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STOCHASTIC HEATING OF ELECTRONS BY ELECTROMAGNETIC WAVES

V.V.Ognivenko

NSC, Kharkov Institute of Physics & Technology, Kharkov, Ukraine,

e–mail: ognivenko@kipt.kharkov.ua

In the present paper the nonlinear dynamics of electrons motion in the fields of two combinational waves or in the fields of one combinational wave and a longitudinal wave is considered. The mechanism of electrons energy grows under an overlap of two nonlinear resonances is investigated and upper limit of the electron energy is determined.

1. Introduction

The investigation of electromagnetic waves interaction with charge particles is of interest for number applications in particular for plasma heating and a generation of electromagnetic radiation [1–4]. In the paper [4] the method of stochastic plasma heating in the field of several (more then three) external electromagnetic waves is considered. A gain in energy of electrons in this method of heating is due to a stochastic instability of electron motion under the overlap of nonlinear resonances of combinational waves, which are formed by nonlinear interaction of pairs electromagnetic waves with electrons of plasma. The stochastic instability of relativistic electron motion in the field of several electromagnetic waves and periodic transverse magnetic field is studied in [3]. The purpose of the present paper is the determination of the upper limit of electron energy, which they can acquire in stochastic heating, when two nonlinear resonances overlap one other. The nonlinear dynamics of electrons in the fields of two combinational waves or in the fields of one combinational wave and a longitudinal wave is considered. It is shown, when two nonlinear resonances are overlap, the maximal energy of electrons is equal to the energy corresponding to the sum of those two nonlinear resonances.

2. The equations of motion

Let us consider a motion of electrons in the fields of several monochromatic electromagnetic waves:

$$\mathbf{E}=\sum_n \mathbf{e}_x A_n^t \cos\left(k_n z-\omega_n t\right)+\mathbf{e}_z A^{\ell}\sin\varphi_l \ , \qquad (1)$$

where A_n^t , A^{ℓ} – are the amplitudes of waves; upper indexes "t" and "ℓ" denote the external transverse electromagnetic waves and a longitudinal one, respectively; $\varphi_\ell = k_\ell z-\omega_\ell t$; $\omega, \mathbf{k}$ are the frequency and wavenumber; $\mathbf{e}_x , \mathbf{e}_z$ are the unit vectors in the *x* and *z* directions.

The energy of electrons, which we shall represent by its relativistic factor $\gamma=\left(1-\beta^2\right)^{-1/2}$ (where $\beta=\mathbf{v}/c$), evolves according to the equations:

$$\frac{d\gamma}{dt}=\frac{e}{mc^2}(\mathbf{vE}) \ , \qquad \frac{d\mathbf{r}}{dt}=\mathbf{v} \ . \qquad (2)$$

A motion of electrons in the fields (1) can be presented as the fast oscillations in the fields of the transverse electromagnetic waves and slow motion in the field of longitudinal wave or combinational wave formed by pairs of transverse waves. Taking the oscillating part of the electron velocity from conservation of canonical transverse momentum in the transverse electromagnetic waves and substituting its in eq. (2), we obtain the following equations for a slow motion of electrons:

$$\frac{d\gamma}{dt}=\frac{1}{2\gamma}\sum_{m,n} a_m a_n\Big[\left(\omega_m-\omega_n\right)\sin\theta_{mn}^{(-)}- -\left(\omega_m+\omega_n\right)\sin\theta_{mn}^{(+)}\Big]+\frac{e\beta_z A^{\ell}}{mc}\sin\varphi_\ell \ . \qquad (3)$$

$$\frac{dz}{dt}=v_z \ , \qquad (4)$$

where $\theta_{mn}^{(\pm)}=\left(k_m \pm k_n\right)z-\left(\omega_m \pm \omega_n\right)t$, $a_n=\dfrac{eA_n^t}{mc\omega_n}$.

If $A^{\ell}=0$ the eq. (3) becomes the same as in [3,4].

The first terms on the right–hand side of eq. (3), proportional to the square of the wave amplitudes, describe a motion of electrons under conditions of combinational resonance of electrons with external transverse electromagnetic waves:

$\omega_m \pm \omega_n = v_z\left(k_m \pm k_n\right)$.

The last term on the right–hand side eq. (3) is responsible for a Cherenkov resonance interaction of the electrons with a longitudinal wave. $\omega_\ell = k_\ell v_z$.

We consider below two cases. In the first case (a) there are only three external electromagnetic waves: the wave E_1 propagates in positive *z* direction, and the both waves E_2 and E_3 propagate in the opposite direction:

$\omega_1-\omega_2=v_z\left(k_1+k_2\right)$, $\omega_1-\omega_3=v_z\left(k_1+k_3\right)$.

In the second case (b) there are two electromagnetic waves E_1, E_2 (the same as in the case (a)) and the longitudinal wave E^{ℓ} :

$\omega_1-\omega_2=v_z\left(k_1+k_2\right)$, $\omega_\ell = k_\ell v$.

For those two cases the equations of motion (3), (4) in dimensionless variables become:

$$\frac{dw}{d\tau}=\varepsilon_1\sin\varphi+\varepsilon_2\sin\left(\kappa\varphi+\Delta_2\tau\right) \ , \qquad (5a)$$

$$\frac{d\varphi}{d\tau} = w + \Delta_1 \ . \qquad (5b)$$

The initial condition for the eqs. (5a), (5b) is: $w(0) = 0$.

Where $w = \gamma - \gamma_0$, $\varphi = k_c z - \omega_c t$, $\tau = ck_c t / (\beta_0 \gamma_0^3)$,

$\varepsilon_1 = \frac{a_1 a_2 \omega_c \beta_0 \gamma_0^2}{2 k_c c}$, $\Delta_1 = \left(1 - \frac{\omega_c}{v_0 k_c}\right)\beta_0^2 \gamma_0^3$ $\omega_c = \omega_1 - \omega_2$, $k_c = k_1 + k_2$.

In the case (a): $\varepsilon_2 = a_1 a_3 \beta_0 \gamma_0^2 \frac{(\omega_1 - \omega_3)}{2c(k_1 + k_3)}$,

$\kappa = \frac{k_1 + k_3}{k_c}$, $\Delta_2 = \left(\frac{\omega_c}{k_c} - \frac{\omega_1 - \omega_3}{k_1 + k_3}\right)\frac{\beta_0 \gamma_0^3 (k_1 + k_3)}{k_c c}$.

In the case (b): $\varepsilon_2 = \frac{eA_\ell \beta_0^2 \gamma_0^3}{mc^2 k_c}$, $\kappa = \frac{k_\ell}{k_c}$,

$\Delta_2 = \left(\frac{\omega_c}{k_c} - \frac{\omega_\ell}{k_\ell}\right)\frac{\beta_0 \gamma_0^3 k_\ell}{k_c c}$.

The nonlinear equations (5a), (5b) have general form and describe a stochastic instability of the charge particles motion in the different electromagnetic waves [3–5].

3. The overlap of resonances

Let us obtain the analytical estimations for the rate of electron energy increase and the upper limit of electrons energy. For the electron moving in the field of the first combinational wave (ε_1) the half–width of the nonlinear resonance in energy $(\delta w)_1$ and in wave number $(\delta\kappa)_1$ (in dimensionless variable) are:

$$(\delta\kappa)_1 = (\delta w)_1 = \sqrt{2\varepsilon_1} \ . \qquad (6a)$$

In the field of second wave (ε_2) the half–width of nonlinear resonance is equal to:

$$(\delta\kappa)_2 = \kappa (\delta w)_2 = \sqrt{2\kappa\, \varepsilon_2} \ . \qquad (6b)$$

The total width of nonlinear resonance in the fields ε_1 and ε_2 is:

$$\delta\kappa = (\delta\kappa)_1 + (\delta\kappa)_2 \ . \qquad (7a)$$

The distance between isolated resonance of those waves is

$$\Delta\kappa = (1 - \kappa)\Delta_1 - \Delta_2 \ . \qquad (7b)$$

The stochastic instability arises when separatrices of nonlinear resonances touch each other [6]:

$$\delta\kappa > |\Delta\kappa| \ .$$

Using (7a) and (7b) the condition for the overlap of nonlinear resonance takes the form:

$$\sqrt{2\varepsilon_1} + \sqrt{2\kappa\, \varepsilon_2} > \left|(1 - \kappa)\Delta_1 - \Delta_2\right| . \qquad (8)$$

When the overlap of nonlinear resonance takes place the motion of electrons becomes random. Assuming that condition (8) is satisfied and phases of electrons are random one can obtain the estimation for the rate of electron energy growth [4]:

$$\langle w^2 \rangle = \pi \left(\frac{a_1 a_2}{2\gamma_0}\right)^2 \omega_c t + \left(\frac{e\beta_0}{mc} A^\ell\right)^2 \frac{t}{\omega_\ell} \ .$$

For the determination of maximal energy of electrons, which they can acquire under the overlap of resonances, we take into account that electrons can move only from one resonance to other. Therefore the energy of electron grows up to the value corresponding to the total width of two nonlinear resonances in energy units:

$$w_{\max} = 2\left(\sqrt{2\varepsilon_1} + \sqrt{2\varepsilon_2 / \kappa}\right) . \qquad (9)$$

Thus the saturation of electron energy grows in the fields of two wave ε_1, ε_2 takes place as a result of the electron trapping in a potential well formed by a total nonlinear resonance of these two waves.

4. Numerical results and discussion

The nonlinear dynamics of individual particle motion, in according to eqs. (5a), (5b), has been carry out numerically for the following value of parameters: $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.06$, $\kappa = 0.2$, $\Delta_1 = 0.02$, $\Delta_2 = 0.3$, and several initial phases $\varphi(0)/2\pi = 0.3$ (fig.1); 0.7 (fig.2); 0.8 (fig.3), 1.1 (fig.4). It is consider the case when criterion of stochastic motion (8) is satisfied and nonlinear resonances are overlap.

In the figures 1–4 the square of normalized energy w^2 as a function of dimensionless time τ is presented.

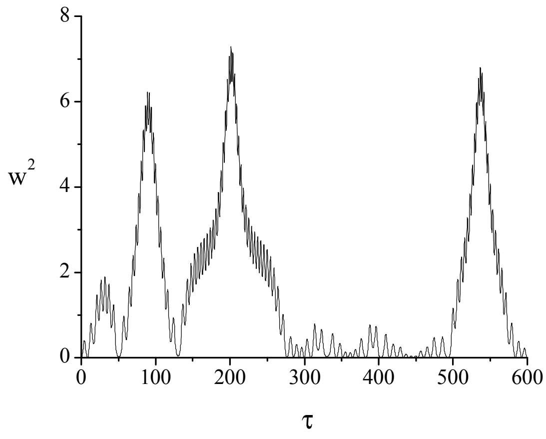

Fig. 1

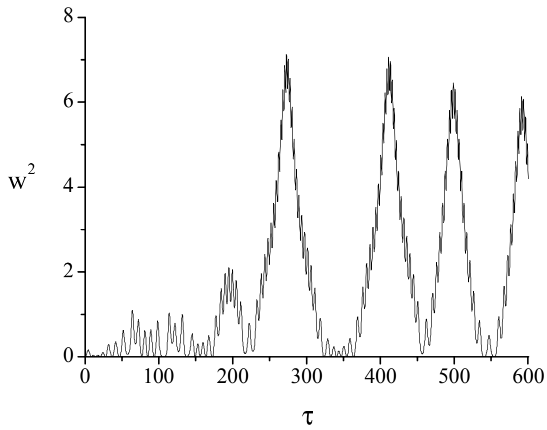

Fig.2

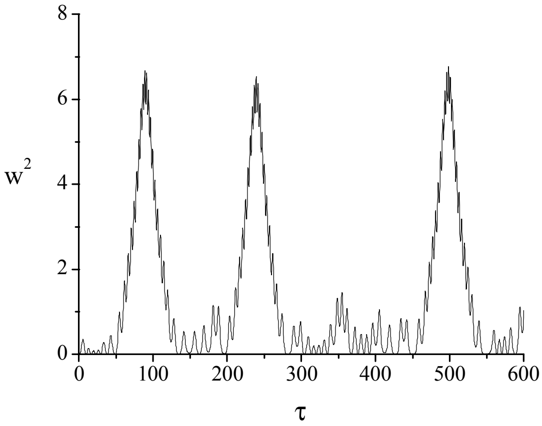

Fig.3

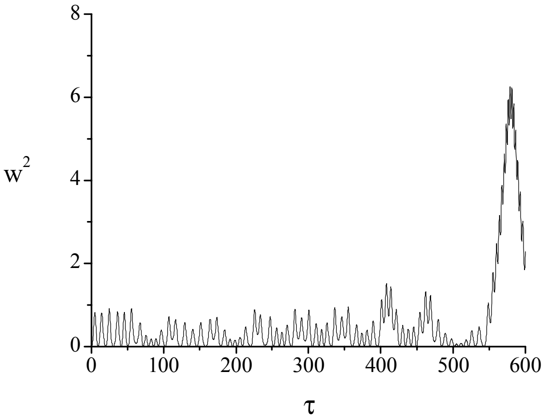

Fig.4

It is shown that motion of particles is substantially irregular. At first the energy of electrons grows slightly that is due to the oscillation of electrons in potential well of first wave ε_1 and a weak influence of the second wave ε_2. When the energy of electron increases it approaches to the separatrix and the influence of the second wave becomes the same as the first one. Since the resonances of these waves are overlap the electron can move from one resonance to other and its energy grows up to the peak value. Then the energy of electrons decreases down to not far from initial value. For the some value of initial phases (fig.2, fig.3) this process is repeated. The results of the numerical calculation depend strongly from initial parameters, as the motion of electrons is stochastic. A gain in energy of electrons in considering case is due to a stochastic motion of electrons in potential well formed by sum of nonlinear resonances of the waves ε_1 and ε_2. Substituting the value of parameters into eq. (9) we obtaine $w_{\max}^2 = 7.92$. The maximal value of energy obtained by numerical calculation well agrees with analytical result.

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