ALGORITHMS AND SOFTWARE FOR INCREASING OF THE CALCULATION SPEED ON PERSONAL COMPUTERS

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Acceleration of time-consuming calculations is discussed.

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1. INTRODUCTION

When improvement of elemental base productivity began approaching to physical limits and a cost factor commenced to play a significant role the multiprocessor systems have started to develop by quick pace. But the world experience shows that at all steps of computing technique development also an important way of reducing the time of obtaining calculation results is a choice of optimal decision algorithms and their programmed realization. Among these are the table-algorithmic methods (TAM) of function calculation being widely applied and at the same time the most effective.

In the present paper considered are the results of application of TAM for increasing the speed of calculations the essence of which consists in the use of a higher-speed function approximation by reference values as compared to direct calculations. The results known in this field [1-5] were obtained mainly for universal computers processing a large memory. However, in connection with significant increasing the operational memory capacity (as a rule exceeding 32 MB) in personal computers of latter generations and due to vigorous widening a sphere of their application, of a great interest is the development and use of TAM for increasing the speed of calculations on PC depending on the features of their technological and system architecture.

Let us consider briefly the problem statement (in more details this is given in authors' paper [4]).

As is well-known, a direct calculation of a wide class of functions in the vicinity of the point $x_0 \in [a, b]$ can be replaced by executing the approximation of a function by its reference values, e.g. by polynomials of a special form or by truncated series expansion of this function. The calculation of the function $f(x)$ is realized by a set of operations {/, *, ±, sampling from the memory} arranged in order of decreasing times of execution $t_1 > t_2 > ... \geq\!\!/\ t_4$ corresponding to the operation. Thus, the calculation time $f(x)$ $t_f = \Sigma\, t_i\, n_i$ for the selected calculation environment is determined by the set $n_f = \{n_1, n_2, ..., n_4\}$, where n_1 equals to the number of division operations, n_2 - to the multiplication operations etc. Therefore, decrease of the calculation time can be achieved by reducing either all or first components at the expense of latter ones of the n_f set.

When selecting the algorithm of high-speed function calculation, the optimization by the n_f set in propagated TAM realizations leads to such significant increase in volumes of function reference tables (RT), and so to increase in time expenditures for search of adequate function nods in RT, that this can result in an effect being directly contrary to the expected speed increasing. Tending to obtain an acceptable accuracy of approximations without increasing table volumes in turn increases the time expenditures being reciprocal to the expected gain. So, for example, it is well known that the use of the best Chebyshev approximation provides the best uniform approximation among polynomials of the set exponent. However, then a significant time is spent for search of alternance points and for calculation of polynomial coefficients. Thus, we have before us the problem of finding a TAM realization with which the effect of increasing the speed of a directly approximating calculation does not lead to the competitive increase of time expenditures to support this approximation. In this respect adaptive aspects of RT applying emerge as a decisive factor.

Let us introduce the set $n^* = \{n_1^*, n_2^*, ..., n_5^*\}$, representing the function approximation operations by the optimally selected series : certainly, this set and the time of its realization $t^* = \Sigma\, t_i\, n_i^*$ should not be dependent on the choice of a function. Let t_m^f is the time necessary for construction by the reference points d_f of the RT function $f(x)$, which should be constructed and used taking in mind assurance of the acceptable accuracy of calculation approximation.

To estimate the effectiveness of TAM realizations developed we determine the coefficients K_a- of reducing the time of proper calculation and K_f - of reducing the time of full calculation. Then for a calculation session in which the function is calculated M_f times, $M_f >> d_f$, the coefficients introduced take the form

$$K_a = t_f / t^*, \quad K_f = M_f\, t_f / (t_m^f + (M_f - d_f)\, t^*) \qquad (1)$$

And with fixed t_f, t_m^f, d_f, t^*, it is evident

$$K_f \to K_a \qquad (2)$$

together with the increase of M_f. Such a situation takes place, of course, in calculation sessions where the number of function calls is accompanied by the essential localization of the function argument values.

A specific feature of universal computers designed for a wide range of problems has in consequence formation of universal software. Therefore, the programmed realization of TAM on a universal computer inevitably increases RT dimensions while embracing classes of problems or extending the range of function determination. This observation is confirmed also by TAM realization for set functions that have been performed in leading computer firms. In [4] one describes the realization allowing obtaining an appreciable acceleration of calculation. The mentioned above realization means in essence a personalization of calculations on universal computers.

In connection with an extensive application of PC and increase of their memory resource it is seem naturally to transfer and develop further TAM on PC. The present paper is aimed at presenting such a direction of calculation speed increase.

2. ADAPTIVE TAM REALIZATIONS

The calculation speed increase is based, according to formula (1), on reducing calculation for a wide class of functions to the standard approximation of functions over its self-stored reference values, so the effect of calculation time reducing within large sessions localized by calculation argument values approaches to the coefficient K_a in correspondence with (2). In this case the speed of this approximation is determined only by the character of RT construction.

The table gives the estimations for K_a obtained in a numerical experiment on PC being analogous to coefficients of calculation time reducing on universal computers obtained earlier. Their realization in C++ language, under conditions when a user has at its disposal practically unlimited resources of the operative memory and PC on the whole, of course, allows one to solve optimally the problem of direct approximation of calculations executed in short time t_m^f.

Function	K_a
$x^{16}/(1-x^8)$	7.
Integral Sine	4.
Gamma- function	2.5
Complete elliptic integral of the first kind	6.5
Generalized complete elliptic integral	8.0
Bessel function	70.

An important constituent part of TAM for increasing the speed of calculation are the operations with RT: their intrinsic organization, technique of finding nods in RT being the most suitable for approximation, ways of completing RT with new nodal values of a given function. As effective are the following mode of adaptive TAM realizations:

–table with intervals (is constructed immediately after as-determined interval),

–discrete RT (RT is completed with as-calculated nods,

–RT with one reference point (when the function argument changes monotonically).

Each of above listed modes of RT use has its own domain of preference determined by the function form and character of distributions required in the course of calculating the function argument values.

If in the calculation mix dominating in time are the parts being accelerated, then the total acceleration in the limit of large sessions, approaching to K_a, can be significant. The degree of approximation depends, as is shown in [4], on the spatial organization of RT, the order of appearance of values in the calculation session completing RT, and on the calculation volume.

A representative bank of results of TAM application for acceleration of calculations and effective algorithms of their realization are formed during development and generalization of experience in their service.

3. DEVELOPMENT OF TAM

On the base of tested algorithms used for increasing the speed of calculation of one-variable functions the following methods should be developed:

–direct methods for table-algorithmic accelerating the calculation of several-variables functions;

–methods of dimensionality model reduction for the available functional dependence or for its required domain of argument changes, and development, on the base of these methods, of TAM for increasing the speed of calculation;

–construction of space filling curves (Peano curves) for many-dimensional domains of function determining. Reducing of TAM for increasing the speed of calculation of many- variable functions to the table-algorithmic accelerating the calculation of one-variable function via superposition of the starting function and Peano mapping;

–TAM for increasing the speed of calculating the «by measure» functions i.e. approximating the given function with an acceptable accuracy, excepting a set of argument values with the measure of a set small value in which the function calculated directly and its "TAM equivalent" can differ considerably.

4. CONCLUSIONS

The necessity of application and development in the PC environment of executed TAM with an experience of using them for universal computers is substantiated. On the base of the expected acceleration it is shown that one should hope for considerable acceleration of time-consuming calculations when localizing domains of their determination.

REFERENCES

1. B.A. Popov, G.S.Tesler *Approximation of functions for technological applications*- Kiev: "Naukova dumka" (in Russian).
2. A.B. Vasil'ev at al. *Table-algorithmic method for function calculation for universal microcomputers*. L.: Publ. H. of Inst. of precise mechanics and optics, 1985, v. 3, p. 621 (in Russian).
3. V.I. Potapov, A.I. Florensov *Table–algorithmic calculation of functions on computers.* Irkutsk: Publ. House, 1985, p. 107 (in Russian).
4. E.V. Krivorukov, I.N. Shapoval. Programmed realization of table-algorithmic acceleration of function calculation. *Programming*. 1989, №3, p.78-87 (R).
5. E.V. Krivorukov, I.N. Shapoval. Development of algorithmic methods of increasing the speed of calculations // *VANT. Ser. Yad. fiz. issledovaniya, №1* (33), 1999, p. 123-124 (in Russian).