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PLASMA SPHERE EQUILIBRIUM IN EXTERNAL R.F.FIELDS

G.A. Bryzgalov, L.N. Nikolaichuk, , N.A. Khizhnyak
National Center, Kharkov Institute of Physics and Technology, Kharkov, 61108, Ukraine
E-mail: khizh@khizh.kharkov.ua

The problem on confinement and equilibrium of the spherical plasma formation in external r.f. electromagnetic fields is solved using the method of the integral equation of macroscopic electrodynamics. Namely, the problem of plasma confinement with r.f. pressure forces (the Gaponov-Miller force) is formulated and self-consistent problem of the external electromagnetic field providing the required configuration of non-disturbance electromagnetic fields e.g. the field in which together with volume charge forces taken into account forms the spherical symmetric potential well is solved. It is shown that such a solution exists only if there is rotating ordered motion of plasma. It is naturally that the non-disturbance confining r.f. field is determined only in the region linear dimensions of which correspond to the sphere ones.

The field value out of these dimensions is defined solving the corresponding non-corrected mathematical problem. This permits to build the source of the confining field with the definite arbitrariness that makes easier to realize the device in practice.

INTRODUCTION

The problem of stable confinement of the limited plasma in the external r.f. fields represents undoubtedly theoretical and practical interest. The theoretical one in the plan that it is only seemed obvious problem, however its realization meets exclusive difficulties especially if it sets in self-consistent conditions. Now among the great number of works on plasma physics it is difficult to find if only one work in which such a problem would be solved relatively correct. The practical interest of such a problem is that stable confinement of the limited three-dimensional plasma formation in the external r.f. electromagnetic fields reveals new perspectives for high temperature plasma confinement in researches on CTE and also for using of atmosphere plasma objects with an aim to control electromagnetic wave propagation.

In this paper only the first attempts to set and solve this problem have been done and the outlined success to build its solution is due to using of the field equations in the form of integrals.

GENERAL PROBLEM OF PLASMA ELLIPSOID EQUILIBRIUM IN R.F. ELECTROMAGNETIC FIELDS

Let us consider the problem of equilibrium and stability of plasma formation of the ellipsoid geometry (in particular the case of a sphere) in the external electromagnetic field. The surface of this ellipsoid in the Cartesian coordinate system combined with the center of the ellipsoid is described by the canonical form equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \qquad (1)$$

where a, b and c are the semiaxes of the ellipsoid and small distortion of the geometry of a plasma bunch we shell describe by the vector of deformations $\bar{\xi}(\bar{r},t)$ defined on its surface if coordinates x, y and z are mutually connected by additional conditions (1). The equation of plasma movement in the vicinity of equilibrium will have the form

$$nM\frac{d^2\bar{\xi}}{dt^2}=-\nabla\left(p+p_{в,г}\right)$$
$$\frac{\partial n}{\partial t}+div\left(n\dot{\bar{\xi}}\right)=0 \qquad (2)$$
$$\frac{d}{dt}\left(pn^{-\gamma}\right)=0$$

where $p=nkT$, $p_{в,г}=\frac{1}{16\pi}\sum_{i,k}E_i^*E_k\frac{\partial\varepsilon_k}{\partial n}n$, and the condition of equilibrium will take the form

$$\left(p+p_{в,г}\right)=const,\quad \bar{\xi}_0(\varepsilon,t)=0 \qquad (3)$$

where index «0» characterizes variables in the initial state (the equilibrium state) on the surface (1). Here, E_i - are the components of the internal electrical field inside the plasma bunch.

Now we formulate the problem on equilibrium of a bunch with r.f. fields in the linear approximation. We suppose

$$n=n_0+n' \qquad p=p_0+p'$$
$$\vec{E}=\vec{\mathrm{E}}+\vec{E}' \qquad (4)$$

where n', p' and $\bar{E}'$ - are the small deviations in equilibrium values.

After linearization of equations (2), we find (deformation ξ has the first order in the small)

$$\frac{\partial n_1}{\partial t}+div\left(n_0\dot{\bar{\xi}}\right)=0,$$

and if n_0 - is the equilibrium then plasma density is assumed independent on time then we have

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$$n_1 = -div\left(n_0 \bar{\xi}\right) \qquad (5)$$

Further,

$$\frac{\partial}{\partial t}\left[p_0 n_0^{-\gamma} + p_1 n_0^{-\gamma} - p_0 \gamma \frac{n_1}{n_0} n_0^{-\gamma}\right] + \\ + \frac{\partial \bar{\xi}}{\partial t} \nabla\left(p_0 n_0^{-\gamma}\right) = 0$$

and if the value $p_0 n_0^{-\gamma}$ is assumed to be independent on time finally we find:

$$p_1 = -\gamma p_0 div\bar{\xi} - \left(\bar{\xi} \nabla p_0\right) \qquad (6)$$

If the internal magnetic field inside the plasma bunch can be expressed by the displacement $\bar{\xi}$, then the closed system of equations describing the considered problem will be obtained. To solve this problem we shall suppose that focusing r.f. fields are harmonic ones dependent on time as $e^{i\omega t}$. Deformations of the bunch are supposed to be variables in time but already having temporary dependence on frequency Ω. Movement of the bunch boundary with the temporary dependence $e^{i\Omega t}$ will result in destruction of rigorously harmonic dependence of the field on time harmonics with frequencies $\omega \pm \Omega, \omega \pm 2\Omega$ will appear together with oscillations of frequency ω and so. on. It is perfectly clear that ponderemotor action of the r.f. field can expressed by some average movement of the electromagnetic field only in cases when oscillations frequency of ellipsoid boundaries Ω is the small value in comparison with the field frequency

$$\Omega << \omega \qquad (7)$$

In this sense to find the self-consistent field in a plasma bunch one can use integral equations of macroscopic electrodynamics in quasistable approximation having the shape (the equation of electrostatics $ka << 1$ [1])

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + \\ + \frac{1}{4\pi} graddiv \int_V \left(\frac{\hat{\varepsilon}_b}{\varepsilon_1} - 1\right) \vec{E}(\vec{r}\,') \frac{d\vec{r}\,'}{|\vec{r} - \vec{r}\,'|} \qquad (8)$$

Let $\bar{E}(\vec{r})$ -is the electrical field inside the plasma bunch with non-disturbance geometry (the volume of a plasma bunch is V_0). In the deformed plasma bunch the internal field is $\vec{E}(\vec{r}) = \vec{\mathrm{E}}(\vec{r}) + \vec{E}\,'(\vec{r})$ where $\vec{E}\,'(\vec{r})$ - is the internal field changing due to bunch deformation:

$$V = V_0 + V'$$

with

$$V' = \oint_S \vec{\xi} d\vec{s} \qquad (9)$$

where S - is the area of the bunch.

Moreover for deformation of an ellipsoid its density changes, i.e. tensor components of dielectric plasma permeability's change also as

$$\hat{\varepsilon}_в = \hat{\varepsilon} + \hat{\varepsilon}' \qquad (10)$$

Consequently, the field disturbance in case of bunch deformation is determined by the integral equation

$$\vec{E}'(\vec{r}) = \vec{E}_0'(\vec{r}) + \\ + \frac{1}{4\pi} graddiv \int_{V_0} \left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right) \vec{E}'(\vec{r}\,') \frac{d\vec{r}\,'}{|\vec{r} - \vec{r}\,'|} \qquad (11)$$

where the integral is taken already over non-deformed (undeformed) ellipsoid volume and all the peculiarities connected with its deformation have been carried into the free term of the equation $\bar{E}_0'(\vec{r})$:

$$\vec{E}_0'(\vec{r}) = \frac{1}{4\pi} graddiv \left\{ \oint_S \left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right) \vec{E}(\vec{r}\,') \frac{\vec{\xi}(\vec{r}\,') d\vec{s}}{|\vec{r} - \vec{r}\,'|} - \\ - \int_{V_0} \frac{\partial \hat{\varepsilon}}{\partial n} \vec{E}(\vec{r}\,') \frac{div\left(n_0 \vec{\xi}(\vec{r}\,')\right) d\vec{r}\,'}{|\vec{r} - \vec{r}\,'|} \right\} \qquad (12)$$

Consequently, equations (8)-(12) permit to express forces influencing on plasma through ellipsoid deformation $\vec{\xi}(\vec{r})$, that is to reduce the considered problem to the system of linear algebraic equations.

The general scheme of calculations is that. Thus, there is the orthonormalized system of functions $\vec{\xi}_{mn}(\vec{r})$, determined on the surface of the ellipsoid (1) (the Lame function) arbitrary disturbances $\vec{\xi}(\vec{r})$ of the surface can be expanded over these functions:

$$\vec{\xi}(\vec{r}) = \sum_{m,n} a_{m,n}(t) \vec{\xi}_{m,n}(\vec{r}) \qquad (13)$$

Consequently,

$$\frac{d^2 a_{mn}}{dt^2} = -\left\{ \oint_{S_0} \frac{1}{n_0 M} \nabla\left(p + p_{‰,‹}\right) \vec{\xi}^{\,*}_{mn}(\vec{r}\,') d\vec{s}\,' \right\} \qquad (14)$$

where the operator $\nabla\left(p + в_{в,г}\right)$ is represented as the linear function $\vec{\xi}(\vec{r})$, so that to find values $a_{mn}(t)$ we have the system of linear differential equations with constant coefficients which at least can be researched always numerically with the given degree of an accuracy.

It is easy to see that the considered system gives the possibility to investigate equilibrium and stability of the plasma bunch not only in the long wave approximation but in case of arbitrary relations between a and λ, and even when the plasma bunch itself is in the limited volume or in the r.f. resonator.

THE CASE OF A SMALL INCOMPRESSIBLE ELLIPSOID

For the incompressible bunch of particles there is

$$div\vec{\xi} = 0, \qquad (15)$$

all the calculations are simplified and therefore such a case is expediently investigated separately.

From (5) and (15) it follows that $n_1 = 0$, and the vector of deformation $\vec{\xi}(\vec{r},t)$ has sense to find in the class of harmonic functions

$$\vec{\xi} = +grad\varphi, \qquad (16)$$
$$\Delta\varphi = 0$$

inside V_0. Then the fist equation of motion (2) is reduced to the elementary equation

$$n_0 M \frac{\partial^2 \varphi}{\partial t^2} = -\left(p + p_{в,г}\right) + C$$

where the constant C characterizes only relativity of a choice of zero level of the effective potential energy. So for the incompressible bunch. $n = const$ then in case of the isothermal bunch

$$\nabla p = 0$$

and therefore one can seek

$$-\left(p + p_{‰,‹}\right) + C =$$
$$= -\frac{1}{16\pi}\left(E_i^* E_k' + E_i^{*\prime} E_k\right)\frac{\partial \varepsilon_{ik}}{\partial n} n \qquad (17)$$

where the disturbance electric field E_k' is expressed through the internal non-disturbance field in plasma E_k by means of the following equation

$$\vec{E}'(\vec{r}) = \frac{1}{4\pi} graddiv \oint_S \left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right)\vec{E}(\vec{r}')\frac{(grad\varphi d\vec{s})}{|\vec{r} - \vec{r}'|} +$$
$$+ \frac{1}{4\pi} graddiv \int_{V_0} \left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right)\vec{E}'(\vec{r}')\frac{d\vec{r}'}{|\vec{r} - \vec{r}'|}$$

If to find $\vec{E}'(\vec{r})$ in the form of

$$\vec{E}'(\vec{r}) = grad\, div\, \vec{A}, \qquad (18)$$

then for $\vec{A}$ we obtain the equation

$$\vec{A} = \frac{1}{4\pi}\oint_S \left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right)\vec{E}(\vec{r}')\frac{(grad\varphi d\vec{s})}{|\vec{r} - \vec{r}'|} +$$
$$+ \frac{1}{4\pi}\int_{V_0}\left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right)\frac{graddiv\vec{A}}{|\vec{r} - \vec{r}'|} d\vec{r}' \qquad (19)$$

The free term of this equation can be also represented in the form of the integral over non-disturbance ellipsoid volume V_0. For example, for the x- component we find:

$$\oint_S \left(\left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right)\vec{E}(\vec{r}')\right)_x \frac{(grad\varphi d\vec{s})}{|\vec{r} - \vec{r}'|} =$$
$$= \frac{1}{4\pi}\int_{V_0} grad\varphi \left\{\frac{grad\left(\left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right)\vec{E}(\vec{r}')\right)_x}{|\vec{r} - \vec{r}'|}\right\} d\vec{r}' -$$
$$- div \int_V \frac{\left(\left(\frac{\hat{\varepsilon}}{\varepsilon_1} - 1\right)\vec{E}\right)_x grad\varphi\, d\vec{r}'}{|\vec{r} - \vec{r}'|}$$

All the further calculations have been built for following known properties of volume potential:

$$W_{n+2}(\vec{r}) = \int_V \frac{\varphi_n(\vec{r}')d\vec{r}'}{|\vec{r} - \vec{r}'|} \qquad (20)$$

If the integrand function of the potential (20) for internal points of ellipsoid volume (1) is the power function of the Cartesian coordinates of the n-th degree, the volume potential itself (20) for internal points of an ellipsoid will be the power function of the Cartesian coordinates of the degree $n+2$ [3]. These potentials have been tabulated and in case of ellipsoids of rotation coefficients of those are expressed by the elementary function of ellipsoid semiaxes.

Thus, if the free term of the integral equation (19) is the power function in the Cartesian coordinates then it is necessary to find the solution of this equation among the power functions of the Cartesian coordinates.

It is clear that one chooses the function $\varphi(\vec{r},t)$ in the shape of power functions (and components of non-disturbance field in a small ellipsoid have already the form of power functions), then in this case the solution of the integral equation (19) has the form of any power functions of coordinates. In this case solution, which have components of the vector $\vec{A}$ being power functions not lower than the second order are of practical interest.

CONCLUSION

The total scheme of investigation of equilibrium and stable configurations with the geometry close to the ellipsoidal one has been developed. As an example of the free incompressible homogeneous plasma ellipsoid it is shown how to build this problem solution taking into account field changes in plasma for self-consistent setting, so far as in a small ellipsoid the internal effective potential energy is the quadratic function of the Cartesian coordinates, to provide the bunch equilibrium it is sometimes necessary to provide its rotation as the whole. Using of integral equations of macroscopic electrodynamics permits to reduce in general case the problem for equilibrium of the ellipsoidal bunch in the external r.f fields to searching of stable solutions of the infinite system of linear algebraic equations in general case.

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