

# STABLE SPATIAL ENVELOPE WAVE STRUCTURES IN PLASMAS

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Conditions for formation of two- and three-dimensional localized envelope soliton-like structures in plasmas are investigated. These structures are described by modified nonlinear Schrödinger equation, including higher order linear and nonlinear effects. It is shown that higher order effects are crucial to explain formation of localized structures, which have been observed in plasmas. Stability of stationary soliton-like structures is investigated. Obtained results are applied to interpretation of the experiments on Langmuir solitons generation in electron-beam plasma systems.

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## INTRODUCTION

Formation of envelope coherent wave structures in plasmas and other nonlinear dispersive media can be rather universally described by nonlinear Schrödinger equation (NSE) with some additional terms. Nonlinear wave structure in the framework of NSE is often a subject to collapse – its size decreases and intensity grows, forming singularity in a finite time. However, for intense and narrow wave packets, the higher-order effects are to be taken into account to describe packet's evolution correctly. In the situation, when dissipation is not very essential, those higher-order effects are usually associated with the saturation of local nonlinearity, nonlinear dispersion (nonlocality) and with high-order wave dispersion. Spatial envelope wave structures with 2D azimuthal or 3D spherical symmetry may be described by the generalized NSE (GNSE) of the form

$$i \frac{\partial \psi}{\partial t} + D \Delta_r \psi + P \Delta_r^2 \psi + B \psi |\psi|^2 + K \psi |\psi|^4 + C \psi \Delta_r |\psi|^2 = 0, \quad (1)$$

where  $\Delta_r$  is the radial part of the Laplacian, terms proportional to D and P describe second and fourth order dispersion effects, terms proportional to B and K represent saturable cubic-quintic nonlinearity (BK<0), term proportional to C describes nonlocal interaction of the main wave with other waves or its self-interaction.

In different physical situations the importance of these higher-order terms may be different. For instance, formation of quasi-one-dimensional upper-hybrid solitons which have been observed experimentally in [1], can not be described in the framework of the model of GNSE (1) with B=K=P=0, DC<0 proposed in [2], since these model solitons were shown to be unstable with respect to collapse in [3]. At the same time, in [4,5] fourth-order linear dispersion was demonstrated to stabilize upper-hybrid solitons in the conditions of experiment [1]. The model, developed in [2] appears again in the theoretical attempts to explain formation of radially-elongated structures – streamers in tokamaks and their influence on the anomalous transport processes [6]. As it is well-known, in the space of more than one dimension, NSE (1) with C=K=P=0 has localized solution which is unstable with respect to collapse if DB>0 and has no localized solutions at all, if DB<0. In the first case (DB>0) any of higher-order effects may be sufficient to prevent collapse and stabilize soliton solutions of Eq. (1). The

stabilizing effect of saturable cubic-quintic nonlinearity [BK<0 in (1)] has been thoroughly investigated in the context of nonlinear optics both in 2D and 3D cases [7]. Effects of nonlocal nonlinearity (with BC>0) in 2D space were first studied in application to upper-hybrid solitons in plasmas in [8] and to formation of matter waves in Bose-Einstein condensates [9]. Simultaneous influence of high-order dispersion and cubic-quintic nonlinearity on the formation of whistler wave stationary waveguides in normal and anomalous dispersive regimes has been studied in [10]. In the case DB<0, the proper combination of higher order linear and nonlinear effects is able to explain formation of localized coherent structures observed in many experiments. Taking into account higher-order linear dispersion together with nonlinear dispersion (such that DP>0, DB<0, BC>0 in (1)), the observations [11] of strongly localized upper-hybrid structures in the anomalous dispersion regime were explained in [12]. A new type of stable soliton solutions, the so-called chirped solitons with nonlinearly changing phase were found analytically and numerically in 1D and in 2D [13-15].

In this paper we consider 2D and 3D soliton solutions of Eq. (1) with BD>0 in the presence of effects which stabilize the collapse. In Section 2 we show that in the 2D space stabilizing effects from all additional terms (proportional to K, C, and P) are summarized in a simple way. We show that in the presence of quintic (BK<0) nonlinearity only the wave packet in the diffractionless case [16] (DH<0, H being the Hamiltonian), can not be contracted to the infinitely small size for any value of wave power N and in the space of any dimension (d=1,2,3). In Section 3 we investigate analytically and numerically the stabilizing effect of nonlocal nonlinearity on 3D Langmuir wave structures in plasmas.

## COLLAPSE ARREST MECHANISMS

The equation (1) has the integrals: number of quanta

$$N = \int |\psi|^2 d\vec{r},$$

Hamiltonian

$$H = D \int |\nabla \psi|^2 d\vec{r} - P \int |A_i \psi|^2 d\vec{r} - \frac{B}{2} \int |\psi|^4 d\vec{r}$$

$$- \frac{K}{3} \int |\psi|^6 d\vec{r} + \frac{C}{2} \int \dot{\psi} \equiv DI_D - PI_P - \frac{B}{2} I_B - \frac{K}{3} I_K + \frac{C}{2} I_C$$

Also we choose in this paper the following signs of coefficients:  $D>0, B>0, P<0, K<0, C>0$ . In this case it is easy to show (similarly to that was done in [16]), that if the Hamiltonian is negative, any wave packet can not decrease it's amplitude  $|\psi_{\max}|$  infinitely. Indeed,

$$|H| < \frac{B}{2} \int |\psi|^4 d\vec{r} < \frac{B}{2} N |\psi_{\max}|^2,$$

and so that

$$|\psi_{\max}|^2 > \frac{2|H|}{BN}.$$

Therefore, for the negative Hamiltonian ( $H<0$ ) wave packet will not disperse. Using the Gulder inequality

$$I_B^2 < I_K N, \quad (2)$$

one estimates

$$DI_D < \frac{B}{2} I_B - \frac{K}{3} I_K < \frac{B}{2} I_B - \frac{K}{3} \frac{I_B^2}{N} < \frac{3}{16} \frac{B^2 N}{K}. \quad (3)$$

Further, employing the "uncertainty relation"

$$\frac{1}{N} \int |\psi|^2 r^2 d\vec{r} \equiv r_{\text{eff}}^2 > \frac{d}{4} \frac{N}{I_D} \quad (4)$$

where  $d$  is the number of space dimensions, we get

$$r_{\text{eff}}^2 > \frac{d}{4} \frac{N}{I_D} > \frac{d}{4} \frac{16}{3} \frac{KD}{B^2} = \frac{4d}{3} \frac{KD}{B^2}. \quad (5)$$

It is interesting to note that the minimum possible square radius of any wave packet in the case  $K \neq 0$  does not depend on  $N$  in space of any dimension, but it increases with number of dimensions. Equation (5) gives also the estimate of minimum square radius for soliton solution if  $H<0$ . Note that this statement is true also for saturable nonlinearity which is typical for plasma waves, when instead of  $B|\psi|^2\psi - K|\psi|^4\psi$  in Eq. (1) we have

$$B\psi f(|\psi|^2) = B\psi \frac{(1 - \exp(-\kappa|\psi|^2))}{\kappa}.$$

Then we get estimate for the Hamiltonian

$$H \geq DI_D - B \int \left[ \int_0^{|\psi|^2} f(x) dx \right] d\vec{r} =$$

$$DI_D - \frac{B}{\kappa^2} \int \left[ \kappa|\psi|^2 - (1 - \exp(-\kappa|\psi|^2)) \right] d\vec{r} > DI_D - \frac{B}{\kappa}.$$

If  $H<0$  then  $DI_D < BN/\kappa$  and thus  $r_{\text{eff}}^2 > d D\kappa/4B$ .

Let us consider in more details 2D space, which corresponds, in particular, to stationary propagation of wave beams. In this case, from the integral inequality

$$I_D \geq 5.84 I_B / N \quad (6)$$

we see that if  $H<0$  then

$$\frac{B}{2} I_B > DI_D \geq D \frac{5.84 I_B}{N} \quad (7)$$

or  $N > 21.36 D/B = N_{\text{cr}}$ . In the other hand, in 2D space and for  $P=0$ , the virial relation takes rather simple form

$$N \frac{d^2 r_{\text{eff}}^2}{dt^2} = 8D \left[ H + \frac{|K|}{3} I_K + \frac{C}{2} I_C \right]. \quad (8)$$

For soliton solutions the r.h.s. of Eq. (8) is equal to zero, which means that  $H<0$  if any of the coefficients ( $K$  or  $C$ ) does not vanish. Thus, the condition (7) is a necessary condition for soliton formation in 2D space. We have seen before that if  $K \neq 0, H<0$ , the wave packet can non contract infinitely.

Suppose that  $N < N_{\text{cr}}$ . In this case  $H>0$  and the relation (8) can be rewritten as

$$N \frac{d^2 r_{\text{eff}}^2}{dt^2} = 8D \left[ 2H - DI_D + \frac{B}{2} I_B \right] < \dot{\psi}$$

$$\dot{\psi} < 8D \left[ 2H - DI_D + \frac{BN}{11.7} I_D \right],$$

where we have used the integral inequality (7). Using also the relation (8), which gives

$$N \frac{d^2 r_{\text{eff}}^2}{dt^2} > 8 DH, \text{ we get}$$

$$8 DH < N \frac{d^2 r_{\text{eff}}^2}{dt^2} < \frac{8D}{N} \left[ 2H - D \left( 1 - \frac{N}{N_{\text{cr}}} \right) I_D \right]$$

For  $N < N_{\text{cr}}$ :

$$8 DH < 8D \left[ 2H - \frac{1}{r_{\text{eff}}^2} D \left( 1 - \frac{N}{N_{\text{cr}}} \right) N \right], \text{ or}$$

$$r_{\text{eff}}^2 > \frac{DN}{2H} \left( 1 - \frac{N}{N_{\text{cr}}} \right). \quad (9)$$

Thus, for  $N < N_{\text{cr}}$ , the wave packet collapse is also impossible.

Let us show that in the 2D space, Hamiltonian is bounded from below if  $D \geq 0, B > 0, C \geq 0, P \geq 0, K \leq 0$ , and at least one of the coefficients  $C, P$  or  $K$  does not vanish. In addition to (7), (8), integrals  $I_C$  and  $I_P$  are to be estimated in terms of integral  $I_B$ . Using Golder inequalities we have

$$\int |\psi|^8 d\vec{r} \geq \frac{1}{N} \left( \int |\psi|^5 d\vec{r} \right)^2 \geq \frac{I_B^4}{N \int |\psi|^3 d\vec{r}} \geq \frac{I_B^3}{N^2}$$

With the help of inequality (7) this gives the estimation

$$I_C \geq \frac{5.84}{I_B} \int |\psi|^8 d\vec{r} \geq 5.84 \frac{I_B}{N^2}. \quad (10)$$

Also we have

$$I_P \geq \frac{1}{N} \left( \int |\nabla \psi|^2 d\vec{r} \right)^2 \geq (5.84)^2 \frac{I_B^2}{N^3}. \quad (11)$$

Thus

$$H \geq D \frac{5.84 I_B}{N} - \frac{B}{2} I_B + \frac{|K| I_B^2}{3 N} + \frac{C}{2} \frac{5.84 I_B^2}{N^2} + \frac{(5.84)^2 |P| I_B^2}{N^3} \geq i - \frac{1}{16} \frac{(BN - 11.7D)^2}{KN/3 + 5.84C + (5.84)^2 |P|/N}$$

which was required to prove. For any soliton solution,  $H < 0$ . Thus, there exists at least one soliton state which gives minimum to the Hamiltonian and hence is stable.

### STABLE LANGMUIR SOLITONS

Here the impact of nonlocal nonlinearity  $C > 0$  will be studied here in the connection with the Langmuir solitons (LS). Formation of LS are connected with the plasma extrusion from the regions of strong high-frequency electric field and trapping of plasmons into the formed density well (caviton). However, in the previous simplified theoretical models, 2D and 3D solitons occur to be unstable with respect to a collapse: above some threshold power. Nevertheless, experimental observations [17,18] of 3D Langmuir collapse demonstrate saturation of wave-packet's spatial scale at some minimum value being of order few tens of Debye radii. It have been observed in Refs. [17,18] that at times  $t > 50\omega_{pi}$ , where  $\omega_{pi}$  is the ion plasma frequency, Langmuir wave packets show considerably slow dynamics (subsonic regime). To our best knowledge, these observations do not meet an appropriate theoretical explanation yet. As it is shown in [19], the local part of electron-electron nonlinearity counteracts the contraction of wave packet. At the same time, the nonlocal contribution of the additional nonlinear term was omitted in [19], though it is of great importance for sufficiently narrow and intense wave packets. As it will be shown below, the role of nonlocal nonlinearity is quantitatively even more significant.

We consider subsonic motions and neglect the terms with time derivatives in the second equation of above set. As result, this set is reduced to the single partial differential equation:

$$i \frac{\partial E}{\partial t} + D \frac{\partial}{\partial r} r^{-2} \frac{\partial}{\partial r} r^2 E + BE |E|^2 + CE \Delta_r |E|^2 - \Gamma \frac{E |E|^2}{r^2} = 0 \quad (12)$$

where the coefficients  $D$ ,  $B$ ,  $C$ ,  $\Gamma$  are given by the expressions:

$$D = \frac{3}{2} \omega_p r_D^2, \quad B = \frac{\omega_p}{32 \pi M n_0 c_s^2}, \\ C = \frac{7}{96 \pi m n_0 \omega_p}, \quad \Gamma = \frac{1}{48 \pi m n_0 \omega_p}.$$

The nonlinear part of this equation includes common cubic nonlinearity (term proportional to  $B$ ) as well as nonlocal

(term proportional to  $C$ ) and local (term with  $\Gamma$ ) parts of electron-electron nonlinearity. Let us show that the effective width  $r_{eff}$  (4) of any stationary and non-stationary wave packet governed by Eq. (12) is bounded from below in the most interesting case of self-trapped wave packets having negative Hamiltonian

$$H = D \int |r^{-2} \partial_r r^2 E|^2 d\vec{r} - 0.5 B \int |E|^4 d\vec{r} + 0.5 C \int (\nabla |E|^2)^2 d\vec{r} + 0.5 \Gamma \int r^{-2} |E|^4 d\vec{r} < 0$$

Using the inequality

$$\int |\nabla E|^2 d\vec{r} \geq 0.25 \int r^{-2} |E|^2 d\vec{r},$$

which is valid in 3D space, one finds for  $H < 0$ :

$$B \int |E|^4 d\vec{r} > C \int (\nabla |E|^2)^2 d\vec{r} + \Gamma \int r^{-2} |E|^4 d\vec{r} \geq i (\Gamma + C/4) \int r^{-2} |E|^4 d\vec{r} \quad (13)$$

On the other hand we have

$$\int (\alpha r |E| - r^{-1} |E|^2) d\vec{r} = \alpha^2 \int r^2 |E|^2 d\vec{r} + \int r^{-2} |E|^4 d\vec{r} - 2\alpha \int |E|^3 d\vec{r} \geq 0$$

for any positive  $\alpha$ . Using also G\ulder inequality

$$\int |E|^3 d\vec{r} \leq \left( \int |E|^2 d\vec{r} \right)^{1/2} \left( \int |E|^4 d\vec{r} \right)^{1/2},$$

we obtain for any  $\alpha$

$$\alpha^2 \int r^2 |E|^2 d\vec{r} - 2\alpha \left( \int |E|^2 d\vec{r} \right)^{1/2} \left( \int |E|^4 d\vec{r} \right)^{1/2} + \int r^{-2} |E|^4 d\vec{r} \geq 0$$

Requirement that the discriminant of the l. h. s. of the last inequality is negative gives

$$r_{eff}^2 \geq \int |E|^4 d\vec{r} / \int r^{-2} |E|^4 d\vec{r}.$$

Finally, using also the relation (13) we get

$$r_{eff}^2 > \frac{\Gamma + C/4}{B}.$$

Soliton solutions have a form  $E = \psi(r) \exp(-i\lambda\tau)$ , where  $\lambda$  is the nonlinear frequency shift of the soliton. To obtain qualitative information about the soliton properties let us consider the variational approach with trial function

$$E(r) = N^{1/2} a^{-3/2} (r/a) \exp(-r^2/a^2) = i N^{1/2} f(\xi) \quad (14)$$

so that  $\int |E|^2 d\vec{r} = N$ . Substitution of (14) into the Hamiltonian gives

$$\frac{H}{N} = \frac{DI_d}{a^2} - \frac{BI_b N}{a^3} + \frac{(CI_c + \Gamma I_\gamma) N}{a^5},$$

where

$$I_d = 4\pi \int_0^\infty [\xi^{-2} \partial_\xi \xi^2 f]^2 \xi^2 d\xi,$$

$$I_b = 2\pi \int_0^\infty f^4 \xi^2 d\xi, \quad I_c = 2\pi \int_0^\infty (\partial_\xi f^2)^2 \xi^2 d\xi,$$

$$I_\gamma = 2\pi \int_0^\infty f^4 d\xi.$$

The standard variational procedure gives connection between number of quanta  $N$  and characteristic size of the soliton  $a$ :

$$N = \frac{2 da^3}{3 ba^2 - 5c}, \quad (15)$$

where  $d=DI_d$ ,  $b=BI_b$ ,  $c=CI_c + \Gamma I_\gamma$ . The dependence  $N(a)$  which follows from the Eq. (15) is similar to that presented in Fig. 1. It follows from (15) that  $a^2 > 5c/(3b) = a_{\min}^2$ . In the other hand,  $N > N_{\min} = (d/b)a_{\max}$ , where  $a_{\max} = (5c/b)^{1/2} > a_{\min}$ . When soliton solution is perturbed, the phase variation with spatial coordinates appears. Taking this into account, one can find for small deviations of parameter  $a$  from soliton's parameter  $a_0$  which satisfies the equation (15) (see, e.g. [20]):

$$\frac{N}{2d} \frac{d^2}{dt^2} (a - a_0) + \frac{\partial^2 H}{\partial a^2} (a - a_0) = 0.$$

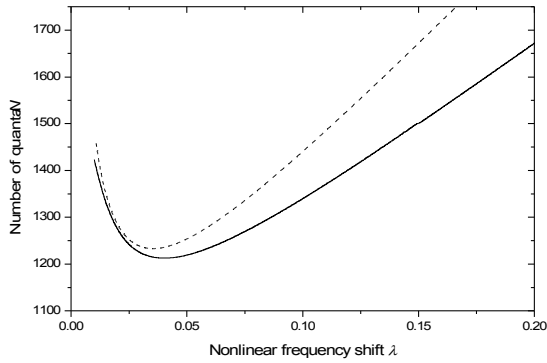


Fig. 1. The number of quanta versus the nonlinear frequency shift for 3D Langmuir solitons. Variational prediction is plotted by the dashed line

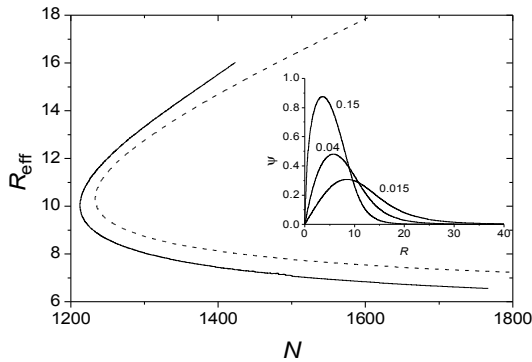


Fig. 2. The effective soliton radius versus the number of quanta. Dashed line presents variational dependence. On the inset, the soliton profiles are presented for different  $\lambda$

Soliton is stable if  $\partial^2 H / \partial a^2 > 0$ . It is easy to show that solitons are stable if they belong to the lower branch of the curve  $N(a_0)$  (15), when  $N < N_{\min}$  and  $a_{\min} < a_0^2 < a_{\max}$ . From our variational approach it follows that

$$\frac{\partial N}{\partial a} = \frac{\partial^2 H / \partial a^2}{-\partial^2 H / \partial a \partial N} = \frac{\partial^2 H / \partial a^2}{\partial \lambda / \partial a},$$

and hence

$$\frac{\partial N}{\partial \lambda} = \frac{\partial N}{\partial a} \frac{\partial a}{\partial \lambda} = \frac{\partial^2 H}{\partial a^2} \left( \frac{\partial a}{\partial \lambda} \right)^2.$$

Thus, in the framework of the variational approach, the soliton stability condition  $\partial^2 H / \partial a^2 > 0$  coincides with Vakhitov-Kolokolov criterion. Variational results were found to be in a very good agreement with our numerical calculations.

The radial soliton profiles  $\psi(R)$  are found from the equation:

$$-\lambda \psi + \frac{\partial^2 \psi}{\partial R^2} + \frac{2}{R} \frac{\partial \psi}{\partial R} - \frac{2}{R^2} \psi + \psi |\psi|^2 + \frac{7}{2} \psi \Delta_R |\psi|^2 - \frac{\psi |\psi|^2}{R^2} = 0,$$

where the dimensionless variables are used:

$$R = \sqrt{\frac{3}{2}} \frac{r}{r_D}, \quad \tau = \frac{9}{4} \omega_p t, \quad \psi = E / \sqrt{72 \pi n_0 T_e}.$$

Stationary states of Eq. (12) were investigated numerically, and the results are summarized in Figures 1 and 2. Figure 1 presents the energy dispersion diagram, number of quanta versus the nonlinear frequency shift for 3D Langmuir solitons, while in the Fig. 2, the effective soliton's radius defined by (4) is plotted versus the soliton power. In 3D, there are two soliton branches, one, corresponding to  $\partial N / \partial \lambda > 0$ , is stable, and the other is unstable. It is clear, that analytical predictions found by the approximate variational approach (dashed lines in figures) are in a good agreement with our exact numerical calculations. With the increase of soliton power, the effective radius decreases and saturates at values being of order  $6r_{De}$ .

## CONCLUSIONS

Saturation of the nonlinearity, as well as non-local wave interaction is able to arrest the wave collapse. The new estimations for minimum possible size of any stationary and non-stationary wave packets are found. Stable solitons may be formed due to any of these higher order nonlinear effects. We have performed analytical and numerical studies of spatial Langmuir solitons in the framework of model based on

generalized nonlinear Schrödinger equation including both local and nonlocal electron-electron nonlinearities. Their influence on intense and narrow Langmuir wave packets are of the same order. Any of them is able to arrest the Langmuir collapse. Both nonlinearities lead to the saturation of soliton width with an increase of the energy, but quantitatively the effect of nonlocal nonlinearity is more significant. In the 3D case, two soliton branches coexist, one is stable and the other is unstable.

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## УСТОЙЧИВЫЕ ПРОСТРАНСТВЕННЫЕ ВОЛНОВЫЕ СТРУКТУРЫ ОГИБАЮЩИХ В ПЛАЗМЕ

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Изучены условия формирования двумерных и трёхмерных солитоноподобных структур огибающих в плазме. Такие структуры описываются модифицированным нелинейным уравнением Шрёдингера, учитывающим линейные и нелинейные эффекты высшего порядка. Показано, что эффекты высшего порядка принципиально важны для объяснения образования локализованных структур огибающих, которые наблюдаются в плазме. Исследована устойчивость стационарных солитонных структур. Полученные результаты применяются для интерпретации экспериментов по возбуждению ленгмюровских солитонов в плазменно-пучковых системах.

## **СТІЙКІ ПРОСТОРОВІ ХВИЛЬОВІ СТРУКТУРИ ОГИНАЮЧИХ В ПЛАЗМІ**

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Вивчено умови формування двовимірних та тривимірних солітоноподібних структур огинаючих в плазмі. Такі структури описуються модифікованим нелінійним рівнянням Шредінгера, що враховує лінійні та нелінійні ефекти вищого порядку. Показано, що ефекти вищого порядку є принципово важливими для пояснення утворення локалізованих квазидвовимірних та тривимірних структур огинаючих, що спостерігаються в плазмі. Досліджено стійкість стаціонарних солітонних структур. Отримані результати використовуються для пояснення експериментів із збудження ленгмюрових солітонів у плазмово-пучкових системах.

