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ION CYCLOTRON RESONANCE FOR FAST MAGNETOSONIC WAVES IN SMALL TOKAMAKS

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The absorption of fast magnetosonic waves in a small-size tokamak has been studied under the multiple cyclotron resonance for bulk ions and under the fundamental resonance for minority ions. The small plasma radius is assumed to be less than or equal to, in the order of magnitude, the transverse wavelength. In this case, the solution to the linearized Vlasov equation for the distribution function of the resonance ions in the form of a power series in their Larmor radius and the expression for the contribution from resonance ions into the wave current density have been obtained. The nonlocal coupling between the wave field and current density in a small azimuth angle has been taken into account, which is associated with the motion of resonance ions along a magnetic line of force while travelling through the cyclotron resonance zone, with consideration for the nonuniform magnetic field along a line of force. Allowances have been also made for the decorrelation effects connected with the Coulomb collisions between the resonance particles during their travelling across the successive zones of the resonance, which effects lead to the dephasing of the particle-wave phase. The expressions for the RF power absorbed in the tokamak plasma have been obtained and analyzed.

1. Introduction

The problem on plasma heating with fast magnetosonic waves (FMSW) due to the ion cyclotron resonance is conventionally considered within the framework of a short-wavelength approximation for the waves with the transverse wavelength $\lambda\!\!\!^{-} \sim v_A/\omega$ considerably below the small plasma radius a_p by analytical and numerical techniques [1,2]. For a tokamak the condition of the cyclotron resonance $\omega \approx n\omega_c(r,\vartheta) + k_{\|}v_{\|}$ is met for thermal resonant particles moving according to a Larmor spiral along magnetic field lines only in a narrow region where the exchange of energy between the wave field and a particle is essential. Here $\omega_{ci}(r,\vartheta)$ is the cyclotron frequency

$$\omega_{ci}(r,\vartheta) = \overline{\omega}_{ci}\left(1 + \frac{r}{R}\cos\vartheta\right) , \qquad (1)$$

where $\overline{\omega}_{ci} = \dfrac{eB_0}{m_i c}$, B_0 is the magnetic field value at the center of the toroidal chamber, r is the distance from this center, ϑ is the small azimuth angle, R is the large radius of the torus, the toroidicity is assumed to be small, $\varepsilon_t = r/R << 1$. As the time a particle requires to traverse a resonant zone τ_0 is small, one can neglect the effect of binary Coulomb collisions on the wave-particle interaction during this period if the collision frequency is sufficiently small. Having left the resonance zone, such a particle travels along the magnetic field line a considerably larger distance (order of qR) before it enters the next zone of cyclotron resonance (q is the safety factor). During this period $\Delta t \sim qR/v_{Ti}$ $(v_{\|} \sim v_{Ti})$, the particle will obtain due to collisions a random increment of the velocity along the magnetic field $\delta v_{\|} \sim v_{Ti}\sqrt{\nu_{col}\Delta t}$. Therefore the wave-particle phase will experience the change $\delta\Phi \sim k_{\|}\delta v_{\|}\Delta t$. For thermal particles $\delta\Phi >> 1$, so that one can regard the passage through the cyclotron resonance zone as random and neglect the effect of preceding passages through cyclotron resonance zones, i.e., the effect of all the particle trajectory at the section t' from $-\infty$ to $t - \tau_0$. This effect happens to be of value only for fast particles for which due to the small value of the collision frequency the quantity $\delta\Phi \le 1$. Collisional decorrelation of the wave-particle phase is determined in papers [3-5]. In what follows we will consider the case of strong decorrelation when $\delta\Phi >> 1$, and take into account nonlocal effects on a small section of the ion trajectory near the point of resonance.

Below there will be considered the problem of the cyclotron resonance $\omega = n\omega_{ci}(r,\vartheta)$ for FMSW in small-size tokamaks, when $\lambda\!\!\!^{-}_{\perp} \ge a_p$. Thus the report takes into account the spatial dispersion related to the resonant particle motion along the magnetic field line in the nonuniform magnetic field of the tokamak leading to the nonlocal dependence of the current density of resonant ions on the small azimuth angle ϑ, and to the nonlocality of current over the small radius.

2. RF current density

As the toroidicity of the tokamak is small in the case considered, we can apply a cylindrical approximation for taking the spatial dispersion into account, and assume that the spatial dispersion of the equilibrium distribution function F_{0i} is determined by the integral of equations of motion

$$X = r^2 + \frac{v_\perp^2}{\varpi_{ci}^2} + 2r\frac{v_\perp}{\varpi_{ci}}\sin(\phi - \vartheta) \quad (2)$$

where ϕ is the azimuth angle in the velocity space and $\omega_{ci}(r,\vartheta)$ differs weakly from $\overline{\omega}_{ci}$ in this case. Then taking into account that the ion Larmor radius $v_\perp/\overline{\omega}_{ci}$ is small compared with the plasma radius, one can develop the function F_{0i} in powers of $v_\perp/\varpi_{ci}$ and keep in the expression for the RF current density **j**=$e_i\int \tilde{f}$ **v**d**v** the leading terms in the small parameter $k_\perp v_\perp/\varpi_{ci} << 1$ (see the corresponding calculations for the nonuniform over radius plasma cylinder [6]). Take also into account that the passage of the resonance zone results in the nonlinear variation of the wave-particle phase against the angle ϑ' (ϑ' if the variation of the angle ϑ when the particle crosses the resonance zone):

$$\Delta\Phi = \int[\omega - n\omega_{ci}(r,\vartheta(t'))]dt' =$$

$$= \left(\zeta_\| \frac{\omega - n\omega_{ci}(\vartheta)}{\overline{\omega}_{ci}\varepsilon_t} - k_\| qR\right)\vartheta' + \frac{1}{2}n\zeta_\| \sin(\vartheta)\vartheta'^2 \quad , \quad (3)$$

where $\zeta_\| = \overline{\omega}_{ci}qr/v_\|$, q(r) is the safety factor.

Then applying the technique of integrating along the trajectories for determining the perturbed distribution function $\tilde{f}$, we obtain for the left-hand polarized component of the RF current density in the FMSW field $j^+(r,\vartheta,\zeta) = j_r + ij_\vartheta$ the following final expression:

$$j^+ = \frac{n}{2^n n!}\sum_{p=0}^{n-1}(-1)^{n-1-p}e^{-i\frac{\pi}{2}p}\begin{pmatrix} n \\ p \end{pmatrix}e^{-i\vartheta(p+1)}r^p \times$$

$$\times\left(\frac{1}{r}\frac{\partial}{\partial r}\right)^p \omega_{pi}^2(r)\rho_i^{2n-2} \times \quad (4)$$

$$\times\int_{-\infty}^{\infty}\frac{dv_\|}{v_\| v_{Ti}(r)}e^{-\frac{v_\|^2}{2v_{Ti}^2}}q(r)R\frac{e^{-i\frac{\pi}{4}}}{\sqrt{\zeta_\| \sin\vartheta}} \times$$

$$\times W(z)\Delta_\perp^{n-1-p}\hat{L}^p e^{i\vartheta}E^+(r,\vartheta)$$

Here $\rho_i = \frac{v_{Ti}}{\overline{\omega}_{ci}}$, $\omega_{pi}^2 = \frac{4\pi e_i^2 n_{0i}(r)}{m_i}$,

$$\begin{pmatrix} n \\ p \end{pmatrix} = \frac{n(n-1)...(n-p)}{p!}.$$

The operators $\Delta_\perp$ and $\hat{L}$ act on the function $e^{i\vartheta}E^+$,

$$\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \vartheta^2},$$

$$\hat{L} = e^{i\vartheta}\left(-i\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial\vartheta}\right)$$

W(z) is the probability integral with a complex argument.

$$W(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right),$$

$$z = \left(\frac{\omega - n\overline{\omega}_{ci}}{\overline{\omega}_{ci}\varepsilon_t} - \frac{k_\| qR}{\zeta_\|}\right)\sqrt{\frac{\zeta_\|}{2\sin\vartheta}}e^{-i\frac{\pi}{4}}. \quad (5)$$

The operator $\left(\frac{1}{r}\frac{\partial}{\partial r}\right)^p$ acts on functions $n_{0i}(r)$ and $T_i(r)$.

3. Conclusions

The expression (4) for j^+ is essentially nonlocal over r and ϑ. The nonlocal dependence over r due to the nonuniformity of density and temperature of ions is important for long wavelength perturbations ($\lambda \geq a_p$). The nonuniformity of the magnetic field and the presence of the rotational transform involve the nonlocality of currents j^+ over ϑ, it being important practically always. It may be neglected only in the region $|z| >> 1$. In this region, using the asymptotic expression $W(z) \approx \frac{i}{\sqrt{\pi}z}$, we obtain for (4) the expression coinciding with the current density for adiabatic traps with the uniform magnetic field [6]. Usually for the quantity z we have $|z| \sim k_\| R\sqrt{q\frac{\rho_i}{r}} \leq 1$ and the nonlocality over ϑ is practically always important. For the short wavelength FMSW ($\lambda << a_p$) one should keep in the expression (4) the largest term with p=0, which coincides in this case with the expression for j^+, obtained by different methods in [3-5]. The value of the RF power absorbed by the plasma

$$P = \mathrm{Re}\int rdr\int d\vartheta j^+\left(E^+\right)^*$$

with the account of the narrow cyclotron resonance zone coincides practically with one obtained neglecting the nonlocality.

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