ON INFLUENCE OF EXTERNAL LOW FREQUENCY HELICAL PERTURBATION ON TOKAMAK EDGE PLASMA

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1. INTRODUCTION

The Dynamic Ergodic Divertor (DED) of TEXTOR is installed to control plasma edge behaviour [1]. The DED helical coils create a specific topology of magnetic field at the plasma edge, where external DED helical perturbations with poloidal number m and toroidal number n are resonant on the magnetic surfaces $q(r_{res}) = m/n$ (q(r) - safety factor) (see, e.g., [2, 3]). However, this topology was investigated using vacuum DED field perturbations without the plasma response. Remind, that the m = 12, n = 4 perturbation field structure is chosen as a standard DED operation regime.

The interaction of an external helical field with a plasma was investigated also in the CSTN- IV tokamak [4].

In the present paper the influence of plasma response to DED helical perturbation penetration is considered in cylindrical geometry. Analytical solutions of perturbations are found and their numerical investigation is carried out.

2. BASIC EQUATIONS

We start from magnetohydrodynamic equations

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V}\right) = -\nabla p + \frac{1}{c}[\mathbf{J}\times\mathbf{B}], \tag{1a}$$

$$rot\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},\quad rot\mathbf{B} = \frac{4\pi}{c}\mathbf{J} \tag{1b}$$

and Ohm's law (σ - conductivity)

$$\mathbf{J} = \sigma\left(\mathbf{E} + \frac{1}{c}[\mathbf{V}\times\mathbf{B}]\right). \tag{2}$$

We consider a current carrying cylindrical plasma whose axis is taken to be as the z direction. The external axial magnetic field B_{z0} is large with respect to the poloidal magnetic field $B_{\theta 0}$ produced by the axial current. The perturbation values depend on the azimuthal angle θ , the coordinate z and the time t as $\exp[i(m\theta - kz - \omega t)]$, $k = n/R$, R plays the role of the tokamak major radius, ω is the frequency of the external perturbation.

For perturbations of radial components of plasma velocity $V_r^{\sim}$ and magnetic field $B(r) = rB_r^{\sim}/\sqrt{4\pi\rho}$ the linearized version of Eqs. (1), (2) take the form:

$$\frac{d}{dr}r\frac{d}{dr}(rV_r^{\sim}) - \left(m^2 + i\frac{r^2}{\delta^2}\frac{F^2(r)}{4\pi\rho\omega^2}\right)V_r^{\sim} = \left\{i\frac{r}{\delta^2}\frac{F(r)}{\sqrt{4\pi\rho\omega^2}} + r\frac{d^2}{dr^2}\frac{F(r)}{\sqrt{4\pi\rho\omega^2}} + 3\frac{d}{dr}\frac{F(r)}{\sqrt{4\pi\rho\omega^2}}\right\}B(r), \tag{3}$$

$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}B(r) - \left(\frac{m^2}{r^2} + k^2 - \frac{i}{\delta^2}\right)B(r) = -\frac{i}{\delta^2}\frac{F(r)}{\sqrt{4\pi\rho\omega^2}}rV_r^{\sim},\ \left(\delta = c/\sqrt{4\pi\sigma\omega},\ F(r) = \mathbf{kB}_0 = \frac{m}{r}B_{\theta 0} - kB_{z0}\right). \tag{4}$$

The perturbations $V_z^{\sim}$ and $B_z^{\sim}$ are small and for simplicity we put $V_z^{\sim} = B_z^{\sim} = 0$. We use approximation of an incompressible plasma motion $div\,\mathbf{V}^{\sim} = 0$, neglect the ∇p term, variations of the plasma density ρ and conductivity σ (compare with [5]).
The value $F(r)$ is equal to zero inside the plasma, $F(r_{res}) = 0$, when $q(r_{res}) = m/n$ ($q(r) = rB_{z0}/RB_{\theta 0}$). The region near $r \approx r_{res}$ is the resonant (interaction) zone.

Inside and near the interaction zone Eq. (3) have the next general solution normalized to the value $V_{rA} = B_r^{vac}(r)/\sqrt{4\pi\rho} = C\,I_m(kr)/\left(r\cdot\sqrt{4\pi\rho}\right)$ ($I_m(kr)$ - modified Bessel and $H_{1/4}^{(1,2)}(z)$ - Hankel functions):

$$V_r^+(r) = \frac{\pi r\sqrt{|z|}}{4\sqrt{2}I_m(kr)}\left\{H_{1/4}^{(1)}\left(z^2\exp\left(i\frac{3\pi}{4}\right)\right)\left[0.5(1+i)\int_{z^2(a)}^{0}du\,uH_{1/4}^{(1)}\left(u\exp\left(i\frac{3\pi}{4}\right)\right)R^+(u) - 0.5(1-i)\int_{z^2(0)}^{0}du\,uH_{1/4}^{(1)}\left(u\exp\left(i\frac{3\pi}{4}\right)\right)R^-(u) - \right.\right.$$
$$\left.\left. - \int_{0}^{z^2}du\,uH_{1/4}^{(2)}\left(u\exp\left(i\frac{3\pi}{4}\right)\right)R^+(u)\right] + H_{1/4}^{(2)}\left(z^2\exp\left(i\frac{3\pi}{4}\right)\right)\int_{z^2(a)}^{z^2}du\,uH_{1/4}^{(1)}\left(u\exp\left(i\frac{3\pi}{4}\right)\right)R^+(u)\right\} \quad \text{for } r \geq r_{res}, \tag{5}$$

$$V_r^-(r)=\frac{\pi\, r\sqrt{|z|}}{4\sqrt{2}I_m(kr)}\left\{H_{1/4}^{(1)}\left(z^2\exp\left(i\frac{3\pi}{4}\right)\right)\left[0.5(1+i)\int\limits_{z^2(0)}^{0}du\,uH_{1/4}^{(1)}\left(u\exp\left(i\frac{3\pi}{4}\right)\right)R^-(u)-0.5(1-i)\int\limits_{z^2(a)}^{0}du\,uH_{1/4}^{(1)}\left(u\exp\left(i\frac{3\pi}{4}\right)\right)R^+(u)-\right.\right.$$

$$\left.\left.-\int\limits_{0}^{z^2}du\,uH_{1/4}^{(2)}\left(u\exp\left(i\frac{3\pi}{4}\right)\right)R^-(u)\right]+H_{1/4}^{(2)}\left(z^2\exp\left(i\frac{3\pi}{4}\right)\right)\int\limits_{z^2(0)}^{z^2}du\,uH_{1/4}^{(1)}\left(u\exp\left(i\frac{3\pi}{4}\right)\right)R^-(u)\right\}\qquad \text{for } r\le r_{res}, \tag{6}$$

where

$$R^\pm(u)=\frac{I_m(kr)}{r}\left\{\pm\left(\frac{r_{res}}{\delta\cdot Q}\right)^{1/2}\frac{1}{u^{3/4}}+\frac{i}{\sqrt{2Q}}\frac{\delta\cdot r_{res}}{r^2}\left[r^2\frac{d^2}{dr^2}\frac{F}{\sqrt{4\pi\rho\,\omega^2}}+3r\frac{d}{dr}\frac{F}{\sqrt{4\pi\rho\,\omega^2}}\right]\frac{1}{u^{5/4}}\right\}, \tag{7}$$

$$z(r)=\left(r_{res}Q/2\delta\right)^{1/2}(r-r_{res})/r_{res}\,,\quad Q=nSV_{zA}/\omega R\,,\quad V_{zA}=B_{z0}\big/\sqrt{4\pi\rho}\,,\quad S=\left(rq'/q\right)_{|r=r_{res}}. \tag{8}$$

In the $R^\pm(u)$ term the radius r is a function of u: $r(u)/r_{res}=1\pm\sqrt{(2\delta/r_{res}Q)\,u}$, a - the minor plasma radius. Outside the resonant zone $V_r^\pm(r)\approx-\sqrt{4\pi\rho\,\omega^2}\big/F(r)$. The same result we obtain from Eqs. (5), (6) in the case $z^2>>1$. We assume that the radial vacuum perturbation of magnetic field $B_r^\sim$ dominates in the plasma and in the right side of Eq. (3) we take $B(r)=CI_m(kr)$ (the vacuum perturbation of the magnetic field).

From Eq. (3), (5), (6) it follows that the half width of the interaction (resonant) zone Δr is of the order of

$$\Delta r\sim\left(2\delta\cdot r_{res}/Q\right)^{1/2}. \tag{9}$$

From Eq. (4) we obtain the contribution to the radial magnetic field perturbation of the plasma motion response ($B_r^\sim(r)=B_r^{vac}(r)+B_{r1}(r)$) with

$$W(r)=1+\frac{F(r)}{\sqrt{4\pi\rho\,\omega^2}}V_r^\pm(r) \tag{10}$$

$$\frac{B_{r1}(r)}{B_r^{vac}(r_{res})}=i\frac{r_{res}}{r}\frac{1}{\delta^2}\left[K_m(kr)\int\limits_0^r dr'\,r'\frac{I_m^2(kr')}{I_m(kr_{res})}W(r')-I_m(kr)\int\limits_a^r dr'\,r'K_m(kr')\frac{I_m(kr')}{I_m(kr_{res})}W(r')\right]. \tag{11}$$

The same for the poloidal component ($B_\theta^\sim(r)=B_\theta^{vac}(r)+B_{\theta 1}(r)$, $K'_m(z)=dK_m/dz$, $I'_m(z)=dI_m/dz$)

$$\frac{B_{\theta 1}(r)}{B_r^{vac}(r_{res})}=\frac{kr_{res}}{m}\frac{1}{\delta^2}\left[I'_m(kr)\int\limits_a^r dr'\,r'K_m(kr')\frac{I_m(kr')}{I_m(kr_{res})}W(r')-K'_m(kr)\int\limits_0^r dr'\,r'\frac{I_m^2(kr')}{I_m(kr_{res})}W(r')\right]. \tag{12}$$

3. COMPUTATIONAL RESULTS

3.1 Tokamak CSTN-IV

First, we present calculations for the CSTN-IV experiment [4] (R =0.4 m, a =0.1 m, r_{res} =7,5 cm, m=6, n=1, B_{z0} =0.086 T, n_{pl} =1.5· 10^{18} m^{-3}).

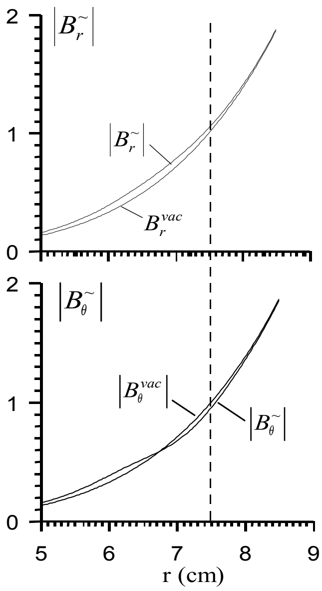

Fig. 1. Profiles $|B_r^\sim(r)|\big/B_r^{vac}(r_{res})$, $|B_\theta^\sim(r)|\big/B_r^{vac}(r_{res})$

The tendency in the $|B_r^\sim|$ and $|B_\theta^\sim|$ behavior is the same as it is in the CSTN-IV experiment (*f*=20 kHz, δ =2 cm).

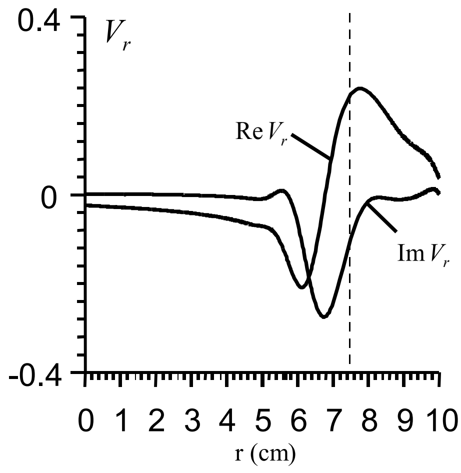

Fig. 2. The radial profile of the velocity V_r

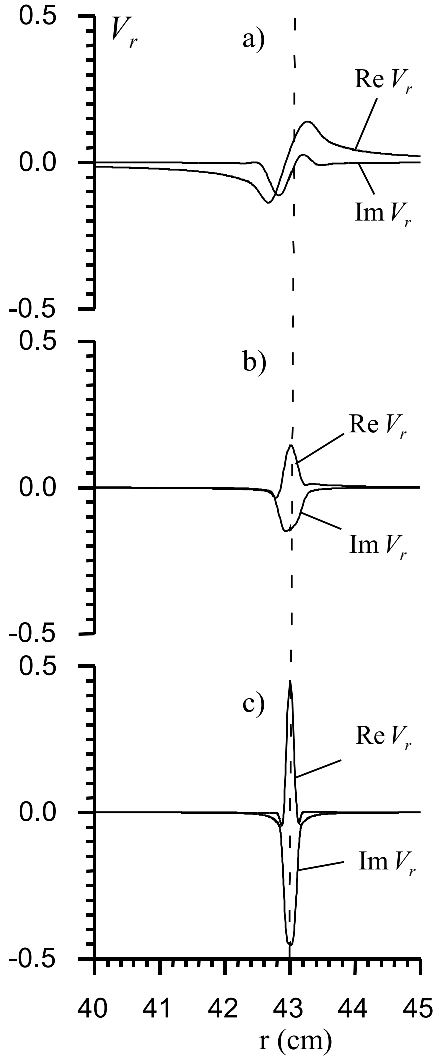

Fig. 3. The radial profile of the velocity V_r :
a) f =10 kHz, δ =0.7 cm; b) f =1 kHz, δ =2.2 cm; c) f =100 Hz, δ =6.96 cm.

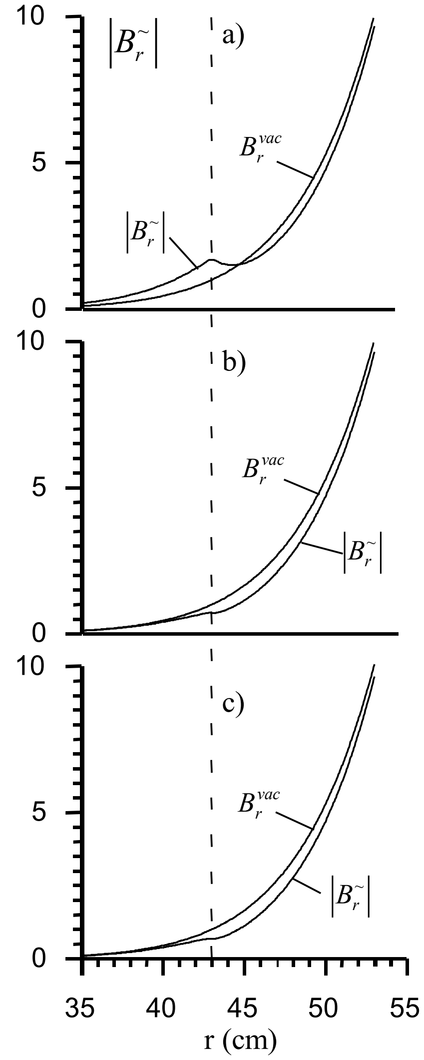

Fig. 4. Profiles $\left|B_r^{\sim}(r)\right| / B_r^{vac}(r_{res})$:
a) f =10 kHz, δ =0.7 cm; b) f =1 kHz, δ =2.2cm;c) f =100 Hz, δ =6.96 cm.

A very wide interaction region width Δr is observed. In Ref. [4] the theoretical estimate $\Delta r \sim$ 4 mm was declared. In the figures the vertical dashed line shows the resonant radius position.

3.2 TEXTOR-DED

Here the calculations for the TEXTOR-DED tokamak are presented (R =1.75 m, a =0.47 m, r_{res} =43 cm, m=12, n=4, B_{z0} =2.25 T, n_{pl} =10^{19} m^{-3}).

CONCLUSIONS

It is shown that for the high frequency ($\gtrsim$ 10 kHz) the radial component of the perturbation field $B_r^{\sim}$ is amplified inward of plasma from the interaction zone. This theoretical result confirms the CSTN-IV tokamak measurements.

For a lower frequency ($\lesssim$ 1kHz) $B_r^{\sim}$ is only attenuated in the plasma between the resonant zone and antenna.

Note, that for TEXTOR-DED the poloidal magnetic field component of the vacuum perturbation is practically completely compensated by the plasma perturbation response at $r = r_{res}$.

The width of the resonant zone Δr for TEXTOR-DED is of the order of 0.5 cm (or larger). It is much larger than the ion gyroradius. For the CSTN-IV experiment the width of the interaction region is very wide.

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