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Illichevskyy S. **УДК 330.46(075.8)**

THE MODEL OF THE FINITE TIME RUIN PROBABILITIES FOR INSURANCE COMPANY WITH INVESTMENT ACTIVITIES

МОДЕЛЬ РАСЧЕТА ВРЕМЕНИ БАНКРОТСТВА ДЛЯ СТРАХОВОЙ КОМПАНИИ С ИНВЕСТИЦИОННОЙ АКТИВНОСТЬЮ

Данная статья посвящена исследованию и разработке актуарной модели расчета времени наступления банкротства для страховой компании. Новизна статьи заключается в том, что анализируется страховая компания, которая осуществляет инвестиционную активность, что в свою очередь выступает в качестве дополнительной статьи ее дохода.

The problem statement. Today it is impossible to imagine a market economy without risks. They are involved almost in every economic activity. There is a great need in measuring, predicting and minimizing risks. Insurance services are one of the industries, which permanently experience risks of bankruptcy. That is why calculating the ruin probabilities for insurance companies are one of the problems that need well-developed mathematical models [1, p. 179]. Nowadays Ukrainian insurance companies are searching for new ways of profitability and competitiveness. Western European insurance companies has an option of investing their fund for additional profit. That is way there is a great necessity of creation and development of the actuarial models for Ukrainian insurance to provide them the possibility of investing their fund for additional profit.

The analysis of main researches and publications. One of the first studies in this area was conducted in the beginning of the twentieth century. Since then, the mathematical methods of ruin probability calculation developed and accumulated a great variety of models and approaches. While the permanent growing of economic needs, insurance services increase steadily in the economies of all developed countries. Insurance services are one of the youngest industries any economy, which experience a stage of active development. In global practice of developed countries, well organized insurance services are involved in many economic sectors like investment activity of insurance companies. This article studies how the actuarial mathematical tools can positively affect the theoretical and practical development of insurance. The development of theoretical, methodological, organizational and legal bases of insurance market have been contributed by many economists, such as: Alexandrova M., Alexandrova T., ArtyukhT., Bazylevych V., Baranovsky A. , Osadets S, Zaruba A., Kolomin E., Klapkiv M., Shah E., Reytman L., Slusarenko E, Yakovlev T., Facil M. and others.

Unsolved issues: one of the main problems at present for actuarial analysis of the Ukrainian insurance market is the lack of large statistical base, which is necessary for any econometric modeling. That is way there is a great necessity of actuarial models that involve fewer statistical information. We analyze methods of calculation of ruin probabilities for insurance company in presents of its investing activity. We consider an insurance company in the case when the premium rate is a bounded by some nonnegative random function and the capital of the insurance company is invested in a risky asset whose price follows a geometric Brownian.

The goals and tasks of the article: the goal of the article is creation new types of actuarial models of the analysis of ruin probabilities that can be helpful for Ukrainian insurance companies under presence of their investment activities. There are different methods for approximating the distribution of aggregate claims and their corresponding stop-loss premium by means of a discrete compound Poisson distribution and its corresponding stop-loss premium. This discretization is an important step in the numerical evaluation of the distribution of aggregate claims, because recent results on recurrence relations for probabilities only apply to discrete distributions. The discretization technique is efficient in a certain sense, because a properly chosen discretization gives raise to numerical upper and lower bounds on the stop-loss premium, giving the possibility of calculating the numerically estimates for the error on the final numerical results.

We consider an insurance company in the case when the premium rate is a bounded nonnegative random function c_t and the capital of the insurance company is invested in a risky asset whose price follows a geometric Brownian motion with mean return α and volatility $\sigma > 0$. If $\beta := 2a/\sigma^2 - 1 > 0$ we find exact the asymptotic upper and lower bounds for the ruin probability as the initial endowment u tends to infinity, i.e. we show that $C_* u^{-\beta} \le \Psi(u) \le C^* u^{-\beta}$ for sufficiently large u. Moreover if $c_t = c^* r^{\gamma t}$ with $\gamma \le 0$ we find the exact asymptotics of the ruin probability, namely $\Psi(u) \sim u^{-\beta}$. If $\beta \le 0$, we show that $\psi(u) = 1$ for any $u \ge 0$. We investigate the problem of consistency of risk measures with respect to usual stochastic order and convex order. It is shown that under weak regularity conditions risk measures preserve these stochastic orders. This result is used to derive bounds for risk measures of portfolios. As a by-product, we extend the characterization of coherent, law-invariant risk measures with the property to unbounded random variables. A surprising result is that the trading strategy yielding the optimal asymptotic decay of the ruin probability simply consists in holding a fixed quantity (which can be explicitly calculated) in the risky asset, independent of the current reserve. This result is in apparent contradiction to the common believe that `rich' companies should invest more in risky assets than `poor' ones. The reason for this seemingly paradoxical result is that the minimization of the ruin probability is an extremely conservative optimization criterion, especially for `rich' companies [2, p. 351].

It is well-known that the analysis of activity of an insurance company in conditions of uncertainty is of great importance [3, p. 62]. Starting from the classical papers of Cramer and Lundberg which first considered the ruin problem in stochastic environment, this subject has attracted much attention. Recall that, in the classical Cramer-Lundberg model satisfying the Cramer condition and, the positive safety loading assumption, the ruin probability as a function of the initial endowment decreases exponentially [4, p. 663]. The problem was subsequently extended to the case when the insurance risk process is a general Levy process.

More recently ruin problems have been studied in application to an insurance company which invests its capital in a risky asset see, e.g., Paulsen [5, p. 139], Kalshnikov and Norberg [6, p. 221], Frolova, Kabanov, Pergamenshchikov [7, p. 231] and many others.

It is clear that, risky investment can be dangerous: disasters may arrive in the period when the market value of assets is low and the company will not be able to cover losses by selling these assets because of price fluctuations. Regulators are rather attentive to this issue and impose stringent constraints on company portfolios. Typically, junk bonds are prohibited and a prescribed (large) part of the portfolio should contain non-risky assets (e.g., Treasury bonds) while in the remaining part only risky assets with good ratings are allowed. The common notion that investments in an asset with stochastic interest rate may be too risky for an insurance company can be justified mathematically.

We deal with the ruin problem for an insurance company investing its capital in a risky asset specified by a geometric Brownian motion:

$$dV_t = V_t(adt + \sigma d\omega_t),$$

where $(\omega_t, t \ge 0)$ is a standard Brownian motion and $a > 0, \sigma > 0$.

It turns out that in this case of small volatility, i.e. $0 < \sigma^2 < 2a$, the ruin probability is not exponential but a power function of the initial capital with the exponent $\beta := 2a/\sigma^2 - 1$. It will be noted that this result holds without the requirement of positive safety loading. Also, for large volatility, i.e. $\sigma^2 > 2a$, the ruin probability equals 1 for any initial endowment.

In all these papers the premium rate was assumed to be constant. In practice this means that the company should obtain a premium with the same rate continuously. We think that this condition is too restrictive and it significantly bounds the applicability of the above mentioned results in practical insurance settings.

The numerical calculation of finite time ruin probabilities for two particular insurance risk models are being analyzed. The first model allows for the investment at a fixed rate of interest of the surplus whenever this is above a given level. Our second model is the classical risk model but with the insurer's premium rate depending on the level of the surplus.

Our methodology for calculating finite time ruin probabilities is to bound the surplus process by discrete-time Markov chains; the average of the bounds gives an approximation to the ruin probability.

Our primary purpose in this paper is to discuss the numerical calculation of finite time ruin probabilities for two particular insurance risk models. Both models are extensions of the classical risk model. For each model, $U(t)$ is a random variable which denotes the surplus at time $t(\ge 0)$, so that $\{U(t)\}_{t\ge 0}$ is a continuous time stochastic process; the aggregate claims in [0,t] are denoted $S(t)$, where $S(t)$ has a compound Poisson distribution with Poisson parameter λ; individual claim amounts have *cdf* $G(x)$, *pdf* $g(x)$ and mean m_1. We assume that $G(0) = 0$, so that all claims are positive. We assume without loss of generality that $\lambda = 1 = m_1$ [8, p. 418.

We denote by T the time to ruin for these processes, where 'ruin' occurs when the surplus first falls below 0, so that:

$$T = \begin{cases} \inf\{t : U(t) < 0\} \\ \infty, if U(t) \ge 0, t \ge 0 \end{cases} \qquad (1)$$

The probability of ruin within finite time t, given that the initial surplus $U(0)$ is equal to u, is denoted $\psi(u,t)$ and defined by:

$$\psi(u,t) = P(T \le t) \qquad (2)$$

Our first model allows for the surplus to be invested at a fixed force, or instantaneous rate, of interest, δ per unit time, whenever the surplus exceeds some level, Δ. Let c denote the fixed rate of premium income per unit time. The surplus process is governed by the stochastic differential equation:

$$dU(t) = I(U(t))dt - dS(t) \qquad (3)$$

where:

$$I(x)\begin{cases} c, 0 \le x < \Delta \\ c + \delta(x - \Delta), x \ge \Delta \end{cases} \qquad (4)$$

An extension of this model can be defined in terms of formula (1) with the following definition of $I(x)$ in place of formula (2):

$$I(x) = \begin{cases} c + \delta_1 x, x < 0 \\ c, 0 \le x < \Delta \\ c + \delta(x - \Delta), x \ge \Delta \end{cases} \qquad (5)$$

so that the surplus process continues even when it is below zero, with interest being paid at rate δ_1 on amounts borrowed. For this model it is natural to define 'ruin' as the event that the surplus falls below the level – c / δ_1, since below that level the premium income is insufficient to pay interest on the deficit and the process cannot subsequently rise above zero, or even above – c / δ_1.

The second model studied in this paper is the classical risk model modified by allowing the rate of premium income to vary through time according to the level of the surplus. Formally, this process is defined by (1) together with:

$$I(x) = \begin{cases} c_1, 0 \le x < B \\ c_2, x \ge B \end{cases} \qquad (6)$$

for some given positive level *B*. It would be possible to have more than two bands for the surplus with a different rate of premium income at time *t* depending on the band in which $U(t)$ lies. However, all our numerical examples assume just two bands and so we have presented the model in this way.

An essential feature of the two models studied in this paper is that they are time-homogeneous Markov processes; the level of the surplus at any given time is sufficient to determine probabilistically its level at any time *h* later. This is the feature that we will exploit in this paper to obtain bounds for the finite time ruin probabilities for our two models. We do not need to assume any form of 'net profit condition' for our two models, but we do need to assume that c, c_1 and $c_2 > 0$.

Our aim is to produce bounds for this probability; approximate values of the probability can be calculated by averaging the upper and lower bounds. However it is not always possible to produce absolute bounds.

The surplus process of an insurance portfolio is defined as the wealth obtained by the premium payments minus the reimbursements made at the times of claims. When this process becomes negative (if ever), we say that ruin has occurred. The general setting is the Gambler's Ruin Problem. We address the problem of estimating derivatives (sensitivities) of ruin probabilities with respect to the rate of accidents. Estimating probabilities of rare events is a challenging problem, since naive estimation is not applicable.

Solution approaches are very recent, mostly through the use of Importance Sampling techniques. Sensitivity estimation is an even harder problem for these situations. We study different methods for estimating ruin probabilities: one via importance sampling (IS), and two others via indirect simulation: the storage process (SP), which restates the problems in terms of a queuing system, and the convolution formula (CF). To estimate the sensitivities, we apply the RPA method to importance sampling, the IPA method to storage process and the Score Function method to convolution formula. Simulation methods are compared in terms of their efficiency, a criterion that appropriately weighs precision and CPU time. As well, we indicate how other criteria such as set-up time and prior formulas development may actually be problem-dependent. The canonical model in Risk Theory assumes that claims due to accidents arrive according to a Poisson process $N(t)$ of rate λ. The successive claim amounts, denoted $\{Y_i\}$, are i.i.d. random variables with general distribution G and premiums are received at a constant rate c. If the initial endowment is $u > 0$, the wealth of the insurance company, known as the *surplus process* is:

$$U(t) u + ct - \sum_{i=1}^{N(t)} Y_i, t \ge 0 \qquad (7)$$

The event epochs of the process $N(t)$ are denoted by $\{T_n n \ge 0\}$, and $W_n = T_n - T_{n-1}$ are the interarrival times. The cumulative claims process $S(t) = \sum_{i=1}^{N(t)} Y_i$ is a compound Poisson process. We shall often write $U_n = U(T_n)$ to denote the embedded discrete event process and $S_n = \sum_{i=1}^{n} Y_i$, with an obvious abuse of notation. If we set $\tau = \min\{n : U(T_n < 0)\} = \min\{n : u + cT_n \le S_n\}$ then the ruin probability is [9, p. 217].

$$\psi(u, \lambda) = P\{\tau < \infty\} \quad (8)$$

and it is a measure of the credit risk of the company. Call $\beta = E[Y_1]$. If $c \le \lambda\beta$,then $\psi(u, \lambda)$ for all initial endowment u. As a consequence of this result, it is common to assume that premiums satisfy $c \le \lambda\beta$.

Conclusions. We analyzed methods of calculation of ruin probabilities for insurance company in presents of its investing activity. We considered an insurance company in the case when the premium rate is a bounded by some nonnegative random function and the capital of the insurance company is invested in a risky asset whose price follows a geometric Brownian.

The weak development of insurance market in Ukraine is explained by the low incomes of Ukrainians and their disinterest in spending money on insurance, although some cases.

The analyzed economic and mathematical models are recommended to be used in for Ukrainian insurance companies for increasing profitability and diversification of ruin risks.

The prospects for further development of the problem. Although there are several methods for calculation the ruin probabilities for insurance companies, this study may enrich existing methods, for cases of investment activities for Ukrainian insurance companies.

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Калмынина Т.В., Андреев А.Б. **УДК 314.143**

ПЕРСПЕКТИВЫ ДЕМОГРАФИЧЕСКОГО РАЗВИТИЯ ПРИГРАНИЧНЫХ ТЕРРИТОРИЙ ЗАБАЙКАЛЬЯ

Забайкальский регион, один из наиболее важных геостратегических плацдармов России в XXI веке может стать объектом демографической экспансии соседних государств. Только, в прилегающих к российско-китайской границе провинциях Китая проживает более ста миллионов человек. В автономном районе Внутренняя Монголия плотность населения достигает 20,2 чел./км^2 , в провинции Хэйлунцзян - 46 чел/км^2, в то время как в Забайкальском крае – 2,59 чел/км^2, в Республике Бурятия – 2,74 чел/км^2.

Возможности демографического развития Забайкальского региона ограничены. На протяжении длительного времени сохранялась тенденция снижения численности населения из-за естественной убыли и миграционного оттока. В настоящее время, несмотря на повышение рождаемости, демографический потенциал региона остается крайне низким. Демографический потенциал отражает совокупность сложных глубинных процессов, которые часто действую разнонаправлено. Это процессы рождаемости, смертности, колебания численности населения в результате миграции, сдвиги в половозрастной структуре, изменения качества жизни населения. Демографический потенциал можно определить как возможности воспроизводства населения и развития человека, характеризующиеся показателями уровня общественного развития, качества жизни, средней продолжительности предстоящей жизни, уровня грамотности взрослого населения и объема реального ВВП на душу населения [1].

Демографический потенциал приграничных территорий является основой конкурентоспособного трансграничного сотрудничества.

Республика Бурятия и Забайкальский край являются приграничными регионами. Приграничный регион определяется как регион в пределах административных или иных государственных территориальных образований, административно-территориальные границы которых совпадают с линией государственной