INSTABILITIES, INDUCED BY A NOISE

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The dynamics of systems, parameters of which are subject to casual forces are considered. The casual action can be as external, and to be generated by own random dynamics of a system. The most important features of such systems dynamics are chosen. First of all, their behavior is characterized by an alternation. In these systems (even linear) the properties, which are characteristic for a stochastic resonance, can be to exist. Instabilities, which are induced by a noise practically are not stabilized by nonlinearities.
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The real physical systems are under action of fluctuations. It is useful to select a special class of oscillatory systems, on which the effect of fluctuations revealed in development of parametrical instability. It happens when the fluctuations are multiplicate, i.e. randomly change parameters of an investigated oscillatory system. Frequently in such systems all moments will became unstable. And the increment of each following moment is more previous. In this case process of instability has an alternated character. It is necessary to mark, that, apparently, this case is unique case, in which the higher moments acquire the concrete physical contents. Really, in overwhelming majority of cases for the description of a physical system it is enough to know a behavior of first two moments. Below we shall formulate the most important features of systems, instability of which are induced by a noise.

1. Dynamics of a linear system, which has an alternated character, is similar to nonlinear dynamics. As an example on Figure 1 the dependence of a linear oscillator amplitude is represented, which frequency is subject to random disturbances. The equation of such oscillator has a form

$$\ddot{x} + \omega^2 \cdot (1 + \xi(t)) \cdot x = 0 \qquad (1)$$

Where $\xi(t)$ - white noise.

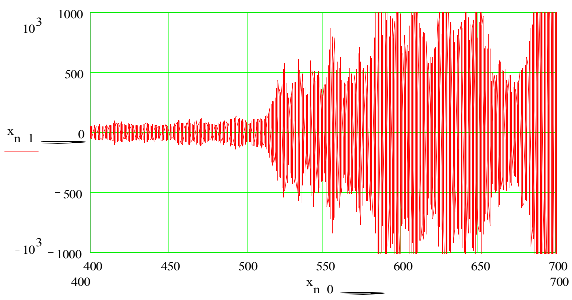

Fig.1 An example of an alternation in oscillations of a unstable linear oscillator.

All moments of such oscillator (beginning with second) are unstable. The increments of each following are more previous. To the equation (1) the analysis of large number of plasma systems is reduced. From Figure it is visible, that dynamics of such linear oscillator has an alternated character. Such feature of linear dynamics is necessary to take into account because it is similar to dynamics generated by nonlinear processes. Let's give the brief proof of the formulated statements. For determinancy we shall be count, that the function ξ has the following properties: $\langle \xi(t) \rangle = 0$, $\langle \xi(t) \cdot \xi(t_1) \rangle = 2D\delta(t - t_1)$. Here angular brackets mean statistical averaging on a casual ensemble $\xi(t)$.

Using variational method (see, for example, [1]) from the equation (1) it is possible to receive the following system $(n+1)$ ordinary differential equations for determination of moments of the order n for displacements $(x(t))$ and velocities $(y = \dot{x})$:

$$\frac{d}{dt}\langle y^p \cdot x^{n-p} \rangle = -p \cdot \omega^2 \langle y^{p-1} \cdot x^{n-p+1} \rangle + p(p-1) \cdot$$
$$\cdot \omega^4 D \langle y^{p-2} \cdot x^{n-p+2} \rangle + (n-p) \langle y^{p+1} \cdot x^{n-p-1} \rangle \quad (2)$$

The analysis of a system (2) shows, that beginning from the second moment all moments are unstable. And the increment of each following moment is more previous. The behaviour of such system, as is known [2], is characterized by an alternation. This statement can be easy to proof for a unstable oscillator $(\omega^2 < 0)$. For the proof it is convenient a set of equations (2) to transform in a vector form. For this purpose we shall enter function $Y_p = \langle y^p \cdot x^{n-p} \rangle$. Using this function, is possible to present set of equations (2) as:

$$\frac{dY_p}{dt} = A_{p,i} \cdot Y_i \qquad p, i = 0,1,2,\ldots..n \qquad (3)$$

The matrix $A_{p,i}$ under condition of $\omega^2 < 0$ is non-negative, undecomposable matrix. From the Perron-Frobenius theorem [3] follows, that the greatest positive eigenvalues of such matrix will increase when anyone from elements $A_{p,i}$ of this matrix will be magnified. Magnification of intensity of fluctuations D, and also the magnification order n of the moments results in magnification of these elements.

2. Important is that fact that the casual modification of parameters can happen as result of dynamic chaos development. The important example of such systems is the movement of a charged particle in an external magnetic field and in a field of an external electromagnetic wave. As is known [4], when the amplitude of the wave is sufficiently large the movement of particles becomes randomly. The magnitude of a

particle energy randomly varies. The energy of a particle is one from main parameters for description of particle dynamics in an external constant magnetic field. The casual modification of this parameter can result in development of instability with features, which we have described above. As an example we shall consider the most simple configuration, when the external flat electromagnetic wave is propagate in a direction that is a perpendicular to the direction of an external homogeneous magnetic field. And, the polarization of this wave is those, that a magnetic component of this wave field coplanar to an external constant magnetic field. Let, besides in an initial time the particles did not have component of a velocity directed along a constant magnetic field. In this case equations describing dynamics of particles in such fields are most simple and look like:

$$\begin{aligned} &\dot{x} = p_x/\gamma, \qquad p_z = 0 \\ &\dot{p}_x = \omega_H \cdot [\alpha\cdot\cos(x-\tau)+1]\cdot p_y/\gamma \\ &\dot{p}_y = -\omega_H \cdot [\alpha\cdot\cos(x-\tau)+1]\cdot p_x/\gamma + \\ &+ H\cdot\cos(x-\tau) \end{aligned} \qquad (4)$$

where $\alpha \equiv H_w/H_0$ - is the ratio strength of wave magnetic field to the strength 0f external uniform magnetic field. In (4) we have used such dimensionless variables: $x \to kx$, $\tau \to \omega t$, $p \to p/mc$,

$H = eH_w/mc\omega$ -is dimensionless parameter of wave force, $\omega_H = eH_0/mc\cdot\omega$.

The set of equations (4) was analyzed numerically for following parameters: $\omega_H = 05$, H=0.9, $\alpha = 1.8$. Characteristic dynamics of a particle impulse changing is represented in fig.2. It is visible, that it has an alternation character. This dynamics, besides is characterized by local instability.

3. The instabilities, induced by a noise, have features of a stochastic resonance. It is revealed, for example, so. Let parameters of a system vary simultaneously under the regular (periodically) law and on noise. Let, besides, the periodic modification of parameters is such, that the parametrical instability can develop. If the amplitude of a periodic modification of parameters is insufficiently great for reaching a threshold of instability, the introduction of casual modulation of these parameters can result in instability arising. Thus, the energy of external casual force can promote development of instability in a considered system. As an example we shall consider dynamics of an oscillator, which is described by the following equation

$$\ddot{x} + \nu\cdot\dot{x} + \omega^2[1+\xi(t)+A\cos(2\omega\cdot t)]\cdot x = 0 \quad (5)$$

In this equation A - amplitude of regular parametrical force; the function $\xi(t)$ characterizes noise action. Using this equation it is easy to find such values of parameters, which under operation of one regular parametrical force does not result in instability; does not result in development of instability and effect only of noise force. However joint operation of these two forces results in development of instability. And, dynamics of this instability is characterized by an alternation. The higher level of casual force, the more clear alternation. As an example on fig.3 the solution of the equation (5) for $\nu = 0.01$, $A = 0.003$, $A_N = 0.11$ is represented.

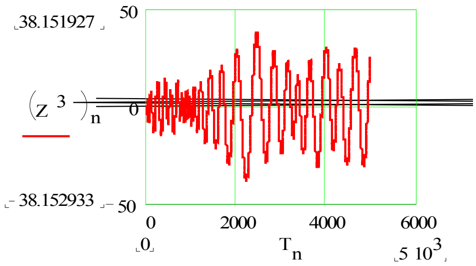

Fig.2. Dependence of transversal impulse on time for $\nu = 0.01$, $A = 0.003$, $A_N = 0.11$

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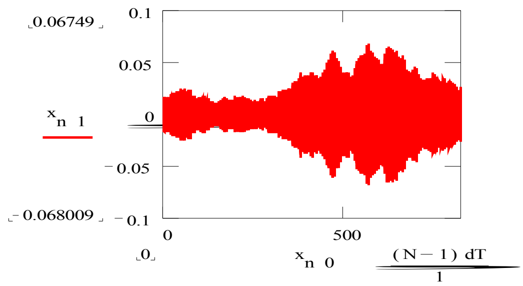

Fig.3 Modification of an oscillator amplitude for simultaneous operation of regular and noise forces $\nu = 0.01$, $A = 0.003$, $A_N = 0.11$

4. The special role the multiplicate fluctuations play when they act on unstable systems. In this case they can radically change dynamics of a system. Practically always in these cases the alternation will be realized. Under it only on the certain time frame (or distance) regular dynamics is saved. Outside of this interval dynamics - is chaotic. The large intensity of a noise, the shorter this interval. It is important, that instabilities, which are induced by a noise are not stabilized by nonlinearities. Really, in this case nonlinear frequency drift is compensated by a broad spectrum of a noise signal.

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