ANGLES OF ROTATIONAL TRANSFORM BEHAVIOR WITH PLASMA PRESSURE VARIATIONS IN THE TORSATRON

Yu.K. Kuznetsov, I.B. Pinos, V.I. Tyupa

National Science Center "Kharkiv Institute of Physics and Technology", Akademichna st.1, Kharkiv, 61108, Ukraine

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1. INTRODUCTION

At present, rather high plasma pressure values are obtained with stellarator systems. Yet, as is known, the plasma pressure can affect both the angle of rotational transform and the mean magnetic well value [1-7]. With an increasing plasma pressure the magnetic axis is displaced to the outward of the torus, and this can lead to an increase in the angle of rotational transform on the magnetic axis, a decrease in the magnetic system shear, and also, to stripping of outer magnetic surfaces (i.e., decrease in the plasma radius). The presence of various resonance perturbations at a small shear of field lines can cause the magnetic surfaces to split into separate rosettes. The existence of rosettes, great in size, essentially reduces the stability of magnetic surfaces in closed helical traps [8, 9]. On the other hand, an increasing displacement, r_c/r_0, gives rise to the magnetic well, and this results in the removal of restriction on the ultimate plasma-stability pressure for some most dangerous MHD instabilities. In view of this, it is of importance to know the distributions of both the angle of rotational, determined by plasma pressure.

For different plasma pressure profiles, the authors have calculated the angles of rotational transform as functions of the parameter α that characterizes the profile of vacuum angles of field line rotation. Three laws of plasma pressure distribution over vacuum magnetic surfaces were considered: $P=P_0$, $P=P_0(1-\psi(r)/\psi(r_0))$; $P=P_0(1-\psi(r)/\psi(r_0))^2$, where P_0 is the plasma pressure on the magnetic axis, ψ(r) is the averaged function of vacuum magnetic surfaces. The distribution of vacuum angle of rotational transform was calculated as $t(r)=t(r_0)[\alpha+(1-\alpha)r^2/r_0^2]$, where $\alpha=t(0)/t(r_0)$ is the ratio of the angle of field line rotation on the magnetic axis to its value at the plasma boundary of radius r_0.

2. ANALYTICAL CALCULATIONS

The angles of rotational transform due to different plasma pressure profiles were calculated by the analytical formulae derived with the use of the formulae for averaging over magnetic surfaces [5, 6, 10]. For the distributions $P=P_0$ and $P=P_0(1-\psi(r)/\psi(r_0))$ the angle of rotational transform was calculated by the following expression

$$\frac{\langle t(r_1)\rangle}{t(r_0)} = a\sqrt{(1+c)^2-b^2}\,\sqrt{1-\frac{4c(1+c)}{b^2}\left(1-\sqrt{1-b^2/(1+c)^2}\right)}\,, \quad (1)$$

where *a, b, c* are written for the $P = P_0$ pressure distribution as

$$a = \alpha + (1-\alpha)\frac{r_1^2}{r_o^2} + 2(1-\alpha)\frac{r_c^2}{r_o^2},$$
$$b = \frac{3(1-\alpha)}{a}\frac{r_c r_1}{r_o^2},\quad c = \frac{1-\alpha}{a}\frac{r_c^2}{r_o^2} \quad (2)$$

For the distribution $P=P_0(1-\psi(r)/\psi(r_0))$, these coefficients have the following form:

$$a = \alpha + (1-\alpha)\frac{r_1^2}{r_o^2} + 2\frac{r_c^2}{r_o^2}\,\frac{1-(1-\alpha)r_c^2/2r_o^2}{1-3r_c^2/2r_o^2};$$
$$b = \frac{3r_1 r_c}{2ar_o^2}\,\frac{\alpha+2(1-\alpha)(1-r_c^2/r_o^2)}{1-3r_c^2/2r_o^2}; \quad (3)$$
$$c = \frac{1-\alpha}{a}\frac{r_c^2}{r_o^2}\,\frac{1-(1-\alpha)r_c^2/2r_o^2}{1-3r_c^2/2r_o^2}.$$

For the distribution $P=P_0(1-\psi(r)/\psi(r_0))^2$ the angle of field line rotation was calculated as

$$\frac{\langle t(r_1)\rangle}{t(r_o)} = $$
$$= \frac{aa_1}{\frac{1}{\sqrt{p+2\sqrt{q}}}\left(A+\frac{B}{\sqrt{q}}\right)+\frac{1}{\sqrt{p_1+2\sqrt{q_1}}}\left(C+\frac{D}{\sqrt{q_1}}\right)} \quad (4)$$

where

$$D = \frac{q_1+\frac{p_1}{q_1}+\frac{q-q_1}{p-p_1}\left(3-p_1-\frac{1}{q_1}\right)-3}{\frac{q-q_1}{p-p_1}\left(1-\frac{q}{q_1}\right)-p+q\frac{p_1}{q_1}},$$

$$A = \frac{D\left(1-\frac{q}{q_1}\right)-3+p+\frac{1}{q_1}}{p-p_1},\quad B = \frac{1-qD}{q_1},$$

$$C = \frac{3-p_1-\frac{1}{q_1}-D\left(1-\frac{q}{q_1}\right)}{p-p_1},$$

where

$$p = \frac{b_1+\sqrt{8y+b_1^2-4c_1}}{2},\; q = y+\frac{b_1 y-d_1}{\sqrt{8y+b_1^2-4c_1}},$$

$$p_1 = \frac{b_1-\sqrt{8y+b_1^2-4c_1}}{2},\; q_1 = y-\frac{b_1 y-d_1}{\sqrt{8y+b_1^2-4c_1}},$$

$$y = \sqrt[3]{-q_o+\sqrt{E_o}}+\sqrt[3]{-q_o-\sqrt{E_o}}+\frac{c_1}{6},\; E_o = q_o^2+p_o^3,$$

where

$$p_o = \frac{b_1 d_1 - 4e_1}{12} - \frac{c_1^2}{36},$$

$$q_o = -\frac{c_1^3}{216} + \frac{c_1 b_1 d_1}{48} - \frac{e_1 b_1^2 + d_1^2}{16} + \frac{e_1 c_1}{6},$$

$$a_1 = 1 - b + c - d + e,$$

$$b_1 = \frac{4 - 2b - 4c + 14d - 28e}{a_1},$$

$$c_1 = \frac{6 - 10c + 70e}{a_1}$$

$$d_1 = \frac{4 + 2b - 4c - 14d - 28e}{a_1}$$

$$e_1 = \frac{1 + b + c + d + e}{a_1},$$

$$a = \alpha + (1-\alpha) r_1^2/r_o^2 + 2(1-\alpha) r_c^2/r_o^2 -$$
$$- 2N r_c/r_o \{-2 + 6M\alpha r_c^2/r_o^2 + 3M(1-\alpha) r_c^4/r_o^4 +$$
$$+ 3M[2\alpha + 3(1-\alpha) r_c^2/r_o^2] r_1^2/r_o^2 + 3M(1-\alpha) r_1^4/r_o^4\},$$

$$b = r_1/a r_o \{3(1-\alpha) r_c/r_o - N[3(-1 + 9M\alpha r_c^2/r_o^2 +$$
$$+ 15/2M(1-\alpha) r_c^4/r_o^4) +$$
$$+ 5M[\alpha + 9/2(1-\alpha) r_c^2/r_o^2] r_1^2/r_o^2$$
$$+ 7/4(1-\alpha) M r_1^4/r_o^4]\},$$

$$c = r_c/a r_o \{(1-\alpha) r_c/r_o -$$
$$- N[-2 + 8\alpha M r_c^2/r_o^2 + 9/2M(1-\alpha) r_c^4/r_o^4 +$$
$$4M[2\alpha + 4(1-\alpha) r_c^2 r_o^2] r_1^2/r_o^2 + 9/2(1-\alpha) M r_1^4/r_o^4]\},$$

$$d = -\frac{3MN r_c^2 r_1}{a r_o^3}\left[\alpha + \frac{5}{4}(1-\alpha)\frac{r_c^2}{r_o^2} + \frac{5}{4}(1-\alpha)\frac{r_1^2}{r_o^2}\right],$$

$$e = -\frac{MN(1-\alpha)}{a}\frac{r_c^3 r_1^2}{r_o^5}, \quad (5)$$

$$N = \frac{[\alpha + (1-\alpha) r_c^2/r_o^2] r_c/r_o}{\frac{2}{3}\frac{2+\alpha}{1+\alpha} - 3\frac{r_c^2}{r_o^2} + M[5\alpha\frac{r_c^4}{r_o^4} + \frac{7}{4}(1-\alpha)\frac{r_c^6}{r_o^6}]} \quad (6)$$

$$= \frac{\beta A_o}{t^2(r_o)(1+\alpha)}, M = \frac{2}{3(1+\alpha)},$$

where $A_0 = R/r_0$ is the aspect ratio, R is the major radius of the torus, $\beta = P_0/(H^2/8\pi)$ is the plasma pressure-to-magnetic pressure ratio.

Figures 1,2 show the angles of rotational transform determined by different plasma pressure profiles as functions of the average radius r_1/r_0 for the magnetic axis displacement $r_c/r_0 = 0.3$. The plots are given for the parameter α that characterizes the profile of the vacuum angle of rotational transform (Fig.1: a- α=0, b- α =0.2, c- α=0.4; Fig.2: a- α=0.6, b- α=0.8, c- α=1.0).

3. CONCLUSIONS

The calculations have shown that for the magnetic systems with a great magnetic shear (α<<1, Uragan-3M) the distributions of angles of field line rotation (Fig.1 a, b) are weakly dependent on the plasma pressure profiles (at the same magnetic axis displacements r_c/r_0).

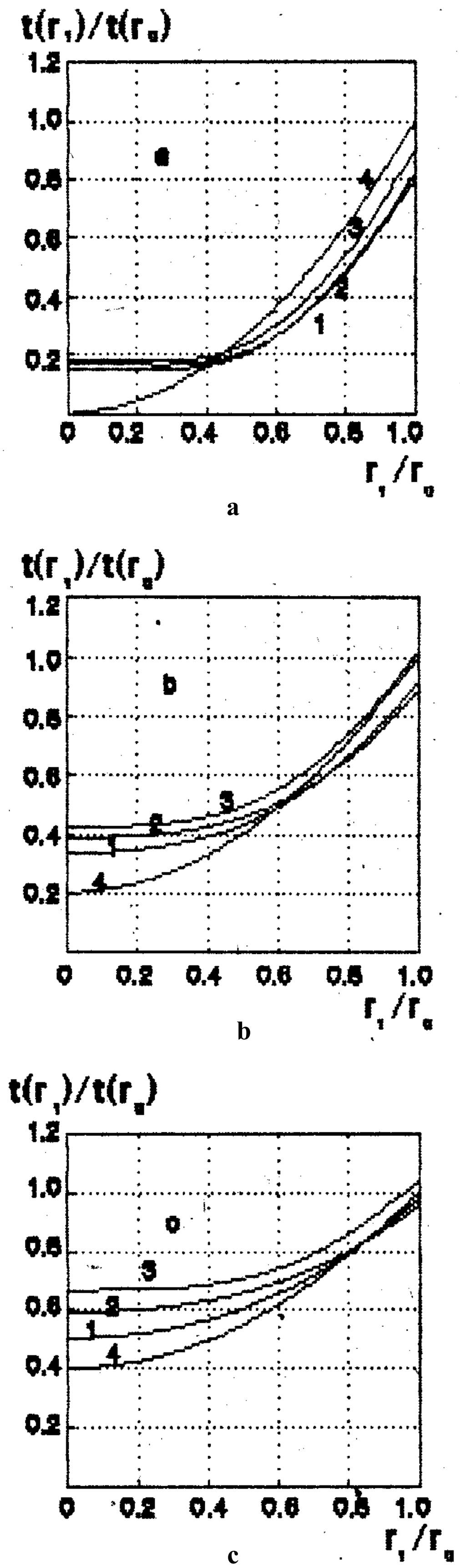
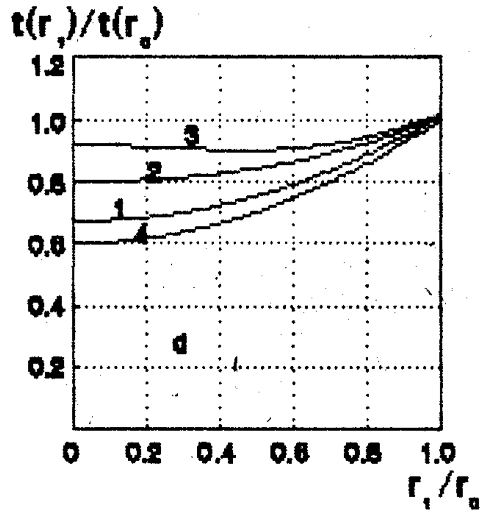
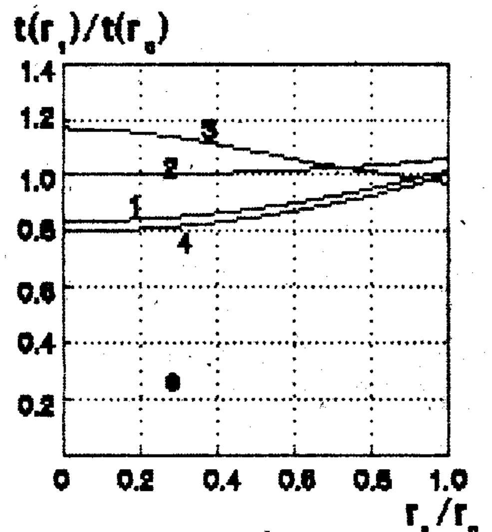


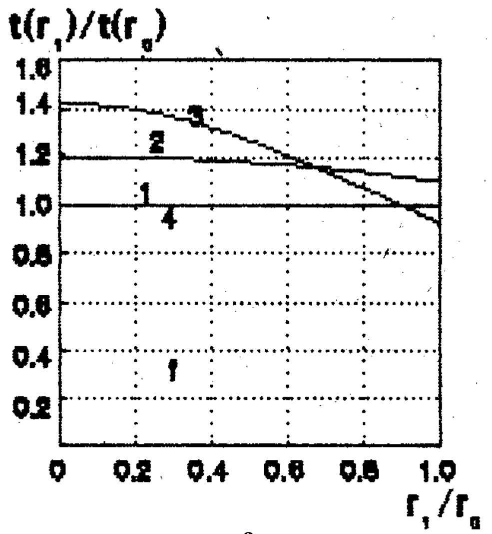
Fig.1. Distributions of angles of rotational transforms resulting from different plasma pressure profiles R/r_0 =8.9 (1: $P=P_0$; 2: $P=P_0(1-\psi(r)/\psi(r_0))$; 3: $P=P_0(1-\psi(r)/\psi(r_0))^2$; 4: P=0)



a



b



c

Fig.2. Distributions of angles of rotational transforms resulting from different plasma pressure profiles R/r_0 =8.9 (1: $P=P_0$; 2: $P=P_0(1-\psi(r)/\psi(r_0))$; 3: $P=P_0(1-\psi(r)/\psi(r_0))^2$; 4: $P=0$)

As α increases (shear of vacuum-configuration field lines decreases), the behavior of angles of field line rotation specified by different plasma pressure profiles becomes different (Fig.2 a, c)/ The sharper distribution $P=P_0(1-\psi(r)/\psi(r_0))^2$ gives rise to large angles of rotational transform in the central region of the magnetic configuration. This reduces the shear and, in the presence of resonance perturbations, can cause the magnetic surfaces to split.

With α approaching unity (small shear of field lines) and at P=P (sloping distribution), the vacuum angle of rotational transform practically retains its initial profile (Fig.2 b, c). This means that the magnetic surfaces are displaced under the plasma pressure to the outward of the torus without changing their form.

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