NONLINEAR DYNAMICS OF RELATIVISTIC ELECTRON BUNCH IN AN UNDULATOR

V.V. Ognivenko
National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

The dynamics of electron bunch motion in a helical undulator for the regime of self–amplification spontaneous undulator radiation is investigated theoretically using the method of self-consistent modeling the motion of electrons in the total field of their undulator radiation. The effects of bunch edge on start up processes are studied. The bunching of electrons within the radiation wavelength and within the full length of the beam is investigated. The efficiency of free–electron lasers using such beam is determined.
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As known [1,2], the regime of self–amplification spontaneous radiation from relativistic electron beams in free–electron lasers (FEL) allows one to obtain frequency tunable coherent ultrashort wavelength radiation by the use of a sufficiently intense relativistic electron beam. For shortening the wavelength of coherent radiation and generation high intensity electromagnetic radiation in FELs the more impotence takes investigations of self–amplification undulator radiation process by finite length relativistic electron beam, in particular, the electron beam the length of which is order of several wavelength of electromagnetic radiation. In this paper the dynamics of the relativistic electrons motion in the field of helical magnetic undulator and the total field of an undulator radiation of these electrons is investigated. The effects of bunch edge are studied and it is shown that self–amplification process start up from the spontaneous coherent undulator radiation of electrons moving at the trailing edge of the bunch.

Let us consider a finite length relativistic electron beam passing through a helical magnetic field of undulator: $\mathbf{H}_u(z) = H_0 \mathrm{Re}\left[\mathbf{e}_- \exp(ik_u z)\right]$, where $\mathbf{e}_- = \mathbf{e}_x - i\mathbf{e}_y$, $k_u=2\pi/\lambda_u$, λ_u is period of undulator, e_x e_y are the unit vectors along the axes OX, OY. The initial velocity of the electron beam is in the axial direction $\mathbf{e}_z v_0$.

We consider one-dimentional model assuming that in x, y directions the bunch is infinite. The pulse duration of the electron beam prior to entering the undulator field is t_b.

The electric and magnetic fields of the electron in the undulator derived from formulae for the field of charge particle moving with an acceleration [3], take the form as given in [4,5]. Assuming the beam density is independent of initial transverse coordinates (x_0, y_0) at the undulator entrance z=0 the expressions for the total field of radiation take the form:

$$\mathbf{E} = 2\pi e v_0 \beta_u \mathrm{Re}\left[\mathbf{e}_- \int dt_0 n_b(t_0) \frac{\exp(i\vartheta)}{1-\beta_0 \operatorname{sgn}(\delta z)}\right], \quad (1)$$

$$\mathbf{H} = -2\pi e v_0 \beta_u \mathrm{Im}\left[\mathbf{e}_- \int dt_0 n_b(t_0) \frac{\operatorname{sgn}(\delta z)\exp(i\vartheta)}{1-\beta_0 \operatorname{sgn}(\delta z)}\right], \quad (2)$$

where $\vartheta = \dfrac{k_u v_0}{1-\beta_0 \operatorname{sgn}(\delta z)}\left[t - \dfrac{z}{c}\operatorname{sgn}(\delta z) - t_0 - \dfrac{\Delta(z,t_0)}{v_0}\right]$, $(\delta z) = z - v_0(t-t_0) - \Delta(z,t_0)$, Δ – is the longitudinal displacement of radiating electron relative of its equilibrium trajectory, $\beta_u = K/\gamma_0$, $\gamma_0 = \left(1-\beta_0^2\right)^{-1/2}$, $K = |e|H_0/mc^2k_u$; e, m are the charge and the rest mass of electron.

The leading edge of the bunch cross the z=0 plane at time t_0=0 and t_0 integration in Eqs. (1) (2) is over region from 0 to $t - z/c$.

The axial dynamics of electrons in the undulator region z>0 determine the corresponding component of Lorentz force: $F_z^{tot} = e\beta_u\left(H_x^{tot}\sin k_u z - H_y^{tot}\cos k_u z\right)$. For the simplification of the formulae we neglect the space–charge force.

The self–consistent nonlinear equations of particle motion are:

$$v_z \frac{dp_z}{dz} = -2\pi e^2 \beta_u^2 v_0 \int dt_0 n_b(t_0) \frac{\operatorname{sgn}(\delta z)\cos\psi}{1-\beta_0 \operatorname{sgn}(\delta z)}, \quad (3)$$

$$\frac{dt_L}{dz} = \frac{1}{v_z}, \quad (4)$$

where $\psi = \dfrac{k_u(\delta z)}{1-\beta_0 \operatorname{sgn}(\delta z)}$.

The efficiency of particle kinetic energy conversion into electromagnetic energy will be defined as the ratio of kinetic beam energy losses to its initial energy (<...> denotes an average over the beam pulse duration):

$$\eta = \left\langle\left[\gamma_0 - \gamma(z,t_0)\right]/(1-\gamma_0)\right\rangle. \quad (5)$$

Below will be considered the motion of electrons under force due to forward traveling radiation field, since it is this force that is responsible for the axial bunching of electrons and leads to the growth of coherent short wavelength radiation.

The density of the electrons at the entrance to the undulator is assumed to be uniform over the beam pulse duration $0<t_0<t_b$. The electrons are monoenergetic $d\tau/d\xi|_{\xi=0} = \tau(0,\tau_0) = 0$ and no input signal in the undulator, where $\tau = \omega\left[t_L(z,t) - z/v_0\right]$.

In the linear regime ($\xi \ll \xi_{sat} = 1/\sqrt{\rho^3\tau_b}$) when longitudinal displacement of electrons relative its equilibrium trajectory is less then wavelength of radiation, from Eqs. (3) (4) can be obtained following expression for the amplitude of this displacement:

$$\tau(\xi,\tau_0) = \tau_0 + R(\xi,\tau_0)\sin(\tau_b - \tau_0)\,, \quad (6)$$

$$R(\xi,\tau_0) = (1/2)\rho^3(\xi + \tau_0 - \tau_b)^2\,\theta(\xi + \tau_0 - \tau_b)\,, \quad (7)$$

where $\rho = \frac{1}{\gamma_0}\left(\frac{\omega_b}{k_u c}K\right)^{2/3}$, $\omega_b = \sqrt{4\pi e^2 n_0/m}$, $\xi = k_u z$, $\tau_0 = \omega t_0$, n_0 is the unperturbed beam density, $\omega = k_u v_0/(1-\beta_0)$, $\lambda = 2\pi c/\omega \approx \lambda_u/2\gamma_0^2$ are the frequency and wavelength of the field radiated along the axis OZ in the direction of bunch motion, $\tau_b = \omega t_b = 2\pi\, l_b/\lambda$, l_b is bunch length, $\theta(x)$ is the Heaviside unit step function.

It follows from Eqs. (6), (7) that at the beginning of selfbunching process there is self–modulation of longitudinal displacement (and longitudinal momentum) of electrons moving at the trailing edge of the beam. The front of modulation propagates from the trailing edge of the beam to its leading edge as beam moves in the undulator.

The results from computer simulation of the electron bunch motion in the undulator are shown in Figs. 1 and 2. Fig. 1 shows the efficiency as a function of axial distance ($\zeta=\rho\xi$) for different bunch length l_b/λ =5 (1), 10 (2), 20 (3) and for ρ=0.02.

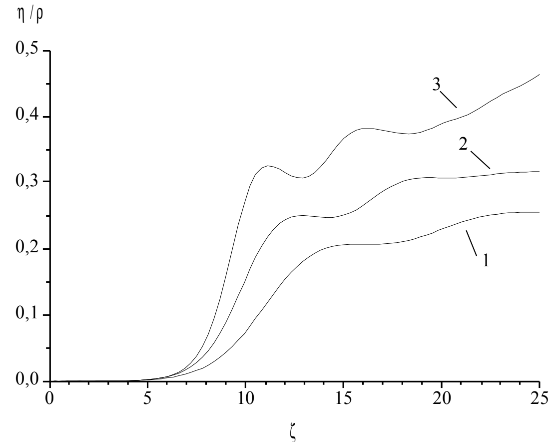

Fig. 1. *The efficiency η as a function of normalized axial distance ζ for several value of bunch length $l_b=5\lambda$ (1), 10λ (2), 20λ (3) and for $\rho=0.02$.*

It is seen from this figure that the spatial growth rate increases and the saturation length decreases with an increase of the beam length. The maximum value of the efficiency grows with an increase of the beam length too.

Fig. 2 shows the phase space ($\tau' \equiv d\tau/d\xi$, τ) of electron beam for the case of $l_b=10\lambda$. The longitudinal momentum as a function of relative time the particles cross the axial positions ζ=1, 9, 12, 15 are plots in this figure. At the initial position ζ=0 the particles have uniform axial velocity $\tau'(0,\tau_0) = 0$.

It can see from this figure that axial velocity of electrons is modulated with spatial period of ponderomotive force, approximately equal λ. The perturbation of axial velocity arises at the trailing edge of the bunch and propagates in forward direction to its leading edge. Physically in considered case only coherent spontaneous undulator radiation of electrons moving at the trailing edge of the bunch is the source of initial perturbation of axial momentum of electrons. Notice that in Fig. 2 the particles at trailing edge have maximum value of entry time. Some electrons have gained energy while others have lost energy depending on their phase relation with ponderomotive force.

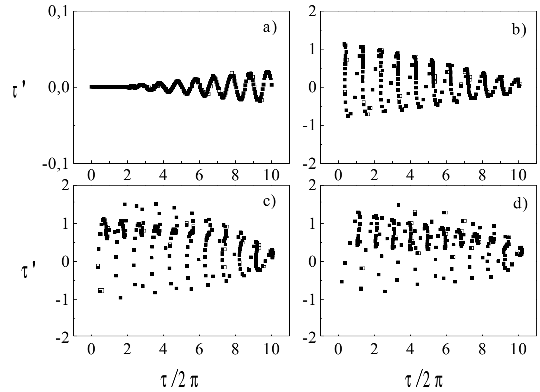

Fig. 2. *The phase space of electron bunch crossing the axial position: ζ=1 (a), 9 (b), 12 (c), 15 (d)*

At the $\xi=\tau$ the front of axial momentum perturbation reaches the leading edge of the bunch. Since the electrons undergo grouping force due to the radiation of electrons moving behind of the considered, the particles placed at the head of bunch have amplitude of displacement in axial momentum larger than the particles at the tail of the bunch. Fig. 2 c, 2 d depict the particles at saturation and show trapping and spatial bunching of electrons. Many of particles within each of wavelength cross the ζ=12 plane (and also ζ=15 plane) at about the same time. Fig. 2d, at ζ=15, shows the formation of microbunches practically within full length of beam. Thus, the performed analytical and numerical simulations show the possibility of development of collective selfbunching process in short pulse relativistic electron beam moving through an undulator.

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