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ON THE PROBLEM OF NEGATIVE DISSIPATION IN MULTI-ION COMPONENT PLASMAS IN NUMERICAL CALCULATIONS AT THE FUNDAMENTAL ICR

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1 Introduction

It is known that finite Larmor radius approximation up to the second order (FLR_2) of Fast Wave (FW) propagation and absorption may give wrong results in numerical simulations of multi-component plasmas in the range of ICR. This is true not only for short wavelength modes (Slow Wave and ion Bernstain wave) but also for long wavelength FW. The main unphysical effect that appears is the negative absorption of high frequency energy of FW. This fact does not depend on a simulation character and takes place in both, wave equation and ray tracing approaches in numerical calculations. This problem is closely connected with the reliability of many past and present high frequency heating calculations, since minority heating, one of the most promising heating methods, has deal with multi-component plasmas and uses absorption at the fundamental ICR for one of the ion sorts. Probably due to that fact, the problem of negative absorption intensively discussed in works since 80s’ years [1-10].

In previous works [1-9] several assumptions on the influence of short wavelength modes description on FW absorption calculations were made. The finiteness of $k_\perp\rho_i$ parameter ($k_\perp$ is transverse wave number, ρ_i is ion Larmor radius), for short wavelength modes, provokes a bad convergence property of the series and makes the FLR_2-approximation fails for them. The main efforts were devoted to weaken that negative influence considering that the wave properties are correctly described for FW due to the smallness of $k_\perp\rho_i$ parameter for FW and, consequently, to the good convergence of expansions for FW.

The works [6,7] are totally devoted to study this problem and, in particular, they deal with the plasma conditions for which negative absorption arises. The possibilities of partially and totally avoiding this problem are also explored. It was found in these works that the main parameter that governs the apparition of negative absorption was the concentration of resonant ions: the negative absorption appears when the concentration becomes high enough. Several studies have been performed in the context of doing calculations avoiding the problem of negative absorption. Those studies play with different variants of the “hot” terms of FLR_2-expansion, that are connected with power absorption at the fundamental ICR frequency. Firstly, the fully omission of these terms gave the so called “SCK”-form. This name comes from the names of their authors: Swanson [8], Colestock and Kashuba [9]. Secondly, the partial omission of them, in which only the imaginary parts of the terms are omitted. And a third group exists that performs the phenomenological modification of these terms, justified by numerical calculations.

Such simplified treatments are usually based on the fact that the “hot” terms must be much smaller than zero-order ones, as a consequence of the smallness of the parameter $k_\perp\rho_i$ for FW. Zero-order terms are themselves resonant at the same frequency and therefore they are dominant in the series. However such a justification can be questioned in the case of quasiperpendicular propagation of FW, when the angle of propagation goes to $\pi/2$, equivalently, longitudinal refractive index, $N_\parallel$, goes to 0.

It is shown in work [10] that the expansion of FW dispersion equation in the parameter $k_\perp\rho_i$ converges in different ways for different propagation angles with respect to magnetic field. In particular, in the case of quasitransverse propagation the convergence of the expansion is slower than in the case of other angles. Therefore, more terms are necessary in the case of calculation of absorption at first harmonic of ICR. This fact implies the necessity to keep more terms of the expansion in the dispersion equation for both, ray tracing and wave equation calculations.

Consequently, the value of the longitudinal refractive index, $N_\parallel$, is proved to be a key parameter for the localization of negative absorption in a plasma. It must be noted that this conclusion is equivalent to the one presented in the works [6,7] in which the main parameter is the concentration of resonant particles. This is because the ion concentration, n_i, and the parallel refractive index, $N_\parallel$, appear in dielectric tensor through the combination $n_i/N_\parallel$ and any arising of the concentration, n_i, may be replaced by decreasing of parallel index $N_\parallel$. Nevertheless, the contrary is not true, since the upper values of the concentration are limited by the known restrictions.

In the work [10] it was also shown that for a correct description of FW in a single-ion component plasma it is necessary and enough to use the FLR_4-expansion in numerical calculations.

In the present report the results of the work [10] derived in the FLR_4 frame are developed and generalized for a multi-ion component plasma.

2. The reasons of the apparition of negative absorption

To display the reasons of appearing of negative absorption it is useful to analyze the structure of whole FLR$_\infty$-expansion of the dispersion relation for FW in the range of the fundamental ICR for the resonant sort of ions. The dispersion equation for FW in the case of neglecting the electron mass has the form:

$$(\varepsilon_{11}-N_\parallel^2)N_\perp^2-(\varepsilon_{11}-N_\parallel^2)(\varepsilon_{22}-N_\parallel^2)-\varepsilon_{12}^2=0, \qquad (1)$$

where ε_{ik} are components of dielectric tensor and $N_\perp=ck_\perp/\omega$ is transverse refractive index (c is the speed of light, ω is the angular frequency of FW). Let us expand the components of dielectric tensor for FW in the small parameter $(k_\perp\rho_\alpha)^2$ (from now on index α denotes the resonant sort of ions). Then let us group the terms proportional to the parameter $Z_{0\alpha}=\omega/(2k_\parallel V_{T\alpha})$ ($V_{T\alpha}= (T_\alpha/m_\alpha)^{1/2}$ is thermal velocity) in the antihermitian part which is connected with absorption at he first harmonic, leaving unchanged the hermitian terms. In this way, the equation (1) can be written as:

$$[\text{hermitian terms}]+Z_{0\alpha}^2[O(k_\perp\rho_\alpha)^6]+Z_{0\alpha}^2[O(k_\perp\rho_\alpha)^4] + Z_{0\alpha}[O(k_\perp\rho_\alpha)^2]+Z_{0\alpha}[O(1)]=0, \qquad (2)$$

The main peculiarity of this expression is the fact that the parameter $Z_{0\alpha}$ appears with first power in two older terms and with second power in all other ones. Since the longitudinal refractive index $N_\parallel =ck_\parallel /\omega$ is contained in $Z_{0\alpha}$ parameter it is easy to see that the value $N_\parallel =0$ is a singular point of the expression (2). When $N_\parallel$ goes to 0 the dominant term will be of order $(k_\perp\rho_\alpha)^4$. Really, we may neglect the terms of higher order since the parameter $(k_\perp\rho_\alpha)^2$ is small for FW. Thus, from the expression (2) it follows that it is necessary and enough to keep in the expression (2) the three older terms for the exact description of absorption at the first harmonic of ICR, i.e. to be limited by frame of FLR$_4$-expansion.

Firstly, let us keep in the expansion (2) only two older terms, i.e. will limit ourself by frame of FLR$_2$-expansion and try to find out the details of the apparition of negative absorption. In this case expansion (2) may be written in the form:

$$N_\perp^2\left(1+i\,\frac{\omega_{p\alpha}^2}{\omega^2}\sqrt{\pi}Z_{0\alpha}\rho_\alpha^2\frac{\omega_{C\alpha}^2}{C^2}W_{-1\alpha}\left[1-\frac{1}{2}\frac{\left(\varepsilon_1+\varepsilon_2-N_{//}^2\right)}{\left(\varepsilon_1-N_{//}^2\right)}\right]\right)= \frac{\left(\varepsilon_1^0-\varepsilon_2^0-N_{//}^2\right)\left(\varepsilon_1+\varepsilon_2-N_{//}^2\right)}{\left(\varepsilon_1-N_{//}^2\right)}, \qquad (3)$$

where $\omega_{p\alpha}$, $\omega_{c\alpha}$ is plasma and cyclotron frequencies of resonant ions, $W_{-1\alpha}=W(Z_{-1\alpha})$ is dispersion function of argument $Z_{-1\alpha}=(\omega-\omega_{c\alpha})/\sqrt{2}\,k_\parallel V_{t\alpha}$, ε_1^0, ε_2^0 are the zero order terms of the expansion of components of dielectric tensor ε_1 and ε_2 ($\varepsilon_1=\varepsilon_{11}$, $\varepsilon_2=i\varepsilon_{12}$) in the parameter $(k_\perp\rho_\alpha)^2$. ε_1, ε_2 contain zero and first order terms of the same expansion. To understand why the imaginary part of perpendicular refractive index changes and even becomes negative in the FLR$_2$-approximation, it is convenient to expand in the equation (3) the ratio $(\varepsilon_1+\varepsilon_2-N_\parallel^2)/(\varepsilon_1-N_\parallel^2)$ up to 2nd order in the parameter $(k_\perp\rho_\alpha)^2$:

$$\frac{\varepsilon_1+\varepsilon_2-N_{//}^2}{\varepsilon_1-N_{//}^2}=(2-x)+i\Big[\Big(\frac{n_\alpha z_\beta}{n_\alpha z_\alpha\left(w-\xi\right)} + \frac{1+w+2w^2}{2w\left(w+1\right)}\Big)(2+2x)-\Big(\frac{n_\beta z_\beta\xi}{n_\alpha z_\alpha\left(w^2-\xi^2\right)} + \frac{1}{2w\left(w+1\right)}\Big)(4+2x)\Big]\frac{w^2}{\sqrt{\pi}Z_{0\alpha}W_{-1\alpha}}, \qquad (4)$$

where $x=(k_\perp\rho_\alpha)^2$, $W=\omega/\omega_{c\alpha}$, $\xi=(Z_\beta m_\alpha)/(Z_\alpha m_\beta)$ and Z_β, Z_α, m_β, m_α, n_β, n_α are charges, masses and concentrations of nonresonant and resonant ions. Now it is easy to see that an imaginary part of the expression (4), multiplication times the clearly imaginary term before square brackets in the equation (3), will be added to the real part of the transverse refractive index and will not present singularities. This product will not have singularities for any value of $N_\parallel$, since $Z_{0\alpha}$ in the expression (3) and imaginary part of expansion (4) will cancel one another. The real part of the expression (4), which after the same multiplication will add to the imaginary part of the refractive index, consists of two addendums: one is of zero order and the second one is of first order in parameter $(k_\perp\rho_\alpha)^2$. The addendum of zero order transforms the expression in square brackets in (3) into zero and, therefore, does not change the imaginary part of $N_\perp^2$. But small second addendum, of first order, as a consequence of multiplication times the big parameter $Z_{0\alpha}$ will essentially change the imaginary part of $N_\perp^2$ when $N_\parallel$ decreases. The sign of this addition will be positive and therefore the addition will make the absorption at first harmonic of ICR to decrease. And even it will make it negative. To get convinced of that fact it is enough to solve a double quadratic equation, which is obtained from equation (3), after substituting the expression (4) into it. It is necessary to note that in right part of the expression (3), instead of $(\varepsilon_1+\varepsilon_2-N_\parallel^2)/(\varepsilon_1-N_\parallel^2)$, it is enough to substitute $(\varepsilon_1^0+\varepsilon_2^0-N_\parallel^2)/(\varepsilon_1^0-N_\parallel^2)$, as a consequence of the smallness of the parameter $(k_\perp\rho_\alpha)^2$ for FW. Then we shall have

$$\frac{1}{2}N_\perp^4\,\mathrm{i}\,\frac{\omega_{p\alpha}^2}{\omega^2}\sqrt{\pi}\,Z_{0\alpha}\,\rho_\alpha^4\,\frac{\omega^4 W_{-1\alpha}}{C^4 w^2} + N_\perp^2-(N_{\perp F}^0)^2=0, \qquad (5)$$

where $(N^0_{\perp F})^2=(\varepsilon_1^0-\varepsilon_2^0-N_\parallel^2)(\varepsilon_1^0+\varepsilon_2^0-N_\parallel^2)/(\varepsilon_1^0-N_\parallel^2)$ is the square of refractive index at the FLR$_0$-approximation. The roots of this equation describe FW and ion Bernstein wave in the FLR$_2$-approximation. For FW we will have

$$\mathrm{Im}N_\perp=\frac{(x-1)\,\mathrm{Re}W_{-1\alpha}-y\,\mathrm{Im}W_{-1\alpha}}{d\left|W_{-1\alpha}\right|^2}$$

$$\mathrm{Re}N_\perp=\frac{(x-1)\,\mathrm{Im}W_{-1\alpha}+y\,\mathrm{Re}W_{-1\alpha}}{d\left|W_{-1\alpha}\right|^2}, \qquad (6)$$

where we have defined the quantities $d=-2\sqrt{\pi}N_{A\alpha}^2 Z_{0\alpha}\,\omega^4/C^4$, $N_{A\alpha}^2=\omega_{p\alpha}^2/\omega_{c\alpha}^2$;

$$X=[\tfrac{1}{2}([1+d^2\left|\left(N_F^0\right)^2\right|^2\left|W_{-1\alpha}\right|^2+2d(\mathrm{Im}(N_F^0)^2\mathrm{Re}W_{-1\alpha}+\mathrm{Re}(N_F^0)^2\mathrm{Im}W_{-1\alpha})]^{1/2}+1+$$
$$d(\mathrm{Im}(N_F^0)^2\mathrm{Re}W_{-1\alpha}+\mathrm{Re}(N_F^0)^2\mathrm{Im}W_{-1\alpha}))]^{1/2},$$
$$Y=[\tfrac{1}{2}([1+d^2|(N_F^0)^2|^2|W_{-1\alpha}|^2+2d(\mathrm{Im}(N_F^0)^2\mathrm{Re}W_{-1\alpha}+\mathrm{Re}(N_F^0)^2\mathrm{Im}W_{-1\alpha})]^{1\backslash 2}-1-$$
$$d(\mathrm{Im}(N_F^0)^2\mathrm{Re}W_{-1\alpha}+\mathrm{Re}(N_F^0)^2\mathrm{Im}W_{-1\alpha}))]^{1/2}$$

At the resonance $\omega=\omega_{c\alpha}$ for not very small values of $N_\parallel$ ($d<<1$) we will have from the expression (6)

$$\mathrm{Im}N_\perp^2=\mathrm{Im}(N_F^0)^2--$$
$$\tfrac{1}{2}|(N_F^0)^2|^2\sqrt{\pi}\,N_{A\alpha}^2 Z_{0\alpha}\omega^4\rho_\alpha^4/C^4$$
$$\mathrm{Re}N_\perp^2=\mathrm{Re}(N_F^0)^2, \qquad (7)$$

From expressions (6),(7) it follows that in the case of FLR_2-approximation for quasiperpendicular FW propagation the imaginary part of $N_\perp$ may decrease and even become negative. Therefore the frame of FLR_2-approximation is not enough for finding an exact value of refractive index.

From the expression (7) it follows that negative absorption appears when $1+2n_\beta z_\beta/n_\alpha z_\alpha(1+\xi)<\sqrt{\pi}\,Z_{0\alpha}(k_\perp\rho_I)^2$ (here $\xi=z_\beta m_\alpha/z_\alpha m_\beta$), since $\mathrm{Im}(N_F^0)^2=\mathrm{Re}(N_F^0)^2(1+2n_\beta z_\beta/n_\alpha z_\alpha(1+\xi)/(2\sqrt{\pi}Z_{0\alpha})$. Thus, if the concentration of resonant ions, n_α, is more or order of the concentration of nonresonant ions, n_β, then negative dissipation appears when $Z_{0\alpha}(k_\perp\rho_I)^2>1$ or $N_{//}<(k_\perp\rho_I)^2C/V_{T\alpha}$. This condition coincides with analogical one for single-ion plasmas [10].

3.About avoiding negative absorption

From the discussion presented above, it follows that one of the possibilities to avoid the negative absorption is to search the solution in the frame of FLR_4-approximation. So, let us keep in expression (2) three older addendums and expand the components of dielectric tensor ε_1 and ε_2 up to 2nd order in the parameter $(k_\perp\rho_\alpha)^2$. In this case the expansion (2) may be reduced into next equation :

$$N_\perp^4[i\frac{\omega_{p\alpha}^2}{\omega^2}\sqrt{\pi}\,Z_{0\alpha}\rho_\alpha^4\frac{\omega^4}{C^4w^2}W_{-1\alpha}(-2+$$
$$\tfrac{5}{8}\frac{(\varepsilon_1^0+\varepsilon_2^0-N_{//}^2)}{(\varepsilon_1^0-N_{//}^2)}+\tfrac{1}{2})]+N_\perp^2-(N_{\perp F}^0)^2=0, \qquad (8)$$

This equation differs from equation (5) in the coefficient of $N_\perp^4$. In this one there appear two new terms which change the sign of the coefficient from plus into minus, since $(\varepsilon_1^0+\varepsilon_2^0-N_{//}^2)/(\varepsilon_1^0-N_{//}^2)=2$. Now the absorption will increase in comparison with the absorption in the case of FLR_0-approximation. So in the frame of FLR_4-approximation the problem of negative absorption disappears due to the fact that the main term in expression (2), which is proportional to $(k_\perp\rho_\alpha)^4$, has the whole line of addendums and, consequently, is correctly described.

Of course, using of the FLR_4-aproximation essentially complicates the numerical calculations, especially those performed in 2-D and 3-D geometry, therefore it is extremely desirable to look for any acceptable phenomenological solution in the frame of FLR_2-approximation for any angle of FW propagation, including the quasiperpendicular ones. However, the strict analysis shows that by means of changing in dielectric tensor of the "hot" terms, connected with the absorption at the first harmonic, it is impossible to obtain the equation (8) with the correct sign at $N_\perp^4$. The sign always is positive that corresponds to decreasing of absorption in comparison with FLR_0-approximation. It follows from the conditions corresponding to receiving of the coefficient at $N_\perp^2$ in equation (8). As to phenomenological modifications of the same "hot" terms used in works [1-9], all they are reduced to receiving the value 2 in the real part of expression (4) instead (2-x). In this case the coefficient at $N_\perp^4$ in the equation (8) automatically equals 0 and, consequently $N_\perp^2=(N_{\perp F}^0)^2$, i.e. it corresponds to formulae received in the FLR_0-approximation, which is not correct in the case of the quasiperpendicular propagation. Thus, the fact that FLR_4-approximation is necessary for correct description of FW absorption for any an angle of propagation may be strictly proved.

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