CALCULATION OF COUPLING COEFFICIENT CAUSED BY VERTICAL DISPERSION

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Sources of horizontal-vertical coupling of transverse oscillations in storage rings are misalignments and tilts of bending magnets, quadrupoles and sextupoles. The coupling arises from two separate factors: spurious vertical dispersion generated by misalignment errors and skew-quadrupole effects generated by tilts of magnet elements. It is sensible to consider the effect of these factors to vertical emittance apparently with the aim to determine an influence of the each factor and to define methods of the coupling correction.

In this paper a method of a coupling coefficient determination caused by spurious vertical dispersion in an electron storage ring is presented. The method is base on a matrix formalism of a coupling motion description and allows take distribution function of the coupling coefficient. The results of the coupling coefficient calculations for the Kharkov source of synchrotron radiation (UNK) are presented.

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1. INTRODUCTION

Projects of electron storage rings created at the end of 60-70 years stimulated an interest in coupling of horizontal and vertical betatron oscillations. First investigations considered the coupling rose from random tilts of quadrupole magnets, which introduce skew quadrupole field errors around a ring. Some time latter an analysis of influence of axial fields of solenoid used as physics detector was carried out. The references [1, 2] are early examples of the analysis of a skew quadrupole effect in electron storage rings. Despite the quite long history, existing literature on the theory of the coupling is not large. It caused by nature of the process, which is partly analytical, but for the main part computational, because the nature of the problem is in the statistical changes of focusing parameters of the storage ring lattice. But there are some useful works by G.Guinard, who had adopted Hamiltonian formalism and used a perturbation theory for estimation of a vertical beam emittance produced by betatron coupling effects [3, 4]. These works shown that the coupling arise from two separate sources, the spurious vertical dispersion generated by displacements of magnet elements of the ring (bending magnets, quadrupole magnets and sextupoles) and by skew quadrupole effects. There means all fields leading to the mixing of oscillation planes. Such fields rise from rotation of magnet elements or when solenoidal fields are present.

The main results by G. Guignard are set out in following:

Skew quadrupoles and longitudinal fields effect to the coupling coefficient is determine by one from the factors:

$$h_{2Q_x} = \frac{1}{32\pi R}\int_0^{2\pi} M^2(\vartheta)\beta_x(\vartheta)e^{i[2(\varphi_x - Q_x\vartheta)+q\vartheta]}\,d\vartheta,$$

$$h_{2Q_z} = \frac{1}{32\pi R}\int_0^{2\pi} M^2(\vartheta)\beta_z(\vartheta)e^{i[2(\varphi_z - Q_z\vartheta)+q\vartheta]}\,d\vartheta,$$

$$h_{Q_x-Q_z} = \frac{1}{32\pi R}\int_0^{2\pi}\left\{K(\vartheta)+\frac{M(\vartheta)}{2}\left[\frac{\beta'_z(\vartheta)}{2\beta_z(\vartheta)}-\frac{\beta'_x(\vartheta)}{2\beta_x(\vartheta)}\right] - i\frac{M(\vartheta)}{2}\left[\varphi'_z+\varphi'_x\right]\right\}e^{i[2(\varphi_x-\varphi_z-(Q_x-Q_z))+q\vartheta]}\,d\vartheta,$$

$$h_{Q_x+Q_z} = \frac{1}{32\pi R}\int_0^{2\pi}\left\{K(\vartheta)+\frac{M(\vartheta)}{2}\left[\frac{\beta'_z(\vartheta)}{2\beta_z(\vartheta)}-\frac{\beta'_x(\vartheta)}{2\beta_x(\vartheta)}\right] + i\frac{M(\vartheta)}{2}\left[\varphi'_z-\varphi'_x\right]\right\}e^{i[2(\varphi_x+\varphi_z-(Q_x+Q_z))+q\vartheta]}\,d\vartheta, \quad (1)$$

where $K(\theta)$ is the skew quadrupoles component of a magnetic field strength, $M(\theta)$ is he longitudinal component of a magnetic field strength, $\beta_x(\theta)$, $\beta_z(\theta)$, $\beta_x'(\theta)$, $\beta_z'(\theta)$ are the amplitude functions of betatron oscillations and their derivatives, φ_x, φ_z, φ_x', φ_z' are the phase of betatron oscillations and their derivatives, R is the average bending radius of the ring, q is the harmonics number.

The parameters h_i are resonance parameters, i.e. its values depend from nearness of working frequencies to one from the resonance conditions $2Q_x - q \sim \delta$, $2Q_z - q \sim \delta$, $Q_x - Q_z - q \sim \delta$, $Q_x - Q_z + q \sim \delta$. G.Guignard shown, that in the case, when parameters h_i are vanished the coupling coefficient K, determined as ratio of vertical emittance ε_z and horizontal emittance ε_x, can be represented by:

$$K = \frac{\langle I_z\rangle}{\langle I_x\rangle}, \quad (2)$$

where $I = (\beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2)$ is Curant Snyder invariant in X and Z planes, β, $\alpha = -\frac{1}{2}\beta'(s)$, $\gamma = \frac{1+\alpha^2}{\beta}$ are Twiss parameters, η, η' are dispersion function and its derivative in X and Z plane respectively. The averages are over all bending magnets.

In other limiting case of vanishing vertical dispersion, the coupling coefficient can write as:

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$$K = \frac{\left|\frac{h_i}{\delta}\right|^2}{\left(\left|\frac{h_i}{\delta}\right|^2 + \frac{1}{2}\right)}. \quad (3)$$

Thus, we can divide the problem of a coupling coefficient determination in two different parts:

1. Determination of the coupling coefficient caused by focusing functions changes and a rise of vertical dispersion and finding of the way to control the coupling.
2. Determination of the coupling coefficient caused by nearness to the working tune of the storage ring to any first order resonance.

In the present paper a method of determination of coupling coefficient caused by vertical dispersion is described.

2. CALCULATION OF COUPLING COEFFICIENT

The idea to use transport matrix 7x7 for calculations of linear electron beam parameters in a storage ring was used in SYNCH [5] program. Subsequently, in SLIM program the same approach was used by A.Chao [6]. These programs were written in thin lens approximation that limited its possibilities.

We used the solutions of a general equation of motion of charged particles in external field in a natural coordinates system. As a result, transport matrixes in thick lens approximation were obtained for main elements of a storage ring lattice and used in DeCA code [7]. For investigation of coupling effects matrixes of following electromagnetic elements will be used: quadrupole magnet, bending magnet, corrector, perturbations matrixes.

For determination of coupling coefficient caused by vertical dispersion, we will to find fifth synchrotron integrals $I_{5x,z}$ in both plans of oscillations:

$$I_{5x,z} = \oint \frac{H_{x,z}(s)}{|R^3|} \mathrm{d}s = \sum_{i=1}^{N} \frac{1}{|\rho_i^3|} \int_{s_i}^{s_i+L_i} H_{ix,z}\left(\eta_{x,z}(s), \eta'_{x,z}(s), \beta_{x,z}(s), \beta'_{x,z}(s)\right) \mathrm{d}s, \quad (4)$$

where N is the number of lattice elements, ρ_i is the bending radius in the i-th lattice element, $H_{ix,z} = \gamma_{ix,z}(s)\,\eta^2_{ix,z}(s) + 2\alpha_{ix,z}(s)\,\eta_{ix,z}(s)\,\eta'_{ix,z}(s) + \beta_{ix,z}(s)\,\eta'_{ix,z}(s)$ is Curant Snayder invariant in the i-th lattice element.

It is necessary to note, since the quadrupole magnets displaced the dipole component of magnetic field on the reference orbit is appeared and integration must to conduct not only in perturbed bending magnets but and in quadrupole magnets too.

Obviously, for finding of invariant (4) $H_{ix,z}$ it is necessary to write a low of dispersion function $\eta_{ix,z}$ transformation and a low of Twiss parameters $\alpha_{ix,z}$, $\beta_{ix,z}$, $\gamma_{ix,z}$ transformation in the each lattice element.

The periodic solution for dispersion function $\eta_{ix,z}$ could be fined from equation system that in matrix form could be wrote by :

$$\begin{pmatrix} \eta_{ix} \\ \eta'_{ix} \\ \eta_{iz} \\ \eta'_{iz} \\ 1 \\ 1 \\ 1 \end{pmatrix} = M_{itot} * \begin{pmatrix} \eta_{ix} \\ \eta'_{ix} \\ \eta_{iz} \\ \eta'_{iz} \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad (5)$$

where M_{itot} is the transport matrix per turn at the i-th azimuth.

In the case of the weak coupling, when the matrix elements of the total transport matrix m_{13}, m_{14}, m_{23}, m_{24}, m_{31}, m_{32}, m_{41}, m_{42} << 1 and when bending magnets in vertical plane and strong solenoids that introduce into lattice big vertical dispersion are absent, equation (5) could be divide in two independent systems of equations:

$$\begin{cases} m_{i11}\eta_{ix} + m_{i12}\eta'_{ix} + m_{i17} = \eta_{ix} \\ m_{i21}\eta_{ix} + m_{i22}\eta'_{ix} + m_{i27} = \eta_{ix} \end{cases} \quad \begin{cases} m_{i33}\eta_{iz} + m_{i34}\eta'_{iz} + m_{i37} = \eta_{iz} \\ m_{i43}\eta_{iz} + m_{i44}\eta'_{iz} + m_{i47} = \eta_{iz} \end{cases}. \quad (6)$$

The solutions of these systems relative to $\eta_{ix,z}$ are follow:

$$\begin{cases} \eta_{ix} = \dfrac{m_{12}m_{27} - m_{22}m_{17} + m_{17}}{2(1-\cos\mu_x)} \\ \eta'_{ix} = \dfrac{m_{21}m_{17} - m_{11}m_{27} + m_{27}}{2(1-\cos\mu_x)} \end{cases}, \quad (7)$$

$$\begin{cases} \eta_{iz} = \dfrac{m_{34}m_{47} - m_{44}m_{37} + m_{37}}{2(1-\cos\mu_z)} \\ \eta'_{iz} = \dfrac{m_{43}m_{37} - m_{33}m_{47} + m_{47}}{2(1-\cos\mu_z)} \end{cases}, \quad (8)$$

In a case of a strong coupling the system (5) must be resolve jointly. In that case a numerical solution must be use for best result, because an analytical solution is too ponderous.

When dispersion functions $\eta_{ix,z}$ are found, it possible to write transformation law of $\eta_{ix,z}$ on i-th lattice element:

$$\begin{pmatrix} \eta_{(i+1)x} \\ \eta'_{(i+1)x} \\ \eta_{(i+1)z} \\ \eta'_{(i+1)z} \\ 1 \\ 1 \\ 1 \end{pmatrix} = M_i * \begin{pmatrix} \eta_{ix} \\ \eta'_{ix} \\ \eta_{iz} \\ \eta'_{iz} \\ 1 \\ 1 \\ 1 \end{pmatrix}. \quad (9)$$

It can receive the law of dispersion function transformation on i-th lattice element by substituting instead M_i a matrix of perturbed magnetic element.

The law of Twiss parameters transformation $\alpha_{ix,z}$, $\beta_{ix,z}$, $\gamma_{ix,z}$ is set by well known expression:

$$\begin{pmatrix} \alpha_{(i+1)x,z} \\ \beta_{(i+1)x,z} \\ \gamma_{(i+1)x,z} \end{pmatrix} =$$

$$\begin{pmatrix} m^2_{i\,\substack{11\\33}} & -2m_{i\,\substack{12\\34}}\,m_{i\,\substack{11\\33}} & m^2_{i\,\substack{12\\34}} \\ -m_{i\,\substack{11\\33}}\,m_{i\,\substack{21\\43}} & m_{i\,\substack{12\\34}}\,m_{i\,\substack{21\\43}} + m_{i\,\substack{22\\44}}\,m_{i\,\substack{11\\33}} & -m_{i\,\substack{12\\34}}\,m_{i\,\substack{22\\44}} \\ m^2_{i\,\substack{21\\43}} & -2m_{i\,\substack{22\\44}}\,m_{i\,\substack{21\\43}} & m^2_{i\,\substack{22\\44}} \end{pmatrix} \quad (10)$$

$$* \begin{pmatrix} \alpha_{ix,z} \\ \beta_{ix,z} \\ \gamma_{ix,z} \end{pmatrix},$$

where m_i are the elements of perturbed matrix of *i*-th lattice elements.

Substituting expressions (9-10) for each element kind into (5) and integrating it can obtain the values of synchrotron integrals $I_{5x,z}$. After that, the coupling coefficient could be obtained from expression (2).

3. CALCULATION OF THE COUPLING COEFFICIENT FOR UNK STORAGE RING

The method of the coupling coefficient determination described above permitted to calculate distribution function of the coupling coefficient caused by displacements, tills and rotations of magnet elements for the Kharkov accelerator-storage complex UNK.

15 000 independent statistical samplings of displacements and rotations of lattice elements were made with Gaussian distribution function. Dispersion of distribution in each case (i.e. RMS errors of installation of lattice elements) was equal 100 mkm for transverse displacements and 200 mkrad for rotary displacements. The coupling coefficient was calculated after each statistical sampling and its value was got in certain numerical interval. After that the positive event in that interval was registered. The distribution function of the coupling coefficient was formed as a result of it. The most probable event (center of the distribution) for the coupling coefficient and width of the distribution function were determined. The calculations were carried out apparently for perturbations of bending magnets, quadrupole magnets and sextupole magnets. The calculations for determination of summarized effect of all magnet elements perturbations were carried out too. Some kinds of errors do not produce vertical dispersion but could affect to coupling that has been generated by others sources. For these reasons it difficult to be precise as to the exact contribution from each source. The distribution functions of coupling coefficient received as a result of calculations shown in Fig. 1-4. It can assume that resonance part of the coupling coefficient is small enough because of operation turns for UNK Kharkov storage ring are $Q_x = 7,202$ and $Q_z = 4,227$, i.e. values of detuning $\Delta = Q_x - Q_z$ and $\Delta = Q_x + Q_z$ is large so as to varnish coefficients h_i in (1). So we can estimate contribution of each kind of the lattice elements errors in the coupling coefficient.

Quadrupole errors.

Quadrupole errors introduce the vertical dispersion by both ways as trough rotations as trough displacements. The quadrupole rotation errors introduce into the ring small skew quadrupoles that couple the horizontal dispersion into the vertical plane. Vertical positioning errors in quadrupoles introduce vertical dispersion straightly because horizontal dipole magnet field appears. In the case of UNK Kharkov lattice the affect of vertical displacement is not so essential. Contribution of rotation errors in the coupling coefficient value is more essential but it summarize value is not too large and is $8.191*10^{-8}$ (Fig.1).

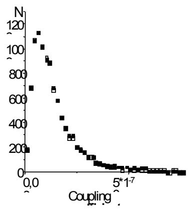

Fig. 1*. Distribution function of the coupling coefficient caused by misalignment errors of quadrupole magnets. RMS of transverse displacement is 100 mkm. RMS of rotary errors is 200 mkrad. The center of distribution (the coupling coefficient value) is* $8.191*10^{-8}$. *Distribution width is* $1.4334*10^{-7}$

Bending magnet errors

Dipole rotation errors generate vertical dispersion that affect the equilibrium close orbit in the ring. The separate value of the coupling coefficient due to dipole rotation error is $6.7039*10^{-7}$. But summarize effect of quadrupoles and bending magnets took into consideration the affect of bending magnet transverse displacement appears and the coupling coefficient value arises up to $6.204*10^{-6}$ (Fig. 3).

Sextupole errors

Transverse displacements of sextupoles have the effect of introducing into the ring skew quadrupole that couple the horizontal dispersion into the vertical plane. Separately this effect is not great (for UNK Kharkov the value of the coupling coefficient is about 10^{-9}) but in combination with other errors the value of the coupling coefficient essentially arise and is $2.591*10^{-4}$ (Fig.4).

that it value could be decreased by procedure of equilibrium close orbit correction. It would appear, that for the UNK Kharkov storage ring with $Q_x=7,202$ and $Q_z=4,227$ a value of the coupling coefficient caused by resonance effects of skew quadrupole fields won't be too large.

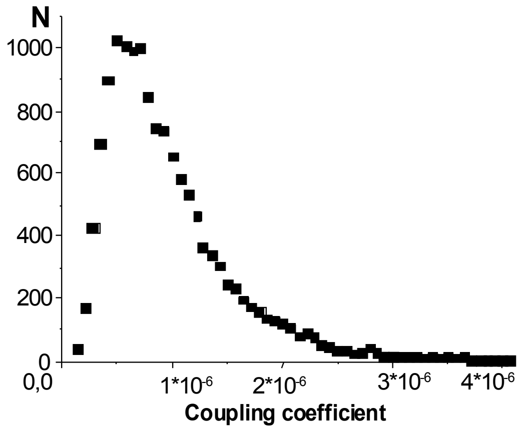

*Fig. 2. Distribution function of the coupling coefficient caused by misalignment errors of bending magnets. RMS of transverse displacement is 100 mkm. RMS of rotary errors is 200 mkrad. The center of distribution (the coupling coefficient value) is 6.7039*10^{-7}. Distribution width is 8.4455*10^{-7}*

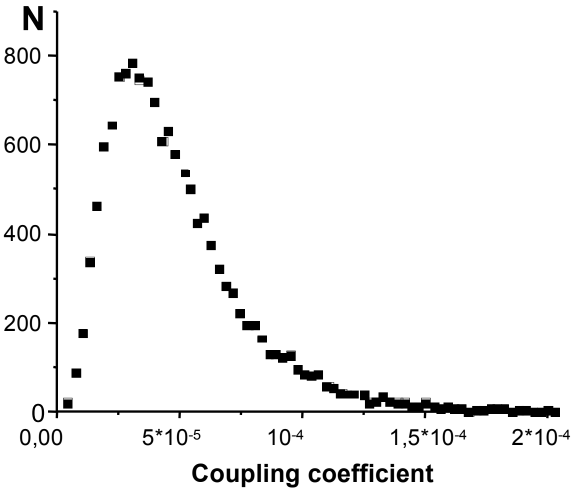

*Fig. 3. Distribution function of the coupling coefficient caused by misalignment errors of bending magnets and quadrupole magnets. RMS of transverse displacement is 100 mkm. RMS of rotary errors is 200 mkrad. The center of distribution (the coupling coefficient value) is 6.204*10^{-6}. Distribution width is 4.5942*10^{-5}*

The final figure (Fig.4) shows the results when all the different possible types of errors are incorporated. This includes both those errors that directly cause coupling and those that enhance it once it has been generated. As we could see for UNK Kharkov storage ring the value of the coupling coefficient caused by generated vertical dispersion is small enough and it could hope

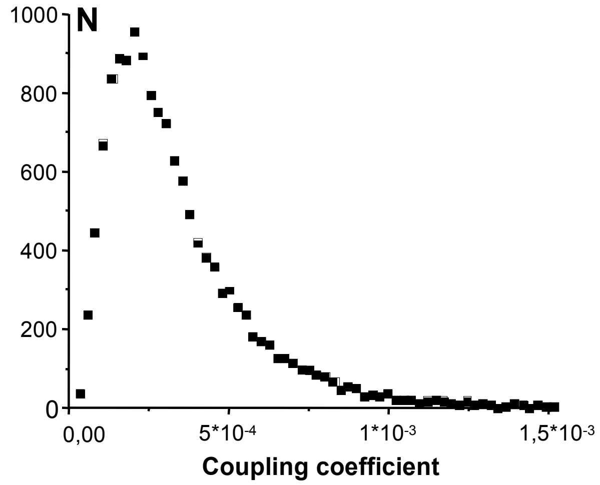

*Fig. 4. Distribution function of the coupling coefficient caused by misalignment errors of quadrupole magnets, bending magnets and sextupole magnets . RMS of transverse displacement is 100 mkm. RMS of rotary errors is 200 mkrad. The center of distribution (epy coupling coefficient value) is 2.591*10^{-4}. Distribution width is 1.5434*10^{-4}*

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