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ESTAFETTE OF RESONANCES, STOCHASTICITY AND CONTROL OF PARTICLE MOTION

Alexander A.SHISHKIN

Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Kharkov - 108, Ukraine and
Plasma Physics Chair, Department of Physics and Technology,
Kharkov "V.N. Karazin" National University, Kharkov-77, Ukraine

A new method of particle motion control in toroidal magnetic traps with rotational transform using the *estafette* of drift resonances and stochasticity of particle trajectories is proposed. The using the word "estafette" (relay race) means here that the particle passes through a set of resonances consistently from one to another during its motion. The overlapping of adjacent resonances can be moved radially from the center to the edge of the plasma by switching on the corresponding perturbations in accordance with a particular rule in time. In this way particles (e.g. cold alpha-particle) can be removed from the center of the confinement volume to the plasma periphery.

1. Introduction

There exist different approaches to control the particle motion and one of them is to apply magnetic field perturbations to produce resonance trajectories - drift islands, overlapping of the islands and stochastic trajectories that lead, for example, to the removal of test particles from the center of the magnetic configuration to the periphery [1-3]. A new method of particle motion control in toroidal magnetic traps with rotational transform using the *estafette* of drift resonances and stochasticity of particle trajectories can be described in the following. If there are some adjacent rational drift surfaces with the drift rotational transform $\iota^* = n/m, \iota^* = n'/m'$, $\iota^* = n''/m''$, then the magnetic perturbations with the wave numbers (m,n), (m',n'), (m'',n'') can lead to some families of drift islands. Overlapping of the adjacent resonance structure is the reason for the stochasticity [2] of particle trajectories. If a particle trajectory passes through the set of perturbations this test particle can escape from the center of the confinement volume to the periphery.

Overlapping of the adjacent resonances in the local space within the plasma can be transferred outward in the process of particle motion by switching on the corresponding perturbations in accordance with a special rule (consequently) in time

The mathematical basis of the description is rather similar to the description of the nonlinear oscillator (pendulum) with a quasi-periodic perturbation [4-6]. In this paper, *estafette* of resonances is studied with both analytical methods and numerical integration of guiding center equations. Stochastic diffusion coefficients $D_{r,r}, D_{r,\vartheta}, D_{\vartheta,\vartheta}$ are introduced. These coefficients are useful to evaluate the deviation of the trajectories in the radial (r) and the poloidal angular (ϑ) directions after a large number of rotations (rounds) along the torus, i.e. in the toroidal angular (φ) direction.

2. Analytical Treatment

2.1. Equation for the Drift Flux Surface Function

As is known, the particle motion can be described with the integrals of the guiding center equations system and one of them is the function of the drift surface $\Psi^*(r,\vartheta,\varphi,V_\parallel,V_\perp,t)$.

We assume that the flux of particle guiding centers is conserved during the particle motion in analogy with the magnetic flux. This leads to

$$\Psi^*(r,\vartheta,\varphi,V_\parallel,V_\perp,t) = const \qquad (1)$$

and the total derivative of the drift surface function is equal to zero

$$\frac{d\Psi^*}{dt} = 0 \qquad (2)$$

The equation (3.2) in variables r,ϑ,φ and $V_\parallel$, $V_\perp$ takes the form

$$\frac{\partial\Psi^*}{\partial t} + \frac{dr}{dt}\frac{\partial\Psi^*}{\partial r} + \frac{rd\vartheta}{dt}\frac{\partial\Psi^*}{r\partial\vartheta} + \frac{Rd\varphi}{dt}\frac{\partial\Psi^*}{R\partial\varphi} + \frac{dV_\parallel}{dt}\frac{\partial\Psi^*}{\partial V_\parallel} + \frac{dV_\perp}{dt}\frac{\partial\Psi^*}{\partial V_\perp} = 0 \qquad (3)$$

After substituting the equations for $\frac{dr}{dt}, \frac{rd\vartheta}{dt}, \frac{Rd\varphi}{dt}$ and $\frac{dV_\parallel}{dt}, \frac{dV_\perp}{dt}$ from guiding center equations system into (3.3) we obtain

$$\frac{\partial\Psi^*}{\partial t} + \frac{V_\parallel}{B}\left(\mathbf{B}\nabla\Psi^*\right) + \frac{Mc}{2eB^3}\left(2V_\parallel^2 + V_\perp^2\right)\left([\mathbf{B}\times\nabla B]\nabla\Psi^*\right) - \frac{V_\perp^2}{B^2}(\mathbf{B}\nabla B)\frac{\partial\Psi^*}{\partial V_\parallel} + \frac{V_\parallel}{4B^2}(\mathbf{B}\nabla B)\frac{\partial\Psi^*}{\partial V_\perp^2} = 0 \qquad (4)$$

We assume that $\frac{\partial\Psi^*}{\partial V_\parallel} = 0$ and $\frac{\partial\Psi^*}{\partial V_\perp^2} = 0$, because only passing particles in the narrow range of $V_\parallel$ and $V_\perp$ are considered. Under the magnetic field that consists of the basic field, perturbing field with "wave" numbers *(m, n)* and additional perturbing field with "wave" numbers (m',n'), namely

$$\mathbf{B} = \mathbf{B_0} + \mathbf{B}_{m,n} + \tilde{\mathbf{B}}_{m',n'}$$

the equation (3.4) reduces to the following one

$$\frac{\partial\Psi^*_{0,m,n}}{\partial t} + \left(\tilde{V}_r + \tilde{V}_r\frac{\partial}{\partial r}\Delta r + \tilde{V}_\vartheta\frac{\partial}{r\partial\vartheta}\Delta r\right)\frac{\partial\Psi^*_{0,m,n}}{\partial r} +$$

$$+\left(\tilde{V}_\vartheta+\tilde{V}_r\frac{\partial}{\partial r}(r\Delta\vartheta)+\tilde{V}_\vartheta\frac{1}{r}\frac{\partial}{\partial\vartheta}(r\Delta\vartheta)-\tilde{V}_r\frac{\Delta\vartheta}{r}\right)\frac{\partial\Psi^*_{0,m,n}}{r\partial\vartheta}$$
$$+D_{r,r}\frac{\partial^2\Psi^*_{0,m,n}}{\partial r^2}+D_{r,\vartheta}\frac{\partial^2\Psi^*_{0,m,n}}{r\partial r\partial\vartheta}+D_{\vartheta,\vartheta}\frac{\partial^2\Psi^*_{0,m,n}}{r^2\partial\vartheta^2}=0 \qquad (5)$$

Here such designations are introduced

$$\tilde{\mathbf{V}}\equiv\frac{V_\parallel}{B}\tilde{\mathbf{B}}+\frac{Mc}{2eB^3}\left(2V_\parallel^2+V_\perp^2\right)\left[\tilde{\mathbf{B}}\times\nabla B\right] \qquad (6)$$

and

$$D_{r,r}\equiv\tilde{V}_r\Delta r,\ D_{r,\vartheta}\equiv\tilde{V}_r r\Delta\vartheta+\tilde{V}_\vartheta\Delta r,\ D_{\vartheta,\vartheta}\equiv\tilde{V}_\vartheta r\Delta\vartheta \qquad (7)$$

$\Psi^*_{0,m,n}$ describes the drift surfaces corresponding to magnetic field $\mathbf{B_0}+\mathbf{B}_{m,n}$.

2.2. Stochastic Diffusion Coefficients

Stochastic diffusion coefficients can be evaluated using

$$D_{r,r}=\tilde{V}_r\Delta r\cong\left(\frac{V_\parallel}{B}\right)^2(B_{m,n,r}\tilde{B}_r+\tilde{B}_r^2)\frac{2\pi R}{V},$$

$$D_{r,\vartheta}=\tilde{V}_r r\Delta\vartheta+\tilde{V}_\vartheta\Delta r\cong 2\left(\frac{V_\parallel}{B}\right)^2(B_{m,n,r}+\tilde{B}_r)(B_{m,n,\vartheta}+\tilde{B}_\vartheta)\frac{2\pi R}{V}, \qquad (8)$$

$$D_{\vartheta,\vartheta}=\tilde{V}_\vartheta r\Delta\vartheta\cong\left(\frac{V_\parallel}{B}\right)^2(B_{m,n,\vartheta}\tilde{B}_\vartheta+\tilde{B}_\vartheta^2)\frac{2\pi R}{V}.$$

It should be noted that the sense of the diffusion coefficients is the following

$$D_{r,r}=\frac{(\delta r)^2}{\tau_{rr}},\ D_{r,\vartheta}=\frac{r\delta r\delta\vartheta}{\tau_{r\vartheta}},\ D_{\vartheta,\vartheta}=\frac{r^2(\delta\vartheta)^2}{\tau_{\vartheta\vartheta}} \qquad (9)$$

From the expressions (9) (after substituting from (8)) we can evaluate, for example, the time τ_{rr} ($\tau_{\vartheta\vartheta}$) necessary for the deviation of particle trajectories from the initial surface in the radial (angular) direction, the time $\tau_{r\vartheta}$ necessary for the deviation of particle trajectories from the initial surface in a certain angular space under a given deviation in the radial direction. Under the perturbations with the "wave" numbers (m',n') the separatrix of the (m,n) resonance is broken and the particle trajectories become stochastic. Thus the particle can wander from one resonance (initial) to another resonance – adjacent to the initial one and then this phenomenon repeats, the particle passes step by step through the set of resonances and can escape from the center of the confinement volume to the periphery and vice versa from the periphery into the center of the confinement volume.

3. Estafette of Resonance and Helium Ion Removal

3.1. Test particle

Estafette of drift resonances of the test particle is demonstrated by the numerical integration of guiding center equations. Helium ions ($_2He^4$) with the energy =100 keV and starting pitch $V_{II}/V=$ 0.9 are taken as the test particles. Starting point coordinates are r_0=3, 7, 11, 15, 19 cm, under ϑ_0=0 and φ_0=0. For analysis here the configuration for the Large Helical Device (LHD) – the most advanced steady-state helical device, which is under successful operation at the National Institute for Fusion Science (Toki, Japan) – is chosen.

3.2 Drift Surfaces without Perturbations

The so-called Inward Shift configuration is taken where the magnetic axis is shifted toward the torus center and the rotational transform varies from $\iota(0)\approx 0.6$ till $\iota(a_p)\approx 1.0$. Among the rational values of the drift rotational transform the following set is chosen $\iota(r^2/a^2)\approx$ 0.75, 0.8, 0.9, 1.0.

3.3. Drift Surfaces with Perturbations

3.3.1. Splitting of Resonant Surfaces

All perturbations act (switched on) simultaneously

Under the set of perturbations: m_p=4, n_p=3; $\varepsilon_{4,3,p}$ = 0.003, $\delta_{4,3,p}=\pi/2$; m_p= 5, n_p=4 $\varepsilon_{5,4,p}$=0.007, $\delta_{5,4,p}=\pi/2$; m_p=9, n_p=10; $\varepsilon_{9,10,p}$=0.03, $\delta_{9,10,p}=0$, - some families of magnetic islands in the vertical cross-section appear (Fig.1)

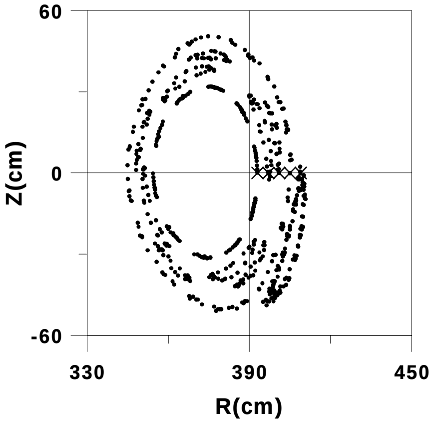

Fig.1. Drift surfaces of the helium ions under the set of perturbations indicated in Section 3.3.1

3.3.2. Stochasticity of Drift Trajectories

Perturbations switched on in the strict sequence

The perturbations are switched on in a strict sequence as is shown in Fig. 2. The amplitudes of the perturbations

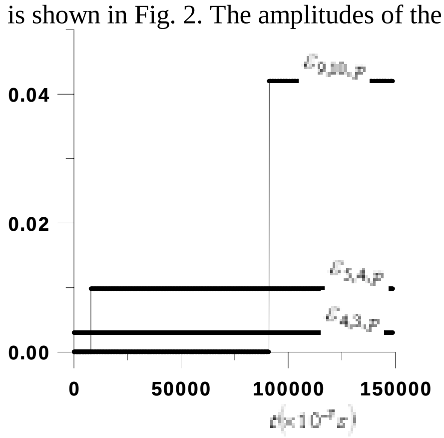

Fig.2. Amplitudes of perturbations in dependence on time

with m_p= 5, n_p =4 and m_p =9, n_p =10 are 1.4 times larger than in Fig. 1. Another very important fact is that the perturbation with $m_p / n_p = 5/4$ is switched on at the moment of time when the radial deviation of test particle is the largest. Therefore the particle starting at the point with $r_0 = 7$ cm, $\vartheta_0 = 0.0$, $\varphi_0 = 0.0$ moves from the initial magnetic surface to the following outward magnetic surfaces (Fig. 3).

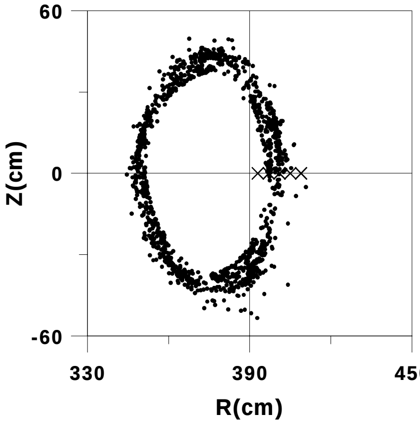

Fig 3.Diffusion of the drift surface of the test helium ion with the starting point r_0 =7 cm

The radial variable of the test particle trajectory increases in time and achieves a value (Fig. 4) where it may be removed by mechanical means. It is possible to mark out the intervals of time when the particle is in resonance with the

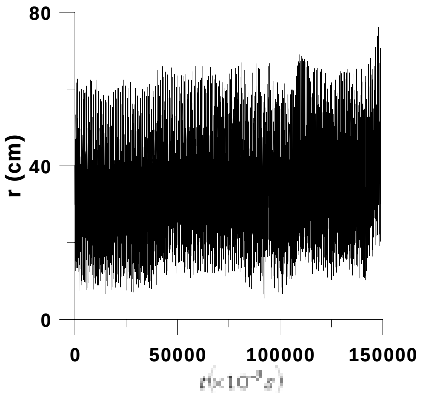

Fig.4,Radial variable of the test particle trajectory in dependence on time

$m_p / n_p = 4/3$ and $m_p / n_p = 5/4$ perturbations and finally the time when it leaves the plasma. Stochasticity of the trajectories with different but rather close starting points is shown on Fig.5.

If a deuterium ion with the energy W =7 keV (the possible thermal energy of the bulk ion plasma in device considered) starts at the same point its drift rotational transform $\iota^* = 4\ /\ 5$ this particle forms 5 islands in the cross-section instead of 4 islands as in the case of the helium ion with W=100 keV. The trajectory of this particle is not stochastic. Electrons with the same energy and the same starting point have $\iota^* = 0.836$. Their drift surfaces have some corrugation but are not stochastic.

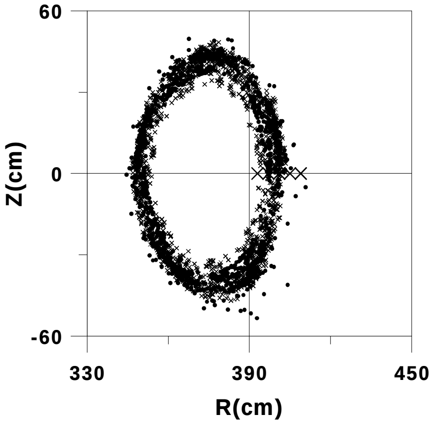

Fig. 5. Diffusion of two drift surfaces for the test helium ions with the starting points r_0 =7 cm (small crosses) and r_0 =11 cm (circles)

4. Conclusions

1. *Estafette* of drift resonances can be used for the removal of the test particle, particularly helium ash, when the perturbing magnetic fields are externally applied. For this purpose in the case of particle trajectories with the drift rotational transform $\iota^* = n_p / m_p$ the perturbing magnetic field with wave numbers m_p, n_p leads to drift island formation. If the adjacent drift islands overlap the particle trajectories become stochastic. A particle passes through a set of adjacent resonances and can leave the confinement volume.
2. The bulk plasma ions do not experience the stochastisation of their trajectories and do not escape the plasma.
3. Perturbing magnetic fields can be produced by a system of coils with specific currents, for example, similar to the coils of the local island divertor [7].
4. The method of the *estafette* of resonances can be applied to other practical physics tasks and may be considered as a new approach in the transition to chaotic states.

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