2)$, δ — Dirac's delta-function, $U_{12} \equiv U(m_1, m_2)$, which describes the merging, permits two nontrivial (here we do not touch the case of $U = const$) exact solutions describing the MF evolution from its initial state localized on small masses [8]. In the second and third terms we have $\delta\,(m_2\text{-}m\text{-}m_1)$ and $\delta\,(m_1\text{-}m_2\text{-}m)$ correspondingly.

For the merging probability proportional to the product of colliding masses the power index of IA is s_1=-5/2. Expressed through the uniformity power u of coagulation coefficient $(U(am) = a^u U(m))$ it equals $s_1 = -(u+3)/2\ (u=2)$. As known (see also below) such index corresponds to the constant mass flux on the spectrum. In the case of $U = c(m_1 + m_2)$ the power of IA is $s_0 = -3/2$ or $s_0 = -(u+2)/2\ \ (u=1)$. The latter corresponds to the constant flux of the number of massive objects. Though at first sight such a conservation law fails with the mergers it is realized in the form of approximate integral in the case of predominant interaction of large objects with the small ones. Such a "nonlocal" situation fits the last solution. The condition of locality, i.e. that of convergence of the collision integral in SE for s_1 - solution is in the form of $|u_2 - u_1| < 1$ [9], where the indexes u_1 and u_2 are defined by the expression for U provided the masses differ strongly $U \propto m_1^{u_1} m_2^{u_2}\ \ (m_1 << m_2)$. Obviously, in the first case $(u_1 = u_2 = 1)$ the locality criterion is fulfilled and in the second the marginal case occurs ($u_1 = 0$, $u_2 = 1$). That is, the galaxy interaction with the most distinguished scales prevails in the latter case and, thus, the conservation law of the "number of particles" is realized.

3. For gravitational galaxy interaction the cross-section of coagulations is usually taken as product of co-factors, which describe, respectively, the geometrical cross-section, gravitational focusing and conditional merging probability at the frontal collision of galaxies (see references in 3]):

$$\sigma = \pi r^2 (1+\gamma)\varphi(\gamma), \quad \gamma \equiv v_g^2 / v^2,$$
$$v_g^2 = 2G\, m/r,\ m = m_1 + m_2,\ r = r_1 + r_2\,.$$

The homogeneity index differs for "large" and "small" masses. Further we will focus on this particular region, regarding a small mass region as contracted to zero. This scheme can be attributed with a more accurate formal meaning. On the assumption that φ decreases as square of relative velocity we can take it in the form of $\varphi = (1 + 1/\gamma)^{-1}$. The resulting cross-section will be a uniform function in all of the mass-changing interval: $\sigma = \pi r^2 \gamma$. By averaging over velocities we come to the coagulation coefficient $U = <\sigma v>$ in the form $U \propto (m_1 + m_2)(m_1^\beta + m_2^\beta)$ where the radius-mass dependence is chosen as $r \propto m^\beta$. Below we employ only the fact that U is the uniform function of masses with

$$u = 1 + \beta, \quad u_1 = 0, \quad u_2 = u \qquad (\beta = 1/3 \div 1/2)$$

For u>1, as known from general theory of SE [1], the evolution of MF has an explosive character and a quasi power asymptotics is established in a wide mass interval between the region of initial mass localization $m \approx m_*$ and the coagulation front which is turned to the infinity mass for the finite time [11,12]. Our goal is to find the power IA of the spatial homogeneous [2] solutions of the SE (1) with the considered kernel U discussed above.

$$J_1(m) = -\int^m dm\ m I_{st}\,, \quad \frac{\partial\, mf}{\partial t} + \frac{\partial J_1(m)}{\partial m} = 0 \qquad (2)$$

Here the I_{st} is the right part of SE (1).

4. Both the numerical solution of SE and modelling by Monte Carlo method show that the power index of IA α lies between $s_0 = -(u+2)/2$ and $s_1 = -(u+3)/2$ (see for example [10] Fig. 2b; [13]). In order to understand what it means consider the symmetry properties of the collision integral of SE in the case of exact uniformity:

$$U_{am_1 am_2 | am} = a^u U_{m_1 m_2 | m}\,.$$

To utilize the similarity of U we must change simultaneously the scale of all three arguments m_1, m_2 and m. But, as one of them (m) is fixed in SE, from the continuous group transformation only two discrete transformations remain (except a trivial one): G_1, transforming $m_1 \to m$, and G_2, transforming $m_2 \to m$. These Zakharov transformations are

[1] In this case the initial distribution localized on small masses within a finite time forms a power "tail" spreading on the region of formally infinite masses [8]. This the so-called kinetic phase transition was first discovered and studied in detail by Stockmayer for the above-mentioned model with $U = cm_1\, m_2$ and was utilized for describing polymerization, in particular, zol–gel transition (in addition to the above-mentioned see also the references in [10]). In the case of the gravitating systems we are interested in, the new phase which emerges at the transition and corresponds to the "infinite" mass is juxtaposed with cD-galaxies in the center of the cluster.

[2] This surely leads to the loss of a number of distribution features, including spatial stratification of galaxy clusters, with a more compact central and less dense periphery, etc. At the same time the chaotization in the systems considered confirms the made assumption.

considered as some change of variables m_1 , m_2 with the fixed mass m conditions:

$$G_1 = m_1 \to \frac{m}{m_1} m_2 , \qquad m_2 \to \left(\frac{m}{m_1} \right)^2 m$$
$$G_2 = m_1 \to \left(\frac{m}{m_2} \right)^2 m, \qquad m_2 \to \frac{m}{m_2} m_1 . \tag{3}$$

They form a symmetry group of SE [7]. For these conformal transformations the integrating paths tending to infinity in the second and third terms in (1) convert into the integrating path with the finite mass variation in the first term of SE. In the issue (using also $x \to m - x$ symmetry) SE acquires the form of:

$$\frac{\partial f(m,t)}{\partial t} = 2 \int_0^{m/2} dx\, U_{m|x,m-x} \{ \ \}, \quad \{ \ \} =$$
$$f(m-x) f(x) - \left(\frac{m}{x} \right)^{2+u} f\left(\frac{m}{x}(m-x) \right) f(m)$$
$$-\left(\frac{m}{m-x} \right)^{2+u} f\left(\frac{mx}{m-x} \right) f(m) \tag{4}$$

If in addition a power-law character of the solution is assumed $\left(f(m) \propto m^s \right)$, then $\{ \ \}$in SE is reduced to [5,14]:

$$\{ \ \} = f(m-x) f(x) \left\langle 1 - \left(\frac{m}{x} \right)^{\nu} - \left(\frac{m}{m-x} \right)^{\nu} \right\rangle,$$
$$\nu = 2 + u + 2s \tag{5}$$

In the stationary case we get the exact power solution $f \propto m^{s_1}$, $\nu = -1$, which corresponds to the constant mass flux P on the spectrum $f_P = c_1 P^{1/2} m^{s_1}$. This can be easily proved by using the definition of mass flux, thus finding the normalization factor and the flux sign [9]. The obtained formal solution, however, is nonlocal: the integrals diverge on small masses, which thus must contribute mainly.

5. Now consider the MF decreasing steeper than the power on the largest masses. With this condition the second term in $\{ \ \}$(3) vanishes in the case of essential contribution of small masses due to nonlocality. In the issue the approximate power solution arises that corresponds to conservation of the number of massive galaxies (if their interaction with the small-mass ones prevails):

$$f \propto m^{s_0}, \quad \nu = 0 \tag{6}$$

Really, the flux of the number of massive galaxies (below – the galaxy flux) on the mass axis is

$$J_0(m) = -\int^{m} dm\, I_{st} \ , \tag{7}$$

where through I_{st} we defined the right part of SE (1). This corresponds rewriting the latter in the form of approximate conservation law

$$\frac{\partial f}{\partial t} + \frac{\partial J_0(m)}{\partial m} = 0 . \tag{8}$$

For the power spectrum the galaxy flux

$$J_0(m) = -\int^{m} dm\, m^{\nu-1} F_0(\nu) = -\frac{m^{\nu}}{\nu} F_0(\nu) ,$$

where $F_0(\nu) =$ (9)

$$2\int_0^{1/2} d\zeta\, U_{1|\zeta,1-\zeta}\, f(\zeta) f(1-\zeta) \left\langle 1 - \left(\frac{1}{1-\zeta} \right)^{\nu} \right\rangle$$

At $(\nu \to +0)$ (onesiding limit corresponds to integrability of the expression for $j_0(m)$ in the origin) in accordance with (5) $F_0 \to 0$ and we obtain the solution with constant galaxy flux $J_0(m) = Q$ >0 (cf. [15]):

$$Q = =2 \int_0^{1/2} d\zeta\, U_{1|\zeta,1-\zeta}\, f(1-\zeta) \ln \frac{1}{1-\zeta} . \tag{10}$$

The positive sign of the flux corresponds with the physics of mergers. Using the definition of particle flux Q we can normalize this distribution too: $f_Q = c_0 Q^{1/2} m^{s_0}$ (cf. [16]). With the two concurrent fluxes it is easy to find the analogous solutions, when one of the fluxes is smaller than the other. The value of this ratio depends on the mass: mQ/P. Obtain in the issue the spectrum with the break at $P/m_{br}Q \sim 1$ which overpasses on its ends to single-flux distributions. However, our whole case is nonlocal in principle [10], and even the additional conservation law is connected with this nonlocality (cf. [1]).

6. Thus, we have to proceed to the differential description primarily accounting for the interaction with multiple dwarf galaxies. With the original SE form (1) this is difficult to do in view of the equal character of divergence on small m_1 and m_2, as well as on the infinity masses. After Zakharov transformations we have only one singular point $(m_1 = 0)$, near-which expanding gives us the equation:

$$\frac{\partial f(M,t)}{\partial \tau} = -A \left(\frac{\partial f(m)}{\partial m} + \frac{2+u}{m} f(m) \right) -$$
$$- f(m) \int_0 dx \left(\frac{m}{x} \right)^{2+u} f\left(\frac{m^2}{x} \right) -$$
$$\frac{f(m)}{m} \int_0 dx\, x^2 \frac{\partial f(x)}{\partial x} ; \tag{11}$$

$$A = \int_0 dx\, x f(x), \quad \left(\tau = t \cdot 2U_{m|0m}\right)$$

The afore-noticed compensation occurs here automatically. For the pure power distribution obtain the equation:

$$-\frac{\partial \Phi}{\partial \tau} = A\frac{\partial \Phi}{\partial m} + B\Phi^2, \qquad (12)$$

$$\Phi \equiv m^{\mu} f(m,t), B = \int_0 dx/x^{\mu}, \mu = 2 + u + s,$$

The formal solution of (12)

$$1/\Phi = Cm + D \quad (C = B/A)$$

(abstract for a moment from its power form) gives us two asymptotics. One corresponds to s_1 ($Cm >> D$):

$$1/\Phi = Cm \quad \left(f \propto m^{s_1}\right); \qquad (13)$$

the other – to s_0 ($Cm << D$):

$$1/\Phi = D \quad \left(f \propto m^{s_0}\right). \qquad (14)$$

Thus we can affirm [13] that if the SE solution is approximated by the power-type function its index appears between these limit values $-2-\beta/2 < s < -3/2 - \beta/2$. The resulting index can be somewhat smaller or larger than -2 and close to the shechter's index $\alpha \cong -1.71 \pm 0.5$, ([4], field galaxies), $\alpha \cong -2.2 \pm 0.3$, [17] rich clusters galaxies)) on the faint end of LF. If $s < -2$, the major part of mass is concentrated in the smallest galaxies. In the two flux MF case as we can see above the divergent of the integral for f_Q on the upper limit and the same for f_P on the lower limit results in the main mass concentration near m_{br} located by the IA of the whole nonstationary solution. Some astrophysical appearances of this question was also discussed in [18].

Note we can not exclude the opposite case when MF is bent out to the bottom.. Because we can only guarantee the existence of two power asymptotics but not of their place in the MF for the assumption we have made of the pure power form of equation (11) solution.

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