NONLINEAR THEORY OF SYMMETRIC SURFACE WAVES EXCITATION AT CYLINDRICAL WAVEGUIDE STRUCTURES 'PLASMA - VACUUM - METAL' BY ELECTRON BEAMS

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This report is devoted to the non-linear theory of symmetrical potential surface waves (SWs) excitation by low-density tubular beam in cylindrical waveguide structure 'plasma - vacuum - metal'. The system of non-linear equations describing the temporary dynamics of plasma - beam interaction is obtained. The influence of beam and waveguide structure parameters on saturation amplitude and SW excitation is investigated both analytically and numerically. The process of beam thermalization in SW field is considered.
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1. INTRODUCTION

The intensive researches of plasma - beam instabilities are important for creation of plasma generators, electromagnetic waves amplifiers, and charged particles accelerators [1-4]. The charged particles beam use is perspective for creation of plasma - beam discharges also [5]. To solve these problems it is necessary to investigate a nonlinear stage of plasma - beam instability development [1, 6] for determination of saturation levels and physical mechanisms controls of excitation efficiency and spectrum of excitation waves.

As opposed to the case of unlimited plasma, the efficiency of wave excitation by electron beam in the bounded plasma essential depends on transverse sizes of plasma and beam [1]. It is due to that SW electric field in bounded plasma depends on a transverse coordinate with respect to the direction of beam propagation. So the potential well, in which the beam particles will be trapped, is nonuniform both in longitudinal and transverse direction. It results in distinction in dynamics of particles with various radial coordinates. It influences on increments, saturation amplitude and efficiency of SW excitation.

This report presents the study of influence of tubular electron beam and waveguide structure parameters on values of increment, saturation amplitude, excitation efficiency and beam thermalization process in symmetric potential surface wave field.

2. TASK STATEMENT

Let us consider the excitation of symmetric high-frequency surface potential waves [7] by an electron tubular beam that propagates in a metal cylindrical waveguide structure of a radius b, partially filled by homogeneous plasma with density n_{oe} and radius $a < b$. We assume that the tubular electron beam with interior R_{1b} and exterior R_{2b} radiuses completely situated in the plasma region ($R_{2b} < a$). The initial beam velocity is equal V_{ob} and its density n_{ob} is much less than the plasma one $n_{ob} << n_{oe}$. There is a vacuum gap between metal and plasma ($a < r < b$).

Assume also that there is an external axial steady magnetic field that magnetizes the beam particles, but is weak for plasma ones, so that the condition $\omega_{pe}^2 >> \omega_{ce}^2 >> \omega_{be}^2$ is valid. Here ω_{ce} is electron cyclotron frequency, ω_{pe}, ω_{be} are Langmuir's frequencies of plasma and beam respectively. In this case it is possible to consider the motion of beam particles as one-dimensional [1].

3. LINEAR THEORY RESULTS

At first let us consider the linear theory results of plasma - beam instability development, which describe the initial stage of symmetric SW excitation by electron tubular beam in a metal waveguide, partially filled with plasma. According to this theory, the greatest increment of symmetric SW excitation:

$$\delta = 2^{-4/3}\sqrt{3}\,(\beta\, n_{ob}/n_{oe})^{1/3}\omega_0(k_3) \qquad (1)$$

is achieved at resonant plasma - beam instability [1]. Thus the excited SW frequency is determined by following expression

$$\omega = \omega_0(k_3)[1 - 2^{-4/3}(\beta\, n_{ob}/n_{oe})^{1/3}], \qquad (2)$$

where

$$\beta = [F(k_3 R_{2b}) - F(k_3 R_{1b})]/[2k_3 a\, I_0(k_3 a) I_1(k_3 a)],$$

$$F(y) = y^2[I_0^2(y) - I_1^2(y)], \quad R_{2b} \le a.$$

The SW wavenumber is determined from resonant condition:

$$k_3 = \omega_0(k_3)/V_{ob}. \qquad (3)$$

In expressions (1) - (3)

$$\omega_0(k_3) = \omega_{pe}\Big[1 - I_0(k_3 a)/I_1(k_3 a) *$$

$$* \frac{I_1(k_3 a)K_0(k_3 b) + K_1(k_3 a)I_0(k_3 b)}{I_0(k_3 a)K_0(k_3 b) - K_0(k_3 a)I_0(k_3 b)}\Big] \qquad (4)$$

is an eigen frequency of a waveguide structure at beam absence [1, 7]; I_m, K_m are modified Bessel and McDonald functions of the order m. The influence of beam geometric parameters on increment (1) and frequency (2) of SW is described by parameter β [1].

The main characteristics of plasma-beam interaction are the wave saturation amplitude and the excitation efficiency. Let us consider the nonlinear theory of SW resonant excitation by electron tubular beam for their determination.

3. NONLINEAR THEORY OF SWS

Let us use a macroparticles method for describing the electron beam. In the considered case the macroparticles are represented as rings with radial r_i and axial z_i coordinates. They fill all beam volume. Let's assume that on a wavelength λ there are N macroparticles. Then the beam volume charge density can be written as follows:

$$\rho(\vec{r},t) = -\frac{e}{r}\sum_{i=1}^{N}\delta[r - r_i(t)]\delta[z - z_i(t)], \qquad (5)$$

where $\delta(\xi)$ is delta-function, e is electron charge.

To find the amplitude and phase of SW let us use the quasihydrodynamics equations and Poisson one, taken into account (5). In the case of resonant wave excitation it is possible to derive the following self-consistent set of equations describing the nonlinear interaction of beam-particles with SW:

$$\left.\begin{aligned}
&\frac{d\varepsilon}{d\tau} = \frac{1}{\sigma}\sum_{i=1}^{N} I_0(x_i)\sin[\xi_i + \varphi],\\
&\frac{d\varphi}{d\tau} = \frac{1}{\varepsilon\sigma}\sum_{i=1}^{N} I_0(x_i)\cos[\xi_i + \varphi],\\
&\frac{d\eta_i}{d\tau} = -\varepsilon I_0(x_i)\sin[\xi_i + \varphi],\\
&\frac{d\xi_i}{d\tau} = \eta_i,\ \frac{dx_i}{d\tau} = 0.
\end{aligned}\right\} \qquad (6)$$

Here

$$\varepsilon = \frac{ek_3^2\psi_0}{m_e\delta_0^2},\ \eta_i = \frac{k_0(V_i - \omega_0/k_3)}{\delta_0},\ \tau = \delta_0 t,\ x_i = k_3 r_i,$$

$$\delta_0^3 = \frac{\Omega_b^2}{\partial D/\partial\omega|_{\omega_0,k_3}}\frac{1}{2k_3 a I_0^2(k_3 a)},\ \xi_i = 2\pi\frac{z_i - t\omega_0/k_3}{\lambda},$$

$N = \sigma(R_2^2 - R_1^2)$, $R_j = k_3 R_{jb}$, $\sigma = n_{ob}\lambda^3/(4\pi)$.

This system has the following motion integrals:

$$\frac{\varepsilon^2}{2} + \frac{1}{\sigma}\sum_{i=1}^{N}\eta_i = const,\quad \varepsilon^2\frac{d\varphi}{d\tau} - \frac{1}{2\sigma}\sum_{i=1}^{N}\eta_i^2 = const. \qquad (7)$$

At nonlinear stage of plasma-beam instability the wave amplitude oscillates nearby steady-state saturation value ε_{sat}.

Let's consider the influence of parameters of tubular beam on ε_{sat}. The analytical expression for ε_{sat} can be written from first equation of (7):

$$\varepsilon_{sat}^2/2 = k_3^2(R_{2b}^2 - R_{1b}^2)(\omega - \omega_0)/\delta_0. \qquad (8)$$

Here the beam particle thermalization at saturation stage is taken into account. It is obtained a good accordance of normalized frequency shift due to beam presence $(\omega - \omega_0)/\delta_0$ obtained from linear theory (2) with its value from numerical integration of the system (6). This circumstance allow to use the linear frequency shift (2) to estimate the SW saturation amplitude:

$$\varepsilon_{sat} = (k_3^2R_{2b}^2 - k_3^2R_{1b}^2)^{1/2}[F(k_3R_{2b}) - F(k_3R_{1b})]^{1/6}. \qquad (9)$$

The results of numerical integration of set (6) and the saturation amplitude value calculated according expression of (9) are represented in fig. 1. The SW saturation amplitude grows with increase of the beam width. Moreover the beam width growth leads to decrease of phase velocity and results in increase of the energy transmitted from beam to SW. Some difference between numerical and analytical results (fig. 1) is caused by deviation of phase velocity at non-linear stage from its value predicted by linear theory.

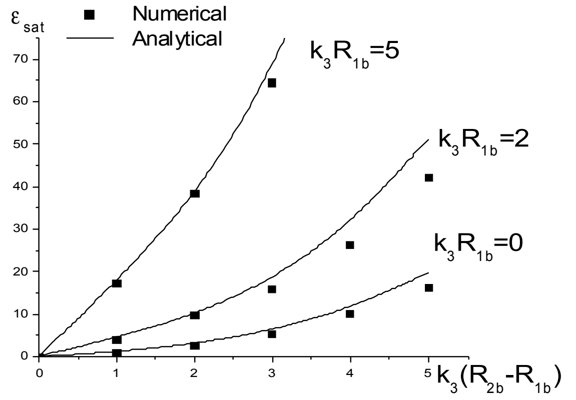

Fig. 1. Dependence of SW saturation amplitude on geometrical beam parameters

The expression (9) becomes simpler in the cases of 'wide' ($k_3R_{1b} >> 1$) or 'narrow' ($k_3R_{2b} << 1$) beams:

$$\varepsilon_{sat} = \begin{cases}\kappa^{2/3},\ k_3R_{2b} << 1,\\ \kappa^{1/2}[(e^{2k_3R_{2b}} - e^{2k_3R_{1b}})/(2\pi)]^{1/6}, k_3R_{1b} >> 1,\end{cases} \qquad (10)$$

where . To large amplitude SW excite it is expedient to use the 'wide' beams. It is caused by growth of beam particles in a strong electrical field region with increase the inner radius of beam and its width. Therefore for SW excitation it is expedient to use an entire beams with $R_{2b} = a$. In this case it is achieved the greatest values of ε_{sat}:

$$\varepsilon_{sat}^{\max} = (k_3a)^{4/3}[I_0^2(k_3a) - I_1^2(k_3a)]. \qquad (11)$$

The SW saturation amplitude grows with beam density increase as $\psi_0 \propto n_{ob}^{2/3}$. It is a result of increasing of energy density that can be transmitted from beam to SW.

The SW saturation amplitude dependence on initial beam velocity and metal waveguide radius has a more complicated character. This is caused by complicated influence these parameters on wavenumber (fig. 2).

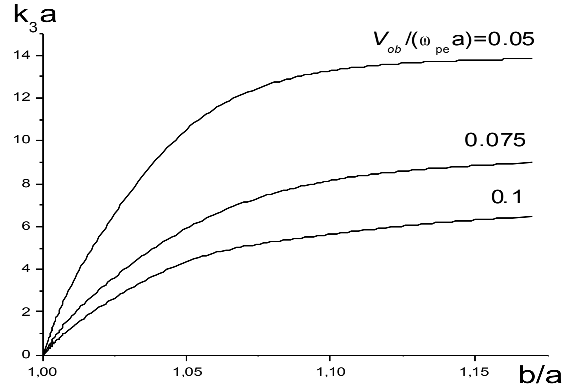

Fig. 2. Dependence of SW wavenumber on initial beam velocity and metal waveguide radius

The analysis of the numerical calculations has shown that decrease of vacuum gap between metal to plasma results in decreasing of k_3. The similar effect takes place at increase of initial beam velocity. Therefore to determine the dependence of saturation amplitude on metal waveguide radius and initial beam velocity we shall use the following dimensionless wave amplitude:

$$\varepsilon_1 = e\psi_0 /(m_e \omega_{pe}^2 a^2)(n_{ob} / n_{oe})^{-2/3} . \qquad (12)$$

The wavenumber decrease caused by metal waveguide coming close to plasma or increase of initial beam velocity leads to increase of saturation amplitude value (fig.3).

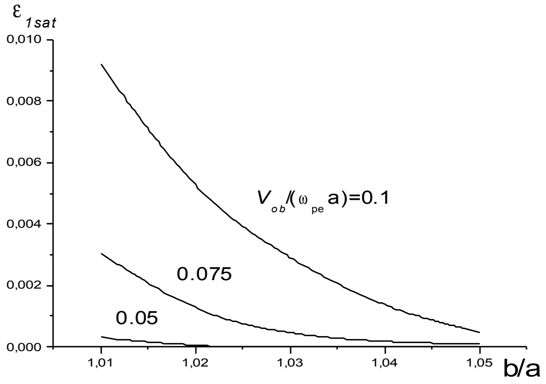

Fig. 3. The influence of initial beam velocity and metal waveguide radius on saturation amplitude

5. EFFICIENCY OF SW EXCITATION

Let us define the parameter E as the ratio of wave energy to initial beam energy. This parameter describes the SW excitation efficiency and can be written in form:

$$E = (\omega_0 - \omega)/\omega_0 = 0.5(\beta\, n_{ob} /(2n_{oe}))^{1/3} \qquad (13)$$

Thus, the SW excitation efficiency grows with beam density increase. The dependence of efficiency E on other parameters of beam and waveguide structure is determined by parameter β .

The analysis of expression (13) has shown that the efficiency increase with growth of tubular beam width. In spite of the fact that 'wide' beams can excite the SW with greater amplitude, the efficiency of this excitation is rather low in comparison with 'narrow' beams.

Thus maximum efficiency (as well as the saturation amplitude value) can be achieved in the case of entire beams that completely occupy the plasma region:

$$E_{\max} = \left(\frac{n_{ob}}{n_{oe}}\right)^{1/3} \left[\frac{k_3 a}{32}\left(\frac{I_0(k_3 a)}{I_1(k_3 a)} - \frac{I_1(k_3 a)}{I_0(k_3 a)}\right)\right]^{1/3} \qquad (14)$$

Thus, the long waves can be more effectively excited than short ones. Therefore to increase the efficiency it is expedient to use the beams with $k_3 a << 1$.

6. BEAM THERMALIZATION

At nonlinear stage of plasma-beam instability occurs beam thermalization process.

During thermalization the beam particles are mixed in phase plane with creation of plateau on electrons distribution function. The plateau width is determined by thermal velocity value V_{Tb}. This process leads to heating initially cold beam. The value of V_{Tb} can be determined from expression (7):

$$V_{Tb} = \sqrt{3}\, V_{ob}\, E . \qquad (15)$$

Thus, according to (15) the electron beam thermal velocity is proportional to excitation efficiency. Therefore, the influence of beam and waveguide structure parameters on value V_{Tb} / V_{ob} is same as well as on SW excitation efficiency. It is necessary to note that the value of V_{Tb} is rather small. It is evidence of the hydrodynamic approach validity to description of resonant plasma-beam instability.

7. CONCLUSIONS

The nonlinear theory of symmetric potential SWs excitation by low-density tubular electron beam in cylindrical waveguide structure 'plasma - vacuum - metal' is considered. The system of the nonlinear equations describing temporal dynamics of plasma - beam interaction is obtained. The influence of beam and waveguide structure parameters on saturation amplitude and SWs excitation efficiency is numerically and analytically investigated. It is shown that to large amplitudes SWs excite it is expedient to use the entire beams, which completely occupy the plasma region. It is obtained that wavenumber decrease caused by coming close of metal to plasma or growth of initial beam velocity result in increase of a maximum accessible value of SW saturation amplitude.

It is obtained that the efficiency of excitation grows with width of tubular beam. It is shown that the 'wide' beams preferably to use for excitation of greater amplitude SW. But in this case the excitation efficiency is small as contrasted with its value for 'narrow' beams.

The thermalization process of beam particles in wave field is investigated. It is shown that the thermal velocity of beam electrons is proportionally to efficiency of plasma - beam interaction.

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