REGGE TRAJECTORIES OF QUARK GLUON BAGS

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Using an exactly solvable statistical model, we discuss the equation of state of large/heavy and short-living bags of the quark gluon plasma (QGP). We argue that the large width of the QGP bags explains not only the observed deficit in the number of hadronic resonances, but also clarifies the reason why the heavy QGP bags cannot be directly observed even as metastable states in the hadronic phase. Also the Regge trajectories of large and heavy QGP bags are established both in vacuum and in a strongly interacting medium. It is shown that, at high temperatures, the average mass and width of the QGP bags behave in accordance with the upper bound of the Regge trajectory asymptotics (the linear asymptotics), whereas, for temperatures below $T_H/2$ (T_H is the Hagedorn temperature), they obey the lower bound of the Regge trajectory asymptotics (the square root one). Thus, for $T < T_H/2$, the spin of the QGP bags is restricted from above, whereas, for $T > T_H/2$, these bags demonstrate the standard Regge behavior consistent with the string models.

1. Introduction

Regge poles have been introduced in particle physics before the QCD era. Since the beginning of the 1960s [1], they are widely used to describe the high-energy interactions of hadrons and nuclei. The Regge approach establishes an important connection between the high-energy scattering and the spectrum of particles and resonances. It served as a starting point to introduce the dual and string models of hadrons. Up to now, a rigorous derivation of Regge poles in QCD remains an unsolved problem, since it is related to the nonperturbative effects in QCD and the problem of confinement.

Nowadays, the Regge trajectories are widely understood as a linear relation between the resonance mass squared and the resonance spin or the radial quantum number, whereas the Regge trajectory $\alpha(S_r)$ contains information about the resonance mass M_r and width Γ_r. Indeed, the resonance spin J is defined in the complex energy plane as $J = \alpha\left((M_r - \frac{i}{2}\Gamma_r)^2\right)$. Moreover, the linear trajectories, i.e. $\alpha(S_r) \sim S_r$, which follow from the string models, are often believed to be the only Regge trajectories of hadrons.

However, it was shown long ago that, under the plausible assumptions, the linear Regge trajectories correspond to the upper bound of the asymptotic behavior, whereas its lower bound is given by a square-root trajectory, i.e. $\alpha_l(S_r) \sim [-S_r]^{1/2}$ [2, 3]. Moreover, there were some indications [3] that the square-root trajectory should give the asymptotic behavior of excited hadronic resonances. The latter means that, for each family of hadronic resonances, the Regge poles do not go beyond some vertical line in the complex spin plane, i.e. the resonances should become infinitely wide in the asymptotic limit $S \rightarrow +\infty$.

Since the linear Regge trajectories of hadrons generate the Hagedorn mass spectrum [4], the square-root ones should lead to a weaker growth of the hadronic mass spectrum. At first glance, it seems that the experimental mass spectrum of hadrons [5] does not show an exponential increase at hadron masses above 2.5 GeV and, hence, it evidences against the linear Regge trajectories of heavy hadrons. Moreover, the best description of particle yields observed in a very wide range of collision energies of heavy ions is achieved by the statistical model which incorporates all hadronic resonances not heavier than 2.3 GeV [6]. Again it looks like that heavier hadronic species, except for the long living ones, are simply absent in the experiments [7]. Thus, we are confronted with a serious conceptual problem between a few theoretical expectations and several experimental facts.

Recently, this conceptual problem was resolved within the finite-width model (FWM) [8]. The FWM introduces the medium dependent finite width of QGP bags into an exactly solvable statistical model. It shows that the large width of the QGP bags explains not only the observed deficit in the number of hadronic resonances, but also clarifies the reason why the heavy QGP bags and strangelets cannot be directly observed even as metastable states in the hadronic phase. In addition, the FWM allows one to establish [9] the Regge trajectories of large and heavy QGP bags both in vacuum and in a strongly interacting medium. As will be shown below, the average mass and width of the QGP bags behave at high temperatures in accordance with the upper bound of the Regge trajectory asymptotics (the linear asymptotics), whereas they obey the lower bound of the Regge trajectory asymptotics (the square-root one) at low temperatures. Thus, at low temperatures, the spin of the QGP bags is restricted from above, whereas these bags demonstrate the typical Regge behavior consistent with the string models at high temperatures.

The work is organized as follows. In Section 2, we discuss the main ideas and results of FWM. The Regge trajectories of QGP bags are established in Section 3, while our conclusions are given in Section 4.

2. FWM of QGP Bags

The main object of the FWM is the mass-volume spectrum of heavy and large QGP bags at a temperature T ($V_0 \approx 1$ fm^3, $M_0 \approx 2.5$ GeV [8, 9])

$$F_Q(s,T) = \int\limits_{V_0}^{\infty} dv \int\limits_{M_0}^{\infty} dm\ \rho(m,v) \exp(-sv) \phi(T,m)\ , \quad (1)$$

where the thermal density of bags of mass m reads

$$\phi(T,m) = \int\limits_0^{\infty} p^2 dp\ \exp\left[-\tfrac{(p^2\ +\ m^2)^{1/2}}{T}\right]. \quad (2)$$

In (1), s denotes the variable of the isobar ensemble which is dual to the system volume. An exponential $\exp(-sv)$ in (1) describes the hard core repulsion between the bags [8], while the density of states $\rho(m,v)$ of bags of mass m and volume v has the form

$$\rho(m,v) = \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^{a+\frac{3}{2}}} \exp\left[\tfrac{m}{T_H} - \tfrac{(m-Bv)^2}{2\Gamma^2(v)}\right], \quad (3)$$

$$\rho_1(v) = f(T)\, v^{-b}\ \exp\left[-\tfrac{\sigma(T)}{T}\, v^{\varkappa}\right]. \quad (4)$$

As one can see from (3), the density of states has a Hagedorn-like parametrization with respect to the mass and the Gaussian attenuation around the bag mass Bv (B is the mass density of a bag of a vanishing width) with the volume-dependent Gaussian width $\Gamma(v)$ or the width hereafter. We will distinguish it from the true width defined as $\Gamma_R = \alpha\,\Gamma(v)$ ($\alpha \equiv 2\sqrt{2\ln 2}\,$). As was shown in [8, 9], the Breit–Wigner attenuation of a resonance mass cannot be used in spectrum (3) because, in this case, the finite width leads to a divergency of the mass integral in (1) above T_H.

The normalization factor obeys the condition

$$N_\Gamma^{-1} = \int\limits_{M_0}^{\infty} \frac{dm}{\Gamma(v)} \exp\left[-\tfrac{(m-Bv)^2}{2\Gamma^2(v)}\right]. \quad (5)$$

The constants $a > 0$ and $b > 0$ are discussed in [8].

Moreover, the volume spectrum in (4) contains the surface free energy ($\varkappa = 2/3$) with the T-dependent surface tension which can be parametrized in a general way [10–12] as

$$\sigma(T,\mu) = \begin{cases} \sigma^- > 0\,, & T \to T_\Sigma(\mu) - 0\,, \\ 0\,, & T = T_\Sigma(\mu)\,, \\ \sigma^+ < 0\,, & T \to T_\Sigma(\mu) + 0\,. \end{cases} \quad (6)$$

For $T \le T_\Sigma(\mu)$, such a parametrization is justified by the usual cluster models like the FDM [13] and SMM [14–17], whereas the general consideration for any T can be driven by the surface partitions of the Hills and Dales model [10].

The recent results obtained within the exactly solvable models [11, 12] also justify parametrization (6) and show that the only physical reason for the degeneration of the first-order deconfinement phase transition at low baryonic densities into a cross-over is the negative surface tension coefficient in this region. Moreover, the existence of negative surface tension at the cross-over region was directly demonstrated from the lattice QCD data very recently within a new phenomenological model of confinement [18].

An actual choice of the continuous functions $\sigma^{\pm}$ of the temperature T and the baryonic chemical potential μ along with parametrization of the nil line of the surface tension coefficient $T_\Sigma(\mu)$ for the tricritical and critical endpoint of the QCD phase diagram can be found in [11] and [12], respectively.

Spectrum (3) has a simple form, but is rather general since both the width $\Gamma(v)$ and the bag mass density B can be medium-dependent. It clearly reflects the fact that the QGP bags are similar to ordinary quasiparticles with the medium-dependent characteristics (lifetime, most probable values of mass and volume). In principle, one could consider various v-dependences $\Gamma(v)$, but it was shown in [8, 9] that the only square-root dependence of the resonance width on its volume, i.e. $\Gamma(v) = \gamma v^{1/2}$, does not lead to the problems with the existence of large QGP bags.

For large bag volumes ($v \gg M_0/B > 0$), factor (5) can be found as $N_\Gamma \approx 1/\sqrt{2\pi}$. Similarly, one can show that, for heavy free bags ($m \gg BV_0$, ignoring the hard core repulsion and thermostat),

$$\rho(m) \equiv \int_{V_0}^{\infty} dv\, \rho(m,v) \approx \frac{\rho_1(\frac{m}{B})}{B\, m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H}\right]. \qquad (7)$$

It originates in the fact that, for heavy bags, the Gaussian in (3) acts like a Dirac δ-function for $\Gamma(v) = \gamma v^{1/2}$. Similarly to (7), one can estimate the width of heavy free bags averaged over bag volumes and get

$$\overline{\Gamma(v)} \approx \Gamma(m/B) = \gamma\sqrt{\frac{m}{B}}\,. \qquad (8)$$

Thus, the mass spectrum of heavy free QGP bags is a Hagedorn-like one with the property that the heavy resonances must have the large mean width. Hence, they are hardly observable. This resolves the conceptual problem of the deficit of observed heavy hadronic resonances as compared with the Hagedorn mass spectrum. Applying these arguments to the strangelets, we conclude that, if their mean volume is a few cubic fermis or larger, they should survive for a very short time, which is similar to the results of [19] predicting an instability of such strangelets.

An analysis of the full spectrum (1) allows one to determine the pressure of the QGP bags [8] which shows two drastically different regimes depending on the value of the most probable mass of a bag ($v \gg V_0$)

$$\langle m \rangle \equiv Bv + \Gamma^2(v)\beta\,, \quad \text{with} \quad \beta \equiv T_H^{-1} - T^{-1}. \qquad (9)$$

For temperatures below (above) $T_{\pm} = c_{\pm}\, T_H$ ($0 < c_{\pm} < 1$), the most probable mass is negative (positive) and spectrum (1) defines the low (high) temperature pressure p^- (p^+)

$$p = \begin{cases} p^+ \equiv T\left[\beta B + \frac{\gamma^2}{2}\beta^2\right], & \langle m \rangle > 0\,, \\ p^{\pm} \equiv \frac{BT\beta}{2}\,, & \langle m \rangle = 0\,, \\ p^- \equiv -T\frac{B^2}{2\gamma^2}\,, & \langle m \rangle < 0\,. \end{cases} \qquad (10)$$

There are two remarkable facts concerning the low-temperature pressure. First, for $\langle m \rangle \le 0$, the resulting mass attenuation of the integrand in (1) decreases at a fixed bag volume so rapidly that the only vicinity of M_0 contributes to the mass-volume spectrum. In other words, all heavy QGP bags are extremely suppressed in this regime, and, as a result, only the smallest bags with the mass M_0 and the width about $\Gamma(V_0)$ can contribute to spectrum (1) [8]. Consequently, such QGP bags would not be distinguishable from the usual low-mass hadrons. Such a regime leads to the *subthreshold suppression of the QGP bags* at low temperatures even in finite systems and, hence, is able to explain the absence of heavy/large QGP bags and strangelets for $T < T_{\pm}$.

Second, for the non-vanishing functions γ and B at low temperatures, i.e. $\gamma_0 = \gamma(T=0) > 0$ and $B_0 = B(T=0) > 0$, the QGP pressure at such temperatures should be negative and linear in temperature $p^-(T \to 0) \approx -T\frac{B_0^2}{2\gamma_0^2}$. Such a linear T-behavior of the QGP pressure at low temperatures, $p_{\rm QGP} = \sigma_p T^4 - A_1 T$, is known for a long time [21] and was reported by several groups (see [9] for details). Using this fact and matching p^+ with $p_{\rm QGP}$, it was possible to estimate the resonance width coefficient γ and the mass density B from the lattice QCD data [9]

$$\gamma^2(T) = 2\,\beta^{-1}[\sigma_p T_H T(T^2 + TT_H + T_H^2) - B(T)]\,, \qquad (11)$$

$$B(T) = \sigma_p T_H^2(T^2 + TT_H + T_H^2)\,, \qquad (12)$$

where $3\sigma_p$ is the Stefan–Bolzmann constant of the QGP. Equations (11) and (12) allow one to determine $T_{\pm} = 0.5T_H$ and show that the resonance width at the zero temperature very weakly depends on the number of elementary degrees of freedom in QGP, but strongly depends on the cross-over temperature T_{co}

$$\Gamma_R(V_0, T=0) \approx C_\gamma\, V_0^{1/2}\, T_{co}^{5/2}\, \alpha\,, \qquad (13)$$

where the constant C_γ, depending on the number of color and flavor states of QGP, varies between 1.22 and 1.3 [9]. The minimal width of the QGP bags strongly increases with temperature. For instance, at the Hagedorn temperature, one obtains $\Gamma_R(V_0, T=T_H) = \sqrt{12}\,\Gamma_R(V_0, T=0)$. Therefore, for $T_{co} \in [170; 200]$ MeV, the minimal width of the QGP bags is $\Gamma_R(V_0, T=0) \in [400; 600]$ MeV and $\Gamma_R(V_0, T=T_H) \in [1400; 2000]$ MeV. These estimates clearly show us that, even without the subthreshold suppression, the heavy/large QGP bags cannot be directly observed at any temperature due to a very short life-time.

We note that one of the most remarkable features of the FWM is that its mass dependence of a mean resonance width can help to determine the Regge trajectories of QGP bags and to resolve a few problems related to them.

3. Asymptotic Trajectories of QGP Bags

Nowadays, there is a great interest in the behavior of the Regge trajectories of higher resonances in the context of the 5-dimensional string theory holographically dual to QCD [20] which is known as the anti-de-Sitter space/conformal field theory (AdS/CFT). However, as was mentioned earlier, the Regge trajectories are widely understood only as the linear relation between the resonance mass squared and the resonance spin or the radial quantum number. We would like to determine the full Regge trajectories of QGP bags. In our analysis, we follow Ref. [3] based on the following most general assumptions: (I) $\alpha(S)$ is an analytical function, having only the physical cut from $S = S_0$ to $S = \infty$; (II) $\alpha(S)$ is polynomially restricted at the whole physical sheet; (III) there exists a finite limit of the phase trajectory at $S \to \infty$. Using these assumptions, it was possible to prove [3] that, for $S \to \infty$, the upper bound of the Regge trajectory asymptotics at the whole physical sheet is

$$\alpha_u(S) = -g_u^2\,[-S]^{\nu}\ , \quad \text{with} \quad \nu \leq 1\,, \tag{14}$$

where the function $g_u^2 > 0$ should increase slower than any power in this limit, and its phase must vanish at $|S| \to \infty$.

On the other hand, it was also shown in Ref. [3] that, if one requires in addition to (I)–(III) that the transition amplitude $T(s,t)$ is a polynomially restricted function of S for all nonphysical $t > t_0 > 0$, then the real part of the Regge trajectory does not increase at $|S| \to \infty$, and the trajectory behaves as

$$\alpha_l(S) = g_l^2\left[-\,[-S]^{1/2} + C_l\right]\,, \tag{15}$$

where $g_l^2 > 0$ and C_l are some constants. Moreover, (15) defines the lower bound for the asymptotic behavior of the Regge trajectory [3]. The expression (15) is a generalization of a well-known Khuri result [2]. It means that, for each family of hadronic resonances, the Regge poles do not go beyond some vertical line in the complex spin plane. In other words, it means that the resonances become infinitely wide in asymptotics $S \to +\infty$, i.e. they are moving out of the real axis of the proper angular momentum J and, therefore, there is only a finite number of resonances in the corresponding transition amplitude.

To compare the FWM results with trajectories (14) and (15), we need to relate the mass and the width of QGP bags, since they are independent variables in this model. Nevertheless, this can be done for their averaged values.

To illustrate this statement, we recall our result on the mean Gaussian width of the free bags averaged with respect to their volume (8) by spectrum (7). Using the formalism of [3], it can be shown that, at zero temperature, the free QGP bags of mass m and mean resonance width $\alpha\,\overline{\Gamma(v)}|_{T=0} \approx \alpha\,\gamma_0\sqrt{\frac{m}{B_0}}$ precisely correspond to the Regge trajectory

$$\alpha_r(S) = g_r^2[S + a_r(-S)^{3/4}] \quad \text{with} \quad a_r = \text{const} < 0\,. \tag{16}$$

Indeed, substituting $S = |S|e^{i\,\phi_r}$ into (16), then expanding the second term on the right-hand side of (16), and requiring $\mathrm{Im}\,[\alpha_r(S)] = 0$, one finds the phase of the physical trajectory (one of four roots of one fourth power in (16))

$$\phi_r(S) \to \frac{a_r \sin\frac{3}{4}\pi}{|S|^{1/4}} \to 0^-\,, \tag{17}$$

which vanishes in the correct quadrant of the complex S-plane. Considering the complex energy plane $E = \sqrt{S} \equiv M_r - i\frac{\Gamma_r}{2}$, one can determine the mass M_r and the width Γ_r,

$$M_r \approx |S|^{1/2}\,, \quad \Gamma_r \approx -|S|^{1/2}\phi_r(S) = \frac{|a_r|M_r^{1/2}}{\sqrt{2}}, \tag{18}$$

of a resonance belonging to trajectory (16).

Comparing the mass dependence of the width in (18) with the mean width of free QGP bags (8) taken at $T = 0$, it is natural to identify them,

$$a_r^{\rm free} \approx -\alpha\,\gamma_0\sqrt{\frac{2}{B_0}} = -4\,\gamma_0\sqrt{\frac{\ln 2}{B_0}}\,, \tag{19}$$

and to deduce that the free QGP bags belong to the linear Regge trajectory (16). Such a conclusion is supported and justified by the well-established results on the linear Regge trajectories of hadronic resonances [22] and by theoretical expectations of the dual resonance model [23], the open string model [24], the closed string model [24], and the AdS/CFT [20]. Moreover, the most direct way to connect the FWM bags with the string models is provided by the recently suggested model of the confinement phenomenon [18] which allows us to relate the string tension of a confining color tube and the surface tension of QGP bags.

We now consider the second way of averaging the mass-volume spectrum over the resonance masses

$$\overline{m}(v) \equiv \frac{\int\limits_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\,\rho(m,v)\ m\ e^{-\frac{\sqrt{k^2+m^2}}{T}}}{\int\limits_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\,\rho(m,v)\ e^{-\frac{\sqrt{k^2+m^2}}{T}}}\,, \tag{20}$$

which is technically simpler than averaging over the resonance volume, but we will make the necessary comments on the other way of averaging in the appropriate places.

Using the results of the preceding section, one can find the mean mass (20) for $T \geq 0.5\,T_H$ (or for $\langle m \rangle \geq 0$) to be equal to the most probable mass of a bag, from which one determines the resonance width:

$$\overline{m}(v) \approx \langle m \rangle \quad \text{and} \tag{21}$$

$$\Gamma_R(v) \approx 2\sqrt{2\ln 2}\,\Gamma\left[\frac{\langle m \rangle}{B+\gamma^2\beta}\right] = 2\,\gamma\sqrt{\frac{2\ln 2\,\langle m \rangle}{B+\gamma^2\beta}}\,. \tag{22}$$

Two last equations lead to a vanishing ratio $\frac{\Gamma_R}{\langle m \rangle} \sim \langle m \rangle^{-1/2}$ in the limit $\langle m \rangle \to \infty$. Comparing (21) and (22) with the mass and the width (18) of the Regge trajectory (16) and applying absolutely the same logic we used for the free QGP bags, we conclude that the location of the FWM heavy bags in the complex energy plane is identical to that of one of the resonances belonging to trajectory (16) with

$$\langle m \rangle \approx |S|^{1/2} \quad \text{and} \quad a_r \approx -4\gamma\sqrt{\frac{\ln 2}{B+\gamma^2\beta}}\,. \tag{23}$$

The most remarkable output of such a conclusion is that the medium-dependent FWM mass and the width of the extended QGP bags obey the upper bound for the Regge trajectory asymptotic behavior obtained for point-like hadrons [3]!

It is also of interest that the resonance width formula (22) follows from that for the most probable volume,

$$v_E(m) \approx \frac{m}{\sqrt{B^2+2\gamma^2 s^*}} = \frac{m}{B+\gamma^2\beta}\,, \tag{24}$$

of heavy resonances of mass $m \gg M_0$ that are described by the continuous spectrum $F_Q(s,T)$ (1). This result can be easily found by maximizing the exponential in $F_Q(s,T)$ with respect to the resonance volume v at a fixed mass m [9].

The extracted values of the resonance width coefficient along with relation (12) for $B(T)$ allow us to estimate a_r as

$$a_r \approx -4\sqrt{\frac{2\,T\,T_H}{2\,T-T_H}\ln 2}\,. \tag{25}$$

This expression shows that, for $T \to T_H/2+0$, the asymptotic behavior (16) breaks down since the resonance width diverges at fixed $|S|$. We hope for that such a behavior can be experimentally observed [25] at NICA (Dubna, Russia) and FAIR (Darmstadt, Germany) energies.

Now we can find the spin of the FWM resonances

$$J = \mathrm{Re}\,\alpha_r(\langle m \rangle^2) \;\approx g_r^2\,\langle m \rangle\left[\langle m \rangle - \frac{a_r^2}{4}\right], \tag{26}$$

which has a typical Regge behavior up to a small correction. Such a property can also be obtained within the dual resonance model [23], the models of open [24] and closed [24] strings, and the AdS/CFT [20]. These models support our result (26) and justify it. Note, however, that, in addition to the spin value, the FWM determines the width of hadronic resonances. The latter allows us to predict the ratio of widths of two resonances having spins J_2 and J_1 and appearing at the same temperature T:

$$\frac{\Gamma_R\left[\frac{\langle m \rangle\big|_{J_2}}{(B+\gamma^2\beta)}\right]}{\Gamma_R\left[\frac{\langle m \rangle\big|_{J_1}}{(B+\gamma^2\beta)}\right]} \approx \frac{\sqrt{v\big|_{J_2}}}{\sqrt{v\big|_{J_1}}} \approx \frac{\sqrt{\langle m \rangle\big|_{J_2}}}{\sqrt{\langle m \rangle\big|_{J_1}}} \approx \left[\frac{J_2}{J_1}\right]^{1/4}, \tag{27}$$

which, perhaps, can be tested at LHC.

We now turn to the analysis of the low temperature regime, i.e. to $T \leq 0.5\,T_H$. Using the previously obtained results from (20), we find

$$\overline{m}(v) \;\approx\; M_0\,, \tag{28}$$

i.e. the mean mass is volume-independent. Taking the limit $v \to \infty$, we get the ratio $\frac{\Gamma(v)}{\overline{m}(v)} \to \infty$ which closely resembles the case of the lower bound of the Regge trajectory asymptotics (15). Similarly to the analysis of the high temperature regime, relation (15) yields the trajectory phase and then the resonance mass M_r and its width Γ_r

$$\phi_r(S) \to -\pi + \frac{2|C_l||\sin(\arg C_l)|}{|S|^{1/2}}\,, \tag{29}$$

$$M_r \approx |C_l||\sin(\arg C_l)| \quad \text{and} \quad \Gamma_r \approx 2|S|^{1/2}\,. \tag{30}$$

Again comparing the averaged masses and widths of FWM resonances with their counterparts in (30), we find a similar behavior in the limit of the large width of resonances. Therefore, we conclude that, at low temperatures, the FWM obeys the lower bound of the Regge trajectory asymptotics of [3].

The other way of averaging, i.e. over the resonance volume, results, in the leading order, in an infinite value of the most probable resonance width [9] defined in this way. Note that such a result is supported by the high-temperature mean width behavior if $T \to T_H/2+0$. As one can see from (25) and (18), in the latter case, trajectory (16) also demonstrates a very large width as compared with a finite resonance mass.

The FWM suggests that the Regge trajectories of QGP bags have a statistical nature. The above estimates demonstrate that, at any temperature, the FWM QGP bags can be regarded as the medium-induced Reggeons which belong, at $T \leq 0.5\,T_H$ (i.e. for $\langle m \rangle \leq 0$), to the Regge trajectory (15). Otherwise, they are described by trajectory (16). Of course, both trajectories (15) and (16) are valid in the asymptotic $|S| \to \infty$, but the most remarkable fact is that, to our knowledge, the FWM gives us the first example of a model which reproduces both of these trajectories and, thus, obeys both bounds of the Regge asymptotics. Moreover, since the FWM contains the Hagedorn-like mass spectrum at any temperature, it shows that such a spectrum is not exclusively related to the linear Regge trajectories. At low temperatures (i.e. for $\langle m \rangle \leq 0$), the large/heavy QGP bags have the square-root trajectory (15) which is in line with expectations of Ref. [3].

4. Conclusions

Here, we briefly describe an exactly solvable statistical model, the FWM, of the QGP equation of state. It accounts for the Hagedorn mass spectrum and for a finite medium-dependent width of large QGP bags. The inclusion of the Gaussian attenuation of the resonance mass leads not only to the partition function convergent at high temperatures, but also it allows us to explain a huge deficit of the experimentally observed hadronic resonances with masses above $M_0 \approx 2.5$ GeV compared with the Hagedorn mass spectrum.

The FWM allows us to establish the full Regge trajectories of large/heavy QGP bags both in vacuum and in a strongly interacting medium. The free QGP bags in vacuum have the linear Regge trajectory and, thus, obey the upper bound of the Regge trajectory asymptotics. A linear Regge trajectory is also found for the in-medium QGP bags at temperatures above $0.5\,T_H$, whereas, for temperatures below $0.5\,T_H$, QGP bags obey the lower bound of the Regge trajectory asymptotics (the square-root one) which is in line with expectations of Ref. [3].

The FWM allows us to connect the statistical description of the QGP equation of state with the Regge poles method and show that the Regge trajectories of large/heavy QGP bags have a statistical nature. These findings bring forward the statistical models of the QGP equation of state to a qualitatively new level of realism.

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РЕДЖЕ ТРАЄКТОРІЇ КВАРК-ГЛЮОННИХ МІШКІВ

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Р е з ю м е

Використовуючи точний розв'язок статистичної моделі, обговорено рівняння стану великих/важких і короткоживучих мішків кварк-глюонної плазми (КГП). Наведено аргументи того, що велика ширина мішків КГП не тільки пояснює дефіцит кількості адронних резонансів, але й причину того, що важкі мішки КГП не можуть безпосередньо спостерігатися навіть як метастабільні стани в адронній фазі. Також знайдено Редже траєкторії великих і важких мішків КГП як у вакуумі, так і в сильновзаємодіючому середовищі. Доведено, що за високих температур середня маса і ширина мішків КГП підкорюються верхній границі асимптотики траєкторії Редже (лінійна асимптотика), тоді як для температур, нижчих за $T_H/2$ (T_H – температура Хагедорна), вони підкорюються нижній границі асимптотики траєкторій Редже (асимптотика кореня квадратного). Таким чином, для $T < T_H/2$ спін мішків КГП обмежено зверху, тоді як для $T > T_H/2$ ці мішки демонструють стандартну Редже поведінку, яка узгоджується з моделями струн.