POLARIZATION CURRENT THRESHOLD MODEL OF NEOCLASSICAL TEARING MODES IN THE PRESENCE OF ANOMALOUS PERPENDICULAR VISCOSITY

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1. The primary theory of neoclassical tearing modes (NTMs) [1-3] predicted that, as a result of the bootstrap drive, magnetic islands can be generated in tokamak discharges with favorable current profile, i.e. for $\Delta' < 0$, where Δ' is the standard parameter of tearing mode theory [4]. Then, the island width W should be smaller than a maximal one W_{max} proportional to the parameter beta (the ratio of plasma pressure to the magnetic field pressure), i.e.

$$W \leq W_{max}, \tag{1}$$

where

$$W_{max} = \frac{\beta C_b}{(-\Delta')}, \tag{2}$$

C_b is a constant.

However, the theory [1-3] did not explain, which β should be substituted into this expression for W_{max}. Then one could suggest that generation of NTMs is possible for arbitrarily low β. Meanwhile, expreimental data on TFTR [5] have shown that these modes are generated only if β exceeds some critical (threshold) value β_{crit},

$$\beta \geq \beta_{crit}. \tag{3}$$

Then the question arose: how one should modify the theory [1-3] in order to explain the critical beta for NTM onset.

As a result, two threshold models of NTMs, allowing one to predict β_{crit}, have been formulated: the polarization current threshold model [6] and the transport threshold model [7] (such a terminology has been introduced in [8]). The present work is addressed to further development of the first of these models, while the companion work [9] summarizes recent developments of the transport threshold model. The goal of the present work is to incorporate anomalous perpendicular viscosity into the polarization current threshold model.

2. The polarization current threshold model of NTM looks as [6]

$$\frac{dW}{dt} \sim \frac{\Delta'}{4} + \Delta_{bs} + \Delta_p, \tag{4}$$

where Δ_{bs} and Δ_p are responsible for the bootstrap drive and polarization current effect, respectively. Starting expression for Δ_p is [10]

$$\Delta_p = \frac{1}{c\widetilde{\psi}}\left(\frac{2L_s}{B_0}\right)^{1/2} \int_{-\widetilde{\psi}+\varepsilon}^{-\infty} d\psi \oint \frac{J_\parallel \cos\xi d\xi}{\left(\widetilde{\psi}\cos\xi - \psi\right)^{1/2}}. \tag{5}$$

Here $J_\parallel$ is the oscillatory part of the parallel current density, ψ is the magnetic flux function introduced by

$$\psi = \widetilde{\psi}\cos\xi - x^2 B_0/2L_s, \tag{6}$$

$\widetilde{\psi}$ is a constant related to W by

$$\widetilde{\psi} = 16W^2 B_0/L_s, \tag{7}$$

ξ is the island cyclic variable, L_s is the shear length, B_0 is the equilibrium magnetic field, c is the speed of light, ε is a positive infinitesimal.

To find $J_\parallel$ one should solve the current continuity equation

$$\nabla_\parallel J_\parallel + \boldsymbol{\nabla}_\perp \cdot \mathbf{j}_\perp = 0, \tag{8}$$

where $\mathbf{j}_\perp$ is the perpendicular current density, $\nabla_\parallel$ and $\boldsymbol{\nabla}_\perp$ are the parallel and perpendicular gradients. For simplicity we neglect drift effects. Then, for obtaining $\boldsymbol{\nabla}_\perp \cdot \mathbf{j}_\perp$ one can use the perpendicular projection of the single-fluid motion equation

$$\rho_0 \frac{d_0 \mathbf{V}}{dt} = \frac{1}{c}\left[\mathbf{j}_\perp \times \mathbf{B}\right] - \boldsymbol{\nabla} p - \boldsymbol{\nabla}\cdot\left(\boldsymbol{\pi}_\perp + \boldsymbol{\pi}_\parallel\right), \tag{9}$$

where

$$d_0/dt = \partial/\partial t + \mathbf{V}\cdot\boldsymbol{\nabla}, \tag{10}$$

$\mathbf{V}$ is the plasma velocity (its structure is explained below), ρ_0 is the equilibrium plasma mass density, p is the plasma pressure, $\boldsymbol{\pi}_\perp$ and $\boldsymbol{\pi}_\parallel$ are the perpendicular and parallel viscosity tensors, respectively, $\mathbf{B}$ is the total magnetic field.

In the Braginskii [11] approximation, the perpendicular viscosity tensor gradient is given by

$$\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_\perp = -\rho_0\mu_\perp\left(\boldsymbol{\nabla}_\perp^2 \mathbf{V}_\perp + 4\mathbf{b}\boldsymbol{\nabla}_\perp^2 V_\parallel\right), \tag{11}$$

where $\mu_\perp$ is the perpendicular viscosity coefficient, $\mathbf{b}$ is the unit vector along the total magnetic field. On the other hand, according to equation (19.6) of [12], the parallel viscosity tensor gradient can be expressed in terms of the parallel viscosity scalar $\pi_\parallel$ by

$$\nabla\cdot\boldsymbol{\pi}_\parallel = \frac{3}{2}\mathbf{b}\left(\mathbf{b}\cdot\nabla\right)\pi_\parallel - \frac{1}{2}\nabla\pi_\parallel + \frac{3}{2}\pi_\parallel\left[\mathbf{b}\nabla\cdot\mathbf{b}+\left(\mathbf{b}\cdot\nabla\right)\mathbf{b}\right]. \quad (12)$$

This scalar satisfies the relation (see, for details, chapter 19 of [12] and, in particular, Eqs. (19.19), (19.73), (19.78) of [12])

$$\frac{3}{2}\left\langle \pi_\parallel \frac{\partial \ln\sqrt{g_m}}{\partial\theta}\right\rangle_\theta = r_s\rho_0\hat{\chi}_\theta\left(V_y+\frac{\epsilon}{q}V_\parallel\right). \quad (13)$$

Here g_m is the metric tensor determinant, $\langle\ldots\rangle_\theta$ is the averaging over the poloidal angle θ,

$$\hat{\chi}_\theta = \frac{q^2}{\epsilon^{1/2}}\left(\frac{d_0}{dt}+\frac{\nu_i}{\epsilon}\right). \quad (14)$$

The y - direction is defined by the unit vector $\mathbf{y}=\mathbf{b}\times\mathbf{x}$, $\mathbf{x}$ is the unit vector along the x - direction, where $x=r-r_s$, r_s is the radial coordinate of the rational magnetic surface, where the island chain is localized, V_y is the y - projection of the cross-field velocity $\mathbf{V}_\perp$ given by

$$\mathbf{V}_\perp = c\left[\mathbf{b}_0\times\nabla\phi\right]/B_0, \quad (15)$$

ϕ is the electrostatic potential. Remaining definitions in (13), (14) are: q is the safety factor, ϵ is the inverse aspect ratio for $r=r_s$, ν_i is the ion collision frequency. The function $V_\parallel$ is the oscillatory part of the parallel plasma velocity.

As a result, according to [12], the value $\nabla_\perp\cdot\mathbf{j}_\perp$ is given by

$$\nabla_\perp\cdot\mathbf{j}_\perp = -\frac{c\rho_0}{B_0}\frac{\partial}{\partial x}\left[\hat{\chi}_\theta\left(V_y+\frac{\epsilon}{q}V_\parallel\right)\right]. \quad (16)$$

To find $V_\parallel$ we use the parallel projection of Eq. (9). Then, allowing for (13), one can find (see, for details, [12, 13])

$$\left(\frac{d_0}{dt}+\frac{\epsilon^2}{q^2}\hat{\chi}_\theta - 4\mu_\perp\frac{\partial^2}{\partial x^2}\right)V_\parallel + \frac{\epsilon}{q}\hat{\chi}_\theta V_y = 0. \quad (17)$$

3. We consider the problem of interest qualitatively, i.e. changing $\partial^2/\partial x^2\to -1/W^2$, $d_0/dt\to -i\omega$. It then follows from (17) that

$$\mathrm{Re}V_\parallel = gqV_y/\epsilon, \quad (18)$$

where

$$g=\frac{\nu_i^2\left[1+\epsilon^{3/2}\left(1+W_\mu^2/W^2\right)W_\mu^2/W^2\right]+\epsilon^{1/2}\omega^2}{\nu_i^2\left(1+W_\mu^2/W^2\right)^2+\omega^2/\epsilon} \quad (19)$$

and

$$W_\mu \simeq \left(\frac{\mu_\perp}{\epsilon^{1/2}\nu_i}\right)^{1/2} \quad (20)$$

is the characteristic island width governed by the perpendicular viscosity. The function

$$g = g\left(\nu_i,\omega,\mu_\perp\right) \quad (21)$$

characterizes the collisionality dependence of the polarization current effect. It is introduced by the relation

$$\Delta_p = g\Delta_p^\infty, \quad (22)$$

where

$$\Delta_p^\infty = \left.\Delta_p\right|_{\nu_i\to\infty}. \quad (23)$$

In the limit of vanishing perpendicular viscosity, $W_\mu/W\to 0$, Eq. (19) reduces to [14]

$$g=\frac{\nu_i^2+\epsilon^{1/2}\omega^2}{\nu_i^2+\omega^2/\epsilon}. \quad (24)$$

Then the function g is given by [14]

$$g=\begin{cases}\epsilon^{3/2}, & \nu_i/\left(\epsilon\omega\right)<C_0,\\ \epsilon\nu_i^2/\omega^2, & C_0<\nu_i/\left(\epsilon\omega\right)<\epsilon^{-3/2},\\ 1, & \nu_i/\left(\epsilon\omega\right)>\epsilon^{-3/2},\end{cases} \quad (25)$$

where

$$C_0\simeq\epsilon^{3/4}. \quad (26)$$

In the limit $W_\mu/W\to\infty$ one has from (19)

$$g=\epsilon^{3/2}, \quad (27)$$

which is the same as the first line of the right-hand side of (25). Thus, for sufficiently large perpendicular viscosity one deals with the minimal value g relevant to the limiting case of weak collisions.

4. The expressions for the function g given by Eq. (25) were found in the linear approximation [14]. Let us show that they are qualitatively valid also in the nonlinear regime.

One can find that Eq. (17) with $\mu_\perp=0$ and $\hat{\chi}_\theta$ of form (14) leads to

$$\left(\epsilon^{1/2}\nu_i-\omega\frac{\partial h}{\partial x}\frac{\partial}{\partial\xi}\right)V_\parallel = \frac{\omega}{k_y}\frac{q}{\epsilon^{1/2}}\left(\nu_i-\epsilon\omega\frac{\partial h}{\partial x}\frac{\partial}{\partial\xi}\right)\left(\frac{\partial h}{\partial x}-\left\langle\frac{\partial h}{\partial x}\right\rangle\right), \quad (28)$$

where $h=h\left(\psi\right)$ is the electrostatic potential profile function [10]. In the limit of weak collisions, $\nu_i\to 0$, it hence follows that

$$V_\parallel = \epsilon^{1/2}q\frac{\omega}{k_y}\left(\frac{\partial h}{\partial x}-\left\langle\frac{\partial h}{\partial x}\right\rangle\right). \quad (29)$$

This corresponds to $g=\epsilon^{3/2}$, see the first line of the equation (25).

In the opposite case of strong collisions, $\nu_i \to \infty$, instead of (29), one has from (28)

$$V_\parallel = \frac{q}{\epsilon}\frac{\omega}{k_y}\left(\frac{\partial h}{\partial x} - \left\langle \frac{\partial h}{\partial x} \right\rangle\right). \qquad (30)$$

This yields $g = 1$, see the third line of (25).

To analyze (28) for finite $\nu_i/(\epsilon\omega)$ we represent $V_\parallel$ as the sum of the cosine and sine parts, i.e. the even and odd parts (with respect to the variable ξ),

$$V_\parallel = V_c + V_s. \qquad (31)$$

Then we arrive at the following two equations

$$\omega\frac{\partial h}{\partial x}\frac{\partial V_c}{\partial \xi} = \epsilon^{1/2}\nu_i V_s + \epsilon^{1/2} q \frac{\omega^2}{k_y}\frac{\partial h}{\partial x}\frac{\partial^2 h}{\partial \xi \partial x}, \qquad (32)$$

$$\omega\frac{\partial h}{\partial x}\frac{\partial V_s}{\partial \xi} = \epsilon^{1/2}\nu_i \left[V_c - \frac{q}{\epsilon}\frac{\omega}{k_y}\left(\frac{\partial h}{\partial x} - \left\langle \frac{\partial h}{\partial x} \right\rangle\right)\right]. \qquad (33)$$

The polarization current is defined by the function V_c.

Note that for $\nu_i/(\epsilon\omega) < \epsilon^{-3/4}$ the contribution of V_s into (32) can be neglected. Then V_c proves to be the same as for $\nu_i/(\epsilon\omega) < 1$, i.e. V_c is given by the right-hand side of (29). On the other hand, if the ratio $\nu_i/(\epsilon\omega)$ lies in the interval $\epsilon^{-3/4} < \nu_i/(\epsilon\omega) < \epsilon^{-3/2}$, the equation system (32) and (33) reduces to

$$\omega\frac{\partial h}{\partial x}\frac{\partial V_c}{\partial \xi} = \epsilon^{1/2}\nu_i V_s, \qquad (34)$$

$$\omega\frac{\partial h}{\partial x}\frac{\partial V_s}{\partial \xi} = -\nu_i \frac{\omega}{k_y}\frac{q}{\epsilon^{1/2}}\left(\frac{\partial h}{\partial x} - \left\langle \frac{\partial h}{\partial x} \right\rangle\right). \qquad (35)$$

It hence follows that in order of magnitude

$$V_c \simeq q\frac{\nu_i^2}{\omega k_y}\left(\frac{\partial h}{\partial x} - \left\langle \frac{\partial h}{\partial x} \right\rangle\right). \qquad (36)$$

This corresponds to $g = \epsilon\nu_i^2/\omega^2$, cf. the second line of (25).

The above nonlinear substantiation of of the collisionality dependence (24) for $\mu_\perp \to 0$ allows one to suggest that by means of more complicated nonlinear analysis for $\mu_\perp \neq 0$, one can justify qualitatively behavior of the function $g(\nu_i, \omega, W)$ given by (19).

5. According to [6], dependence of β_{crit} on the function g, $\beta_{crit}(g)$, characterizing the collisionality dependence of NTMs, is given by

$$\beta_{crit} \sim g^{1/2}. \qquad (37)$$

Such a collisionality dependence was the subject of experimental studies [15]. Following [6], it was assumed in [15] that the function g has the step-like form with a jump in a region of sufficiently large $\nu_i/\epsilon\omega$. Then the authors of [15] have concluded that their experimental data corroborate the theory [6]. However, according to [15], the form of the function g suggested in [6] is unadequate, so that, instead of g given by [6], one should use g of form (25). Then one can find that experimental data of [15] is in disagreement with the polarization current threshold model.

According to (19), (27), such a disagreement is redoubled in the presence of anomalous perpendicular viscosity. As a whole, this decreases attractiveness of the polarization current threshold model. Then, in order to find β_{crit} one should appeal to the transport threshold model [9] or to the theory of β-limiting sub-Larmor modes [16].

Acknowledgments

This work was supported by the Russian Federal Program on Support of Leading Scientific Schools, Grant No. 00-15-96526, the Research Support Foundation of the State of São Paulo (FAPESP), University of São Paulo, and Excellence Research Programs (PRONEX) RMOG 50/70 Grant from the Ministry of Science and Technology, Brazil.

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