MAGNETIZATION OF DENSE NEUTRON MATTER IN A STRONG MAGNETIC FIELD

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Spin polarized states in neutron matter at a strong magnetic field up to 10^{18} G are considered in the model with the Skyrme effective interaction. Analyzing the self-consistent equations at zero temperature, it is shown that a thermodynamically stable branch of solutions for the spin polarization parameter as a function of the density corresponds to the negative spin polarization when the majority of neutron spins are oriented oppositely to the direction of the magnetic field. In addition, beginning from some threshold density dependent on the magnetic field strength, the self-consistent equations have also two other branches of solutions for the spin polarization parameter with the positive spin polarization. The free energy corresponding to one of these branches turns out to be very close to the free energy corresponding to the thermodynamically preferable branch with the negative spin polarization. As a consequence, at a strong magnetic field, the state with the positive spin polarization can be realized as a metastable state at the high density region in neutron matter which changes into a thermodynamically stable state with the negative spin polarization with decrease in the density at some threshold value. The calculations of the neutron spin polarization parameter, energy per neutron, and chemical potentials of spin-up and spin-down neutrons as functions of the magnetic field strength show that the influence of the magnetic field remains small at the field strengths up to 10^{17} G.

1. Introduction

Neutron stars observed in the nature are magnetized objects with the magnetic field strength at the surface in the range $10^9 - 10^{13}$ G [1]. For a special class of neutron stars such as soft gamma-ray repeaters and anomalous X-ray pulsars, the field strength can be much larger and is estimated to be about $10^{14} - 10^{15}$ G [2]. These strongly magnetized objects are called magnetars [3] and comprise about 10% of the whole population of neutron stars [4]. However, in the interior of a magnetar, the magnetic field strength may be even larger, reaching the values about 10^{18} G [5, 6]. The possibility of the existence of such ultrastrong magnetic fields is not yet excluded, because what we can learn from the magnetar observations by their periods and spin-down rates, or by hydrogen spectral lines, is only their surface fields. There is still no general consensus regarding the mechanism to generate such strong magnetic fields of magnetars, although different scenarios were suggested such as, e.g., a turbulent dynamo amplification mechanism in rapidly rotating neutron stars [2] or the possibility of the spontaneous spin ordering in the dense quark core of a neutron star [7].

Under such circumstances, the issue of interest is the behavior of a neutron star matter in a strong magnetic field [5, 6, 8, 9]. In the recent study [9], the neutron star matter was approximated by a pure neutron matter in the model considerations with the effective Skyrme and Gogny forces. It has been shown that the behavior of the spin polarization of neutron matter in the high density region at a strong magnetic field crucially depends on whether neutron matter develops a spontaneous spin polarization (in the absence of a magnetic field) at several times nuclear matter saturation density, as is usual for the Skyrme forces, or the appearance of a spontaneous polarization is not allowed at the relevant densities (or delayed to much higher densities), as in the case with the Gogny D1P force. In the former case, a ferromagnetic transition to a totally spin polarized state occurs.

While in the latter case, a ferromagnetic transition is excluded at all relevant densities, and the spin polarization remains quite low even in the high density region. Note that the issue of the spontaneous appearance of spin polarized states in neutron and nuclear matter is a controversial one. On the one hand, the models with the Skyrme effective nucleon-nucleon (NN) interaction predict the occurrence of spontaneous spin instability in nuclear matter at densities in the range from ϱ_0 to $4\varrho_0$ for different parametrizations of the NN potential [10–22] ($\varrho_0 = 0.16$ fm^{-3} is the nuclear saturation density). For the Gogny effective interaction, a ferromagnetic transition in neutron matter occurs at the density larger than $7\varrho_0$ for the D1P parametrization and is not allowed for D1 and D1S parametrizations [23]. However, for the D1S Gogny force, an antiferromagnetic phase transition happens in symmetric nuclear matter at the density $3.8\varrho_0$ [24]. On the other hand, for the models with the realistic NN interaction, no sign of spontaneous spin instability at any isospin asymmetry has been found so far up to densities well above ϱ_0 [25–31].

Here, we will study the thermodynamic properties of spin polarized neutron matter at a strong magnetic field in the model with the Skyrme effective forces. As a framework for consideration, we choose a Fermi liquid approach for the description of nuclear matter [32–34]. Proceeding from the minimum principle for the thermodynamic potential, we get the self-consistent equations for the spin order parameter and the chemical potential of neutrons. In the absence of a magnetic field, the self-consistent equations have two degenerate branches of solutions for the spin polarization parameter corresponding to the case where the majority of neutron spins is oriented along or oppositely to the spin quantization axis (positive or negative spin polarization, respectively). In the presence of a magnetic field, these branches are modified differently. A thermodynamically stable branch corresponds to the state with the majority of neutron spins aligned oppositely to the magnetic field. At a strong magnetic filed, the branch corresponding to the positive spin polarization splits onto two branches with the positive spin polarization as well. The last solutions were missed in the study of Ref. [9]. We perform a thermodynamic analysis based on the comparison of the respective free energies and arrive at the conclusion about the possibility of the formation of metastable states in neutron matter with the majority of neutron spins directed along the strong magnetic field. The appearance of such metastable states can be possible due to the strong spin-dependent medium correlations in neutron matter with the Skyrme forces at high densities.

Note that we consider the thermodynamic properties of spin polarized states in neutron matter at a strong magnetic field up to the high density region relevant for astrophysics. Nevertheless, we take into account the nucleon degrees of freedom only, although other degrees of freedom, such as pions, hyperons, kaons, or quarks could be important at such high densities.

2. Basic Equations

The normal (nonsuperfluid) states of neutron matter are described by the normal distribution function of neutrons $f_{\kappa_1\kappa_2} = \mathrm{Tr}\,\varrho a^+_{\kappa_2} a_{\kappa_1}$, where $\kappa \equiv (\mathbf{p}, \sigma)$, $\mathbf{p}$ is the momentum, σ is the projection of a spin on the third axis, and ϱ is the density matrix of the system [20, 21]. Further, it will be assumed that the third axis is directed along the external magnetic field $\mathbf{H}$. The energy of the system is specified as a functional of the distribution function f, $E = E(f)$, and determines the single-particle energy

$$\varepsilon_{\kappa_1\kappa_2}(f) = \frac{\partial E(f)}{\partial f_{\kappa_2\kappa_1}}. \tag{1}$$

The self-consistent matrix equation for determining the distribution function f follows from the minimum condition of the thermodynamic potential [32, 33] and is

$$f = \{\exp(Y_0\varepsilon + Y_4) + 1\}^{-1} \equiv \{\exp(Y_0\xi) + 1\}^{-1}. \tag{2}$$

Here, the quantities ε and Y_4 are matrices in the space of κ variables,

$$Y_{4\kappa_1\kappa_2} = Y_4\delta_{\kappa_1\kappa_2},$$

$Y_0 = 1/T$, and $Y_4 = -\mu_0/T$ are the Lagrange multipliers, μ_0 is the chemical potential of neutrons, and T is the temperature.

Given the possibility for the alignment of neutron spins along or oppositely to the magnetic field $\mathbf{H}$, the normal distribution function of neutrons and the single-particle energy can be expanded in the Pauli matrices σ_i in the spin space

$$f(\mathbf{p}) = f_0(\mathbf{p})\sigma_0 + f_3(\mathbf{p})\sigma_3,$$

$$\varepsilon(\mathbf{p}) = \varepsilon_0(\mathbf{p})\sigma_0 + \varepsilon_3(\mathbf{p})\sigma_3. \tag{3}$$

Using Eqs. (2) and (3), one can express evidently the distribution functions f_0, f_3 in terms of the quantities ε:

$$f_0 = \frac{1}{2}\{n(\omega_+) + n(\omega_-)\},$$

$$f_3 = \frac{1}{2}\{n(\omega_+) - n(\omega_-)\}. \quad (4)$$

Here, $n(\omega) = \{\exp(Y_0\omega) + 1\}^{-1}$ and

$$\omega_\pm = \xi_0 \pm \xi_3,$$

$$\xi_0 = \varepsilon_0 - \mu_0,\ \xi_3 = \varepsilon_3. \quad (5)$$

As follows from the structure of the distribution functions f, the quantity $\omega_\pm$, being the exponent in the Fermi distribution function n, plays the role of a quasiparticle spectrum. The spectrum is twofold split due to the spin dependence of the single-particle energy $\varepsilon(\mathbf{p})$ in Eq. (3). The branches $\omega_\pm$ correspond to neutrons with spin up and spin down.

The distribution functions f should satisfy the normalization conditions

$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_0(\mathbf{p}) = \varrho, \quad (6)$$

$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_3(\mathbf{p}) = \varrho_\uparrow - \varrho_\downarrow \equiv \Delta\varrho. \quad (7)$$

Here, $\varrho = \varrho_\uparrow + \varrho_\downarrow$ is the total density of neutron matter, $\varrho_\uparrow$ and $\varrho_\downarrow$ are the neutron number densities with spin up and spin down, respectively. The quantity $\Delta\varrho$ may be regarded as the neutron spin order parameter. It determines the magnetization of the system $M = \mu_n\Delta\varrho$, μ_n being the neutron magnetic moment. The magnetization may contribute to the internal magnetic field $B = H + 4\pi M$. However, we will assume, analogously to Refs. [6, 9], that the contribution of the magnetization to the magnetic field B remains small for all relevant densities and magnetic field strengths, and, hence,

$$B \approx H. \quad (8)$$

This assumption holds true due to the tiny value of the neutron magnetic moment $\mu_n = -1.9130427(5)\mu_N \approx -6.031\times10^{-18}$ MeV/G [35] (μ_N being the nuclear magneton) and is confirmed numerically by finding solutions of the self-consistent equations in two approximations, corresponding to preserving and neglecting the contribution of the magnetization.

In order to get the self-consistent equations for the components of the single-particle energy, one has to set the energy functional of the system. In view of the approximation (8), it reads [21, 33]

$$E(f) = E_0(f,H) + E_{\rm int}(f) + E_{\rm field}, \quad (9)$$

$$E_0(f,H) = 2\sum_{\mathbf{p}} \underline{\varepsilon}_0(\mathbf{p}) f_0(\mathbf{p}) - 2\mu_n H \sum_{\mathbf{p}} f_3(\mathbf{p}),$$

$$E_{\rm int}(f) = \sum_{\mathbf{p}}\{\tilde\varepsilon_0(\mathbf{p}) f_0(\mathbf{p}) + \tilde\varepsilon_3(\mathbf{p}) f_3(\mathbf{p})\},$$

$$E_{\rm field} = \frac{H^2}{8\pi}\mathcal{V},$$

where

$$\tilde\varepsilon_0(\mathbf{p}) = \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_0^n(\mathbf{k}) f_0(\mathbf{q}),\ \mathbf{k} = \frac{\mathbf{p}-\mathbf{q}}{2},$$

$$\tilde\varepsilon_3(\mathbf{p}) = \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_1^n(\mathbf{k}) f_3(\mathbf{q}). \quad (10)$$

Here, $\underline{\varepsilon}_0(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_0}$ is the free single-particle spectrum, m_0 is the bare mass of a neutron, $U_0^n(\mathbf{k}), U_1^n(\mathbf{k})$ are the normal Fermi liquid (FL) amplitudes, and $\tilde\varepsilon_0, \tilde\varepsilon_3$ are the FL corrections to the free single-particle spectrum. In this study, we will not be interested in the total energy density and the pressure in the interior of a neutron star. By this reason, the field contribution E_{field}, being the energy of the magnetic field in the absence of matter, can be omitted. Using Eqs. (1) and (9), we get the self-consistent equations in the form

$$\xi_0(\mathbf{p}) = \varepsilon_0(\mathbf{p}) + \tilde\varepsilon_0(\mathbf{p}) - \mu_0,\ \xi_3(\mathbf{p}) = -\mu_n H + \tilde\varepsilon_3(\mathbf{p}). \quad (11)$$

To obtain numerical results, we utilize the effective Skyrme interaction (SLy4 and SLy7 parametrizations). Using the same procedure as in Ref. [33], it is possible to find expressions for the normal FL amplitudes in terms of the Skyrme force parameters t_i, x_i, β:

$$U_0^n(\mathbf{k}) = 2t_0(1-x_0) + \frac{t_3}{3}\varrho^\beta(1-x_3) +$$

$$+\frac{2}{\hbar^2}[t_1(1-x_1) + 3t_2(1+x_2)]\mathbf{k}^2, \quad (12)$$

$$U_1^n(\mathbf{k}) = -2t_0(1-x_0) - \frac{t_3}{3}\varrho^\beta(1-x_3) +$$

$$+\frac{2}{\hbar^2}[t_2(1+x_2) - t_1(1-x_1)]\mathbf{k}^2 \equiv a_n + b_n\mathbf{k}^2. \quad (13)$$

Further, we do not consider the effective tensor forces which lead to coupling of the momentum and spin degrees of freedom [36–38] and, respectively, to the anisotropy in the momentum dependence of FL amplitudes with respect to the spin quantization axis. Then

$$\xi_0 = \frac{p^2}{2m_n} - \mu, \tag{14}$$

$$\xi_3 = -\mu_n H + (a_n + b_n \frac{\mathbf{p}^2}{4})\frac{\Delta\varrho}{4} + \frac{b_n}{16}\langle \mathbf{q}^2\rangle_3, \tag{15}$$

where the effective neutron mass m_n is defined by the formula

$$\frac{\hbar^2}{2m_n} = \frac{\hbar^2}{2m_0} + \frac{\varrho}{8}[t_1(1-x_1) + 3t_2(1+x_2)], \tag{16}$$

and the renormalized chemical potential μ should be determined from Eq. (6). The quantity $\langle \mathbf{q}^2\rangle_3$ in Eq. (15) is the second-order moment of the distribution function f_3:

$$\langle \mathbf{q}^2\rangle_3 = \frac{2}{V}\sum_{\mathbf{q}} \mathbf{q}^2 f_3(\mathbf{q}). \tag{17}$$

In view of Eqs. (14) and (15), the branches $\omega_\pm \equiv \omega_\sigma$ of the quasiparticle spectrum in Eq. (5) read

$$\omega_\sigma = \frac{p^2}{2m_\sigma} - \mu + \sigma\big(-\mu_n H + \frac{a_n\Delta\varrho}{4} + \frac{b_n}{16}\langle \mathbf{q}^2\rangle_3\big), \tag{18}$$

where m_σ is the effective mass of a neutron with spin up ($\sigma = +1$) and spin down ($\sigma = -1$)

$$\frac{\hbar^2}{2m_\sigma} = \frac{\hbar^2}{2m_0} + \frac{\varrho_\sigma}{2}t_2(1+x_2)+$$

$$+\frac{\varrho_{-\sigma}}{4}[t_1(1-x_1) + t_2(1+x_2)],\ \varrho_{+(-)} \equiv \varrho_{\uparrow(\downarrow)}. \tag{19}$$

For totally spin polarized neutron matter, we have

$$\frac{m_0}{m^*} = 1 + \frac{\varrho m_0}{\hbar^2}t_2(1+x_2), \tag{20}$$

where m^* is the effective neutron mass in the fully polarized state. Since $t_2 < 0$ usually for Skyrme parametrizations, we have the constraint $x_2 \leq -1$ which guarantees the stability of totally polarized neutron matter at high densities.

It follows from Eq. (18) that the effective chemical potential μ_σ for neutrons with spin-up ($\sigma = 1$) and spin-down ($\sigma = -1$) can be determined as

$$\mu_\sigma = \mu + \sigma\big(\mu_n H - \frac{a_n\Delta\varrho}{4} - \frac{b_n}{16}\langle \mathbf{q}^2\rangle_3\big). \tag{21}$$

Thus, with account of expressions (4) for the distribution functions f, we obtain the self-consistent equations (6), (7), and (17) for the effective chemical potential μ, spin order parameter $\Delta\varrho$, and second-order moment $\langle \mathbf{q}^2\rangle_3$.

3. Solutions of Self-Consistent Equations at $T = 0$. Thermodynamic Stability

Here, we directly solve the self-consistent equations at zero temperature and present the neutron spin order parameter as a function of the density and the magnetic field strength. In solving numerically the self-consistent equations, we utilize SLy4 and SLy7 Skyrme forces, which were constrained originally to reproduce the results of microscopic neutron matter calculations (pressure versus density curve) [39]. Note that the density dependence of the nuclear symmetry energy, calculated with these Skyrme interactions, gives the neutron star models in a good agreement with the observables such as the minimum rotation period, gravitational mass-radius relation, the binding energy released in a supernova collapse, *etc.* [40]. In addition, these Skyrme parametrizations satisfy the constraint $x_2 \leq -1$ obtained from Eq. (20).

We consider magnetic fields up to the values allowed by the scalar virial theorem. For a neutron star with mass M and radius R, equating the magnetic field energy $E_H \sim (4\pi R^3/3)(H^2/8\pi)$ with the gravitational binding energy $E_G \sim GM^2/R$, one gets the estimate $H_{\max} \sim \frac{M}{R^2}(6G)^{1/2}$. For a typical neutron star with $M = 1.5M_\odot$ and $R = 10^{-5}R_\odot$, this yields the maximum value of the magnetic field strength $H_{\max} \sim 10^{18}$ G. This magnitude can be expected in the interior of a magnetar while the recent observations report the surface values up to $H \sim 10^{15}$ G, as inferred from the hydrogen spectral lines [41].

In order to characterize the spin ordering in neutron matter, it is convenient to introduce a neutron spin polarization parameter

$$\Pi = \frac{\varrho_\uparrow - \varrho_\downarrow}{\varrho} \equiv \frac{\Delta\varrho}{\varrho}. \tag{22}$$

Fig. 1 shows the dependence of the neutron spin polarization parameter on the density normalized to the nuclear saturation density ϱ_0, at zero temperature in the absence of a magnetic field. The spontaneous polarization develops at $\varrho = 3.70\varrho_0$ for the SLy4 interaction ($\varrho_0 = 0.16$ fm^{-3}) and at $\varrho = 3.59\varrho_0$ for the SLy7 interaction ($\varrho_0 = 0.158$ fm^{-3}), which reflects the instability of neutron matter with the Skyrme interaction at such densities against spin fluctuations. Since the self-consistent equations at $H = 0$ are invariant with respect to the global flip of neutron spins, we have two branches of solutions for the spin polarization parameter, $\Pi_0^+(\varrho)$ (upper) and $\Pi_0^-(\varrho)$ (lower) which differ only by sign, $\Pi_0^+(\varrho) = -\Pi_0^-(\varrho)$.

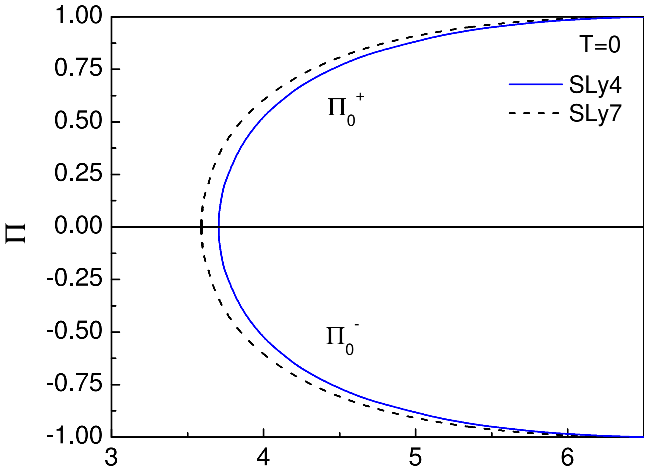

Fig. 1. Neutron spin polarization parameter as a function of the density at vanishing temperature and magnetic field

Figure 2 shows the neutron spin polarization parameter as a function of the density for a set of fixed values of the magnetic field. The branches of spontaneous polarization are modified by the magnetic field differently, since the self-consistent equations at $H \neq 0$ lose the invariance with respect to the global flip of spins. At nonvanishing H, the lower branch $\Pi_1(\varrho)$, corresponding to the negative spin polarization, extends down to the very low densities. There are three characteristic density domains for this branch. At low densities $\varrho \lesssim 0.5\varrho_0$, the absolute value of the spin polarization parameter increases with decrease in the density. At intermediate densities $0.5\varrho_0 \lesssim \varrho \lesssim 3\varrho_0$, there is a plateau in the $\Pi_1(\varrho)$ dependence, whose characteristic value depends on H, e.g., $\Pi_1 \approx -0.08$ at $H = 10^{18}$ G. At densities $\varrho \gtrsim 3\varrho_0$, the magnitude of the spin polarization parameter increases with the density, and neutrons become totally polarized at $\varrho \approx 6\varrho_0$.

Note that the results in the low-density domain should be considered as the first approximation to the real complex picture, since, as discussed in details in Ref. [9], the low-density neutron-rich matter in β-equilibrium possesses a frustrated state, "nuclear pasta" arising as a result of the competition of Coulomb long-range interactions and nuclear short-range forces. In our case, where a pure neutron matter is considered, there is no mechanical instability due to the absence of the Coulomb interaction. However, the possibility of the appearance of a low-density nuclear magnetic pasta and its impact on the neutrino opacities in the protoneutron star early cooling stage should be explored in a more detailed analysis.

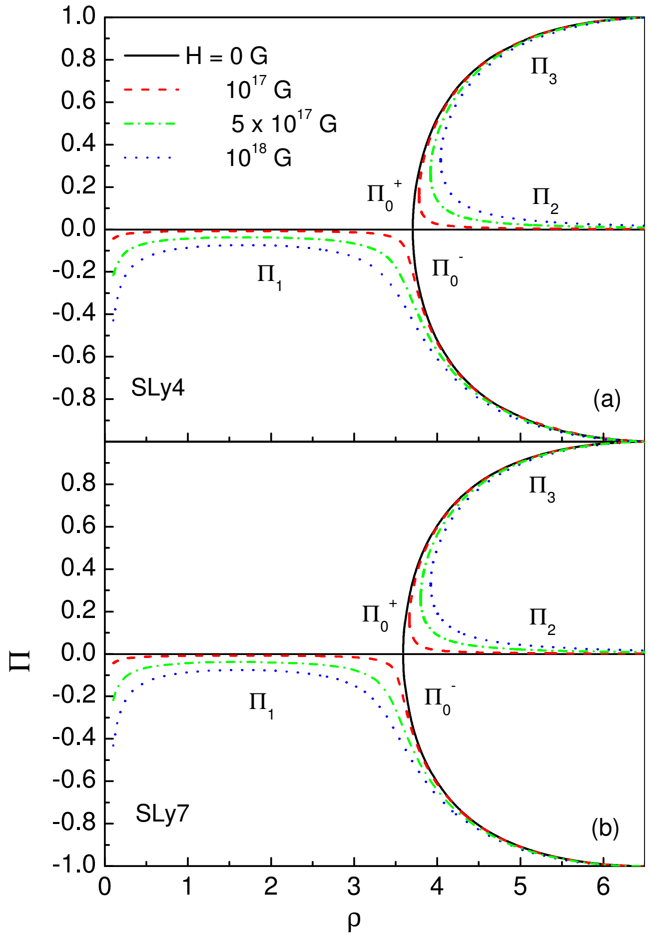

Fig. 2. Neutron spin polarization parameter as a function of the density at $T = 0$ and different magnetic field strengths for SLy4 (top) and SLy7 (bottom) interactions

At a nonzero magnetic field, the upper branch of spontaneous polarization $\Pi_0^+(\varrho)$, corresponding to positive values of Π, turns into two branches with different dependences on the density. For one of these branches, $\Pi_2(\varrho)$, the spin polarization parameter decreases with the density and tends to zero. For the other branch, $\Pi_3(\varrho)$, it increases with the density and is saturated. It is important that these branches appear step-wise at the same threshold density $\varrho_{\rm th}$ dependent on the magnetic field and being larger than the critical density of spontaneous spin instability in neutron matter. For example, for SLy7 interaction, $\varrho_{\rm th} \approx 3.80\,\varrho_0$ at $H = 5 \times 10^{17}$ G, and $\varrho_{\rm th} \approx 3.92\,\varrho_0$ at $H = 10^{18}$ G. The magnetic field, due to the negative value of the neutron magnetic moment, tends to orient the neutron spins oppositely to the magnetic field direction. As a result, the spin po-

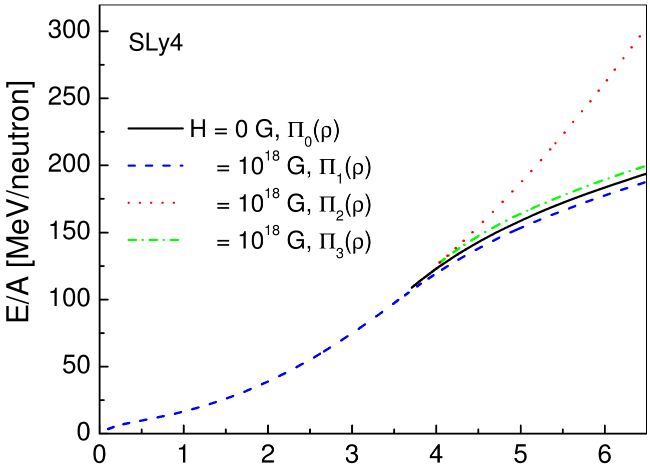

Fig. 3. Energy per neutron as a function of the density at $T = 0$ for different branches $\Pi_1(\varrho)$-$\Pi_3(\varrho)$ of solutions of the self-consistent equations at $H = 10^{18}$ G for the SLy4 interaction, including a spontaneously polarized state

larization parameter for the branches $\Pi_2(\varrho)$, $\Pi_3(\varrho)$ with the positive spin polarization is smaller than that for the branch of spontaneous polarization Π_0^+, and, *vice versa*, the magnitude of the spin polarization parameter for the branch $\Pi_1(\varrho)$ with the negative spin polarization is larger than the corresponding value for the branch of spontaneous polarization Π_0^-. Note that the impact of even such strong magnetic field as $H = 10^{17}$ G is small: the spin polarization parameter for all three branches $\Pi_1(\varrho)$-$\Pi_3(\varrho)$ is either close to zero or close to its value in the state with spontaneous polarization which is governed by the spin-dependent medium correlations.

Thus, at densities larger than $\varrho_{\rm th}$, we have three branches of solutions: one of them, $\Pi_1(\varrho)$, with the negative spin polarization and two others, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, with the positive polarization. In order to clarify, which branch is thermodynamically preferable, we should compare the corresponding free energies. Because the results of calculations with SLy4 and SLy7 Skyrme forces are very close, we will present the obtained dependences for one of these parametrizations. Figure 3 shows the energy per neutron as a function of the density at $T = 0$ and $H = 10^{18}$ G for these three branches, compared with the energy per neutron for a spontaneously polarized state [the branches $\Pi_0^{\pm}(\varrho)$]. It is seen that the state with the majority of neutron spins oriented oppositely to the direction of the magnetic field [the branch $\Pi_1(\varrho)$] has a lowest energy. This result is intuitively clear, since the magnetic field tends to direct the neutron spins oppositely to $\mathbf{H}$, as mentioned above. However, the state, described by the branch $\Pi_3(\varrho)$ with the positive spin polarization, has the energy very close to that of the thermodynamically stable state. This means that, despite the presence of a strong magnetic field $H \sim 10^{18}$ G, the state with the majority of neutron spins directed along the magnetic field can be realized as a metastable state in the dense core of a neutron star in the model consideration with the Skyrme effective interaction. In this scenario, since such states exist only at densities $\varrho \geqslant \varrho_{\rm th}$, under decreasing the density (going from the interior to the outer regions of a magnetar), a metastable state with the positive spin polarization at the threshold density $\varrho_{\rm th}$ changes into a thermodynamically stable state with the negative spin polarization.

The existence of a metastable state with the positive spin polarization in a neutron star at a strong magnetic field could lead to the following effect. As is apparent from our consideration, the *p*-*h* interaction in the Skyrme model for neutron matter becomes large and attractive in the spin channel with increase in the density. As a result, the neutrino cross sections are enhanced at the high-density region, and this drastically reduces the neutrino mean free paths [14]. Since the magnitude of the spin polarization parameter in a metastable state with the positive spin polarization is less than that in the thermodynamically stable state with the negative spin polarization, one can expect that the above-mentioned effects will be less pronounced in a neutron star possessing a metastable state with the positive spin polarization. Hence, this could lead to the accelerated cooling of a neutron star with a metastable positive spin polarized state as compared to the cooling rates of a neutron star possessing a stable thermodynamic configuration of neutron spins at a nonzero magnetic field.

At this point, we note some important differences between our results and those in Ref. [9]. First, while studying the neutron matter in a strong magnetic field [9], only one branch of solutions for the spin polarization parameter was found in the model with the Skyrme interaction (for the same SLy4 and SLy7 parametrizations). However, in fact, we have seen that the degenerate branches of spontaneous polarization (at zero magnetic field) with the positive and negative spin polarizations are modified differently by the magnetic field. As a result, there are generally three different branches of solutions of the self-consistent equations at a nonvanishing magnetic field in the Skyrme model. In addition, the only branch which was considered in Ref. [9] and corresponds to our thermodynamically stable branch Π_1 is characterized by the positive spin polarization, con-

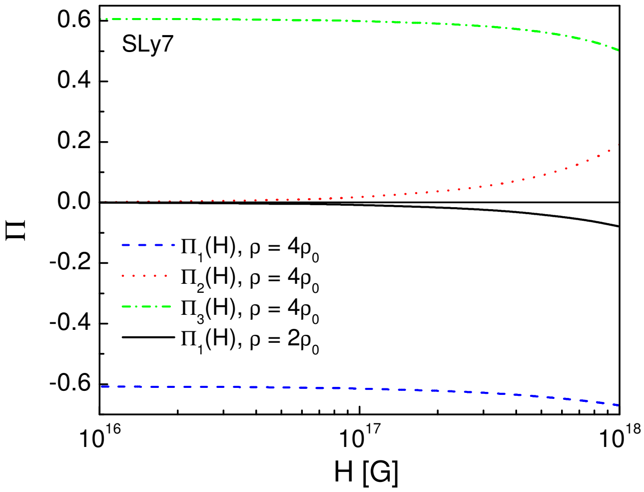

Fig. 4. Spin polarization parameter as a function of the magnetic field strength at $T = 0$ for different branches $\Pi_1(H)$-$\Pi_3(H)$ of solutions of the self-consistent equations at $\varrho = 4\varrho_0$ and for the branch $\Pi_1(H)$ at $\varrho = 2\varrho_0$ for the SLy7 interaction

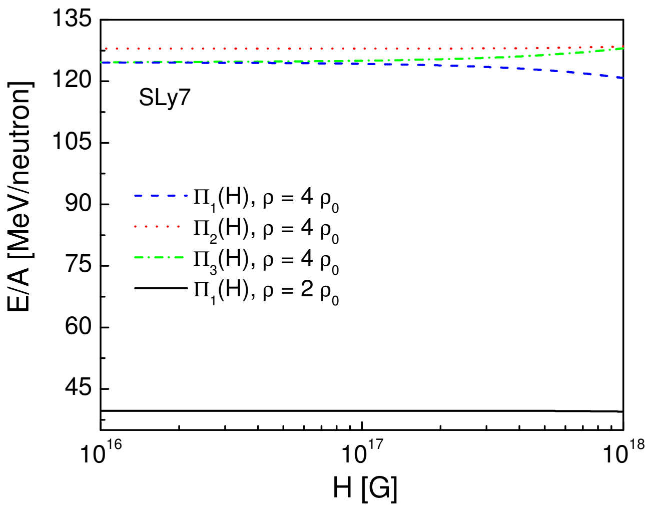

Fig. 5. Same as in Fig. 4 but for the energy per neutron

trary to our result with $\Pi_1 < 0$. This disagreement is explained by the incorrect sign before the term involving the magnetic field in the equation describing the single-particle spectrum in Ref. [9] (analog to Eq. (18) in our case). Clearly, the majority of neutron spins in the equilibrium configuration are aligned oppositely to the magnetic field.

Figure 4 shows the spin polarization parameter as a function of the magnetic field strength at zero temperature for different branches $\Pi_1(H)$-$\Pi_3(H)$ of solutions of the self-consistent equations at $\varrho = 4\varrho_0$ compared with that for the branch $\Pi_1(H)$ at $\varrho = 2\varrho_0$. It is seen that, up to the field strengths $H = 10^{17}$ G, the influence of the magnetic field is rather marginal. For the branches $\Pi_1(H)$ and $\Pi_2(H)$, the value of the spin polarization parameter increases with the field strength, while it decreases for $\Pi_3(H)$.

Figure 5 shows the energy of neutron matter per particle as a function of the magnetic field strength at $T = 0$ under the same assumptions as in Fig. 4. It is seen that the state with the negative spin polarization [branch $\Pi_1(H)$] becomes more preferable with increase in the magnetic field, although the total effect of changing the magnetic field strength by two orders of magnitude on the energy corresponding to all three branches $\Pi_1(H)$-$\Pi_3(H)$ remains small. It is also seen that the increase in the density by a factor of two leads to the increase in the energy per neutron roughly by a factor of three reflecting the fact that the medium correlations play a more important role in building the energetics of the system than the impact of a strong magnetic field.

Figure 6 shows the chemical potentials of spin-up and spin-down neutrons as functions of the magnetic field strength, where the chemical potentials are calculated according to Eq. (21). It is seen that the splitting of the spin-up and spin-down chemical potentials is very small up to the field strengths 10^{17} G. Hence, we arrive at the previous conclusion that the impact of a magnetic field remains small for such field strengths. Another point is that the spin-up and spin-down chemical potentials for branches Π_1 and Π_2 decrease, as H increases. But, for the branch Π_3, they increase with H. Hence, their dependence on the magnetic field strength is basically determined by the dependence of the effective chemical potential μ on H which is found from the normalization condition (6). In addition, the comparison of the spin-up and spin-down chemical potentials for the branch Π_1 at $\varrho = 2\varrho_0$ and $\varrho = 4\varrho_0$ implies that the in-medium effects strongly increase the effective chemical potentials.

In summary, we have considered the spin polarized states in neutron matter at a strong magnetic field in a model with the Skyrme effective NN interaction (SLy4 and SLy7 parametrizations). The self-consistent equations for the spin polarization parameter and the chemical potential of neutrons have been obtained and analyzed at zero temperature. It has been shown that the thermodynamically stable branch of solutions for the spin polarization parameter as a function of the density corresponds to the case where the majority of neutron spins is oriented oppositely to the direction of the magnetic field (negative spin polarization). This branch

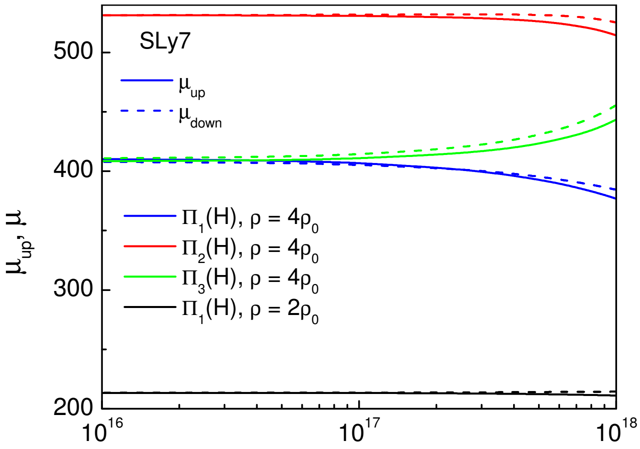

Fig. 6. Chemical potentials of spin-up (solid curves) and spin-down (dashed curves) neutrons as functions of the magnetic field strength at $T = 0$ for different branches $\Pi_1(H)$-$\Pi_3(H)$ of solutions of the self-consistent equations at $\varrho = 4\varrho_0$ and for the branch $\Pi_1(H)$ at $\varrho = 2\varrho_0$ for the SLy7 interaction

extends from very low densities to the high-density region, where the spin polarization parameter is saturated, and, respectively, neutrons become totally spin polarized. In addition, beginning from some threshold density ϱ_{th} dependent on the magnetic field strength, the self-consistent equations have also two other branches (upper and lower) of solutions for the spin polarization parameter corresponding to the case where the majority of neutron spins is oriented along the magnetic field (positive spin polarization). For example, for the SLy7 interaction, $\varrho_{th} \approx 3.80\,\varrho_0$ at $H = 5 \times 10^{17}$ G, and $\varrho_{th} \approx 3.92\,\varrho_0$ at $H = 10^{18}$ G. The spin polarization parameter along the upper branch increases with the density and is saturated, while it decreases and vanishes along the lower branch. The free energy corresponding to the upper branch turns out to be very close to that corresponding to the thermodynamically preferable branch with the negative spin polarization. As a consequence, at a strong magnetic field, the state with the positive spin polarization can be realized as a metastable state in the high-density region in neutron matter which changes with decrease in the density (going from the interior regions to the outer ones of a magnetar) at the threshold density ϱ_{th} into a thermodynamically stable state with the negative spin polarization. The calculations of the neutron spin polarization parameter, energy per neutron, and chemical potentials of spin-up and spin-down neutrons show that the influence of the magnetic field remains small at the field strengths up to 10^{17} G. Note that the consideration also has been done in Ref. [9] for the Gogny effective NN interaction (D1S and D1P parametrizations) up to densities $4\varrho_0$. Since there is no spontaneous ferromagnetic transition in neutron matter for all relevant densities for the D1S parametrization, and this transition occurs for the D1P parametrization at a density larger than $7\varrho_0$ [23], no sign of a ferromagnetic transition at a strong magnetic field was found in Ref. [9] up to densities $4\varrho_0$ for these Gogny forces. According to our consideration, one can expect that the metastable states with the positive spin polarization in neutron matter at a strong magnetic field could appear at densities larger than $7\varrho_0$ for the D1P parametrization, while the scenario with the only branch of solutions corresponding to the negative spin polarization would be realized for the D1S force.

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НАМАГНІЧЕНІСТЬ ГУСТОЇ НЕЙТРОННОЇ МАТЕРІЇ В СИЛЬНОМУ МАГНІТНОМУ ПОЛІ

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Р е з ю м е

Розглянуто спінові поляризовані стани в нейтронній матерії у сильних магнітних полях до 10^{18} Гс у моделі з ефективною взаємодією Скірма. На підставі аналізу рівнянь самоузгодження при нульовій температурі показано, що термодинамічно стійка гілка розв'язків для параметра спінової поляризації як функції густини відповідає від'ємній спіновій поляризації, коли більшість нейтронних спінів орієнтується протилежно магнітному полю. Крім цього, починаючи з деякої граничної густини, що залежить від напруженості магнітного поля, рівняння самоузгодження мають також дві інших гілки розв'язків для параметра спінової поляризації з додатною спіновою поляризацією. Вільна енергія, що відповідає одній із цих гілок, виявляється дуже близькою до вільної енергії, що відповідає термодинамічно стійкій гілці з від'ємною спіновою поляризацією. Як наслідок, у сильному магнітному полі стан з додатною спіновою поляризацією може реалізовуватися як метастабільний стан за високими густинами нейтронної матерії, який змінюється на термодинамічний стійкий стан з від'ємною спіновою поляризацією при зменшенні густини, починаючи з деякої граничної густини.