UDC 539.4

Size Effect in Contact Fatigue

L. A. Sosnovskii,[a] V. A. Zhmailik,[b] and V. V. Komissarov[c]

[a] Scientific and Production Group "Tribofatigue Ltd.," Gomel, Belarus

[b] Production Group "Gomselmash," Gomel, Belarus

[c] Belarusian State University of Transport, Gomel, Belarus

The results of special experiments on the size effect in contact fatigue are presented. It is established that under constant contact loading conditions the durability is higher, the larger is the diameter of a tested element. The methods for estimation of contact fatigue resistance of gear wheels, which is based on the statistical model for a deformable solid body having a critical volume, are proposed. The limiting stresses of a gear wheel are estimated using a regulated base for this machine parts.

Keywords: size effect, contact fatigue, gear wheels.

The durability and fatigue resistance of components operating under cyclic loading by bending, tension, compression, twisting, etc. appear to be the lower, the larger are the component dimensions [1, 2] Since the phenomena of contact fatigue are based on the same processes as those occurring under other types of fatigue, it is natural to expect that increasing the absolute dimensions of a component would decrease its contact fatigue limit. However, by analyzing the available results of studies on the size effect in contact fatigue it is impossible to get a definite opinion on this problem [2–13].

Some researchers state that the size effect in contact fatigue undergoes inversion, i.e., as the diameter of contacting parts is increased, the durability grows [4–6]. However, others assert that increase in a component size leads to reduction of bending and contact fatigue limits [7, 10].

For the basic regularities of the size effect in contact fatigue to be elucidated, special experimental studies have been made. The test scheme is shown in Fig. 1. Cylindrical sample *2* serves as a tooth of a gear wheel. A counterbody – roller *1* – is pressed to the surface of sample *2* by a contact load F_N in the contact zone x. Roller *1* serves as a tooth of the second gear wheel that transmits the contact load F_N to sample-model *2*.

Sample *2*, which is fastened in spindel *3*, is rotated with an angular velocity ω_1. Counterbody *1* is rotated with an angular velocity ω_2, its rotation axis being parallel to that of sample *2*. Regulating the ratio of the velocities ω_1 and ω_2 allows one to obtain the required slip coefficient, imitating the slip in gearing. The contact load F_N provides a simultaneous excitation both of contact and bending stresses in the corresponding zones, whereas the distance between these zones is chosen tobe equal to that between the pitch point and the tooth root.

Using the counterbody (roller) with a constant diameter D =100 mm and sample-models of various diameters d (Fig. 2) makes it possible to change two main curvatures and to obtain the size ratio of the contact area (a/b) within the range 0.4–0.8, which is satisfactory for practical purposes.

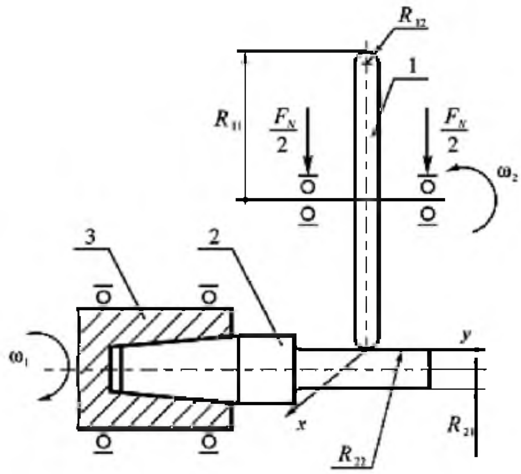

Fig. 1. Testing scheme for a toothed gearing model: (*1*) sample-model of the tooth; (*2*) counterbody (roller); (*3*) testing machine spindle.

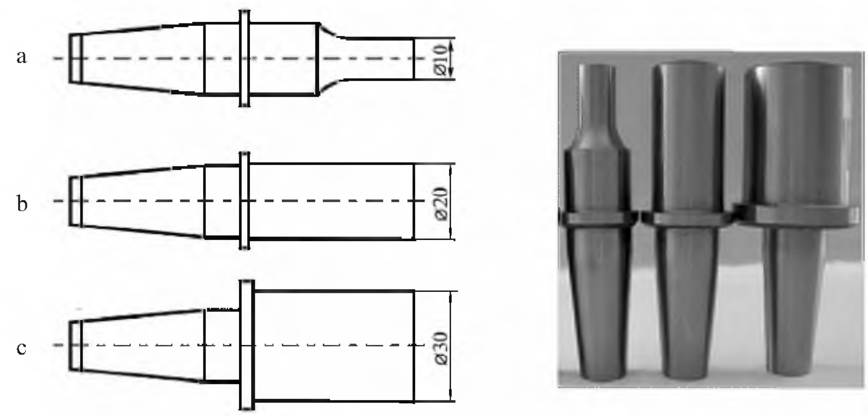

Fig. 2. Tested tooth models.

Samples 10, 20, and 30 mm in diameter were made from steel 18KhGT, invoking the technology of manufacturing gear wheels at the Production Group "Gomselmash." Working surfaces of counterbodys and samples were cemented at a depth of 1.0–1.5 mm with subsequent hardening up to 59–63 *HRC* and polished ($R_a \geq 0.32$ μm). The run-out of the samples in the working zone was not more than 10 μm. Tests were performed using a wear fatigue testing machine of UIM type [20] at a constant linear velocity in the contact $v_{circ} = 1.57$ m/s. The error of keeping the shaft rotation frequency within the steady regime is $\pm 3\%$ of the measured value [20].

When the sample and the counterbody are tested in the contact zone, the slip degree is controlled to be equal to 3%. In the course of tests, a lubricant (oil TAD-17 I) is supplied to the contact zone with a feeding speed of 2–4 drops per minute. The tests were interrupted after occurrence of the limiting state conditions corresponding to the real service ones for a particular gearing (limiting convergence of the axes of the sample and the counterbody $\delta_{lim} = 100$ μm). The contact load F_N (see Fig. 3) serves as a parameter controlling the model loading.

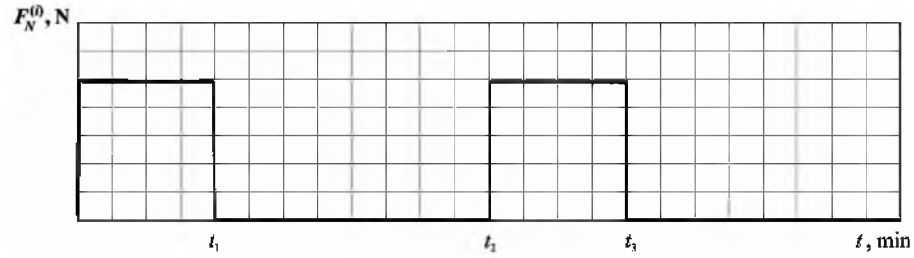

Fig. 3. Loading program scheme.

The basic test results are shown in Figs. 4 and 5.

From Fig. 4 it is seen that under constant contact loading conditions the durability is higher, the larger is the diameter of a tested element. For the base of $N_b = 4\cdot 10^6$ cycles, we have established the dependences of limiting contact stresses p_f and limiting contact load F_{lim} on the diameter of the tested models, d (Fig. 5). The analysis of Fig. 5 yields that:

1) in terms of the limiting contact load, if the tested model diameter is increased from 10 to 30 mm, this load grows from 780 to 1520 N.

2) in terms of the limiting contact fatigue, if the tested model diameter is increased from 10 to 30 mm, the contact fatigue limit decreases from 5150 MPa to 4900 MPa.

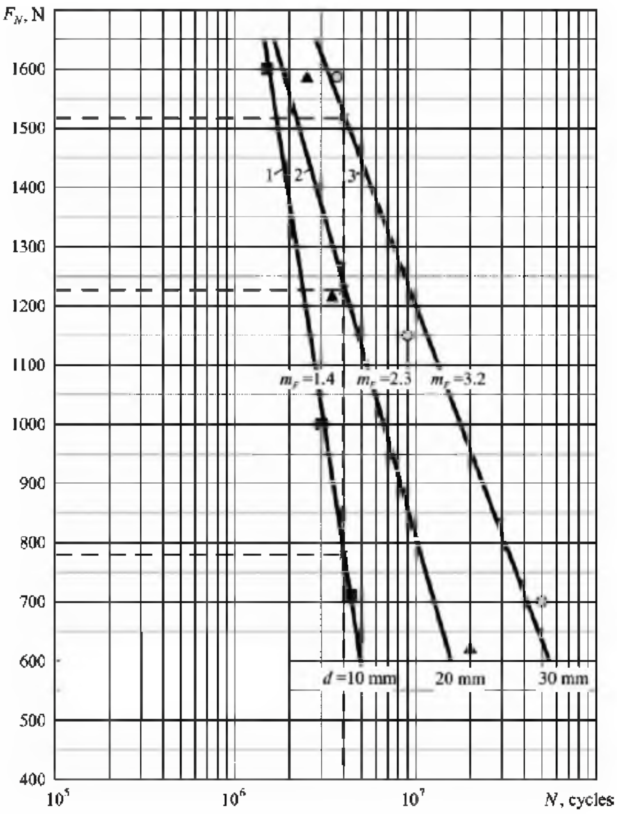

Fig. 4. The size effect on the contact fatigue resistance [(*1*) $\log F = 7.8009 - 0.7434\log N$; (*2*) $\log F = 5.9047 - 0.4266\log N$; (*3*) $\log F = 5.2255 - 0.3097\log N$].

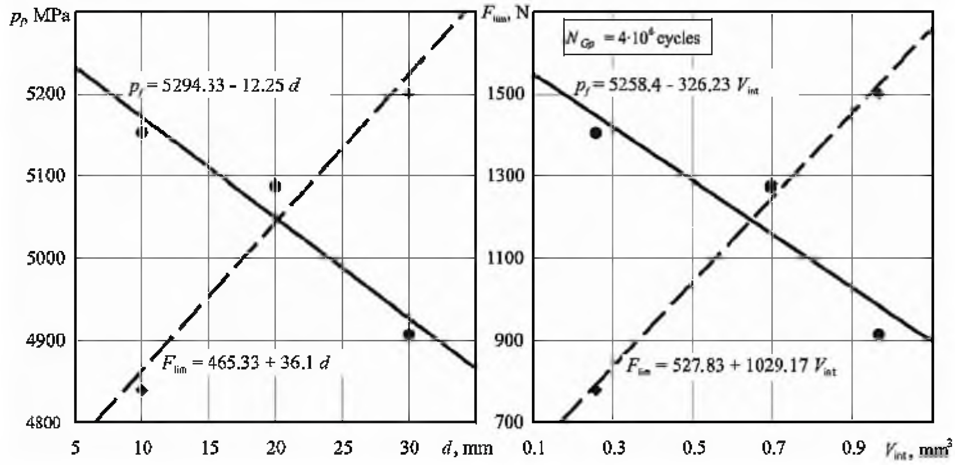

Fig. 5. Limiting contact stresses and limiting contact load vs. sample diameter (a) and critical volume size (b).

Based on the above experimental results, it is possible to formulate the following basic regularity of the size effect in friction: under constant contact loading conditions the durability is higher, the larger is the diameter of a tested component.

For the size effect in contact fatigue to be analyzed theoretically, the model of a solid body with a critical volume [14] was used. According to this model, if a deformable solid body is characterized by such a stressed state that its fatigue failure is possible, then it is composed of two regions: safe and critical volumes. Similarly, assume that in contact deformation by a critical volume V_{int} is understood the region of a loaded body, at each point of which the value of the stress intensity σ_{int} is less than the limiting value of $\sigma_{int}^{(lim)}$ $(\sim p_f^{(lim)})$

$$V_{int} = \iiint\limits_{\sigma_{int}(x,y,z)\geq\sigma_{int}^{lim}} dxdydz. \qquad (1)$$

The critical volume is an absolute measure of damage; it is statistical in character and contains the geometrical sizes of tested elements. This permits using it as the parameter controlling the size effect in contact fatigue.

The proposed procedure is applied for the assessment of critical (limiting) stresses as a limitation criterion for the corresponding critical regions [15]. It consists in determining the limiting intensity of stresses $\sigma_{int}^{(*lim)}$ when the tested system is subjected to the limiting load F_{*lim}:

$$\sigma_{int}^{(*lim)} = \max_{dV}(\sigma_{int}(F_{*lim}, dV),) \qquad (2)$$

where dV is the elementary volume of the loaded body.

Then the criterial condition for limitation of critical volumes will be of the form

$$V_{int} = \{dV / \sigma_{int} \geq \sigma_{int}^{(*lim)},\ dV \subset V_k\}, \tag{3}$$

where V_k is the working volume of a deformable solid body.

As critical volumes can have arbitrary and complex shapes, it is difficult to determine them by formula (1). Therefore, calculations were made using the program package "Mathematics" by the numerical Monte-Carlo method.

Figure 6 presents the calculation results on the critical volumes formed due to normal and tangential contact stresses.

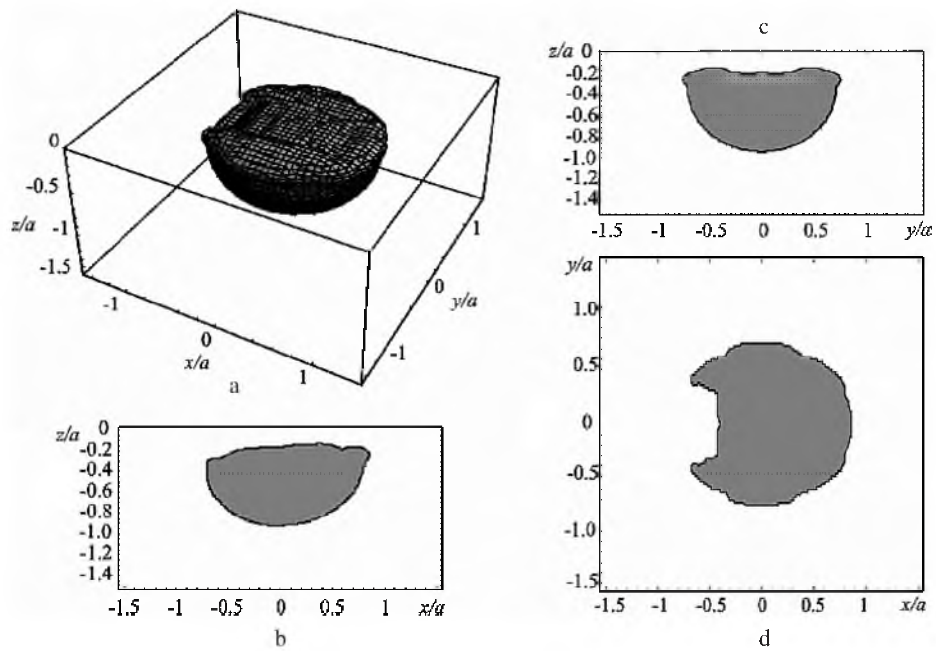

Fig. 6. The critical volume (a) developed due to normal and tangential contact stresses: (b) section by the plane $y = 0$; (c) section by the plane $x = 0$; (d) section by the plane $z = 0.5a$.

Since critical volumes are used as the damage measure of deformable bodies, the analysis of Fig. 6 specifies the particular regions (zones) where internal cracks can initiate and propagate.

Using the proposed method, the dependences of limiting contact stresses p_f and limiting contact load F_{lim} on the critical volume size (V_{int}) (Fig. 5) are constructed for the base $N_b = 4 \cdot 10^6$ cycles. The obtained dependences are seen to have a qualitatively similar character both in analyzing the tested models in terms of the diameter and the critical volume in terms of size. Thus, the proposed model is valid and does not contradict the results obtained. In this case, a critical volume to be calculated includes the geometrical dimensions of tested components and has a statistical character [limiting contact stresses $p_f^{(lim)}$ serve as a limitation criterion]. In this respect, this is a more preferable parameter for description of the size effect in contact fatigue.

Using the given approach, it is possible to execute a settlement estimation of rolling fatigue resistance of particular toothed gear wheels. The input data for calculations are given in Table 1.

T a b l e 1

Input Data for Calculation of Gear Wheels

Gear	Combined torque M_k, N·m	Reference diameter d, mm	Radius of tooth profile evolvent in a pitch point		Effective face width B, mm
			ρ_1, mm	ρ_2, mm	
PKK 0135684	290	92	15.73	19.15	20
PKK 0135661	450	108	18.47	21.89	40

For the rolling fatigue limit estimation for a particular type cogwheel it is necessary to pass to base of tests equal to $1.2 \cdot 10^8$ cycles which is regulated for tooth gearings [10, 17]. As a first approximation, such transition can be executed, accepting the slope indicator of the left branch of rolling fatigue curve $m_p = 3–6$ [17] or on the average $m_p = 4.5$. Calculation results are presented in Fig. 7 and in Table 2.

T a b l e 2

Results of Calculation of Gear Wheels

Gear	V_{int}, mm^3	p_0, MPa	p_f, MPa	n_p	σ_H, MPa	n_{σ_H}
PKK 0135684	0.00201	1171.4	2340.8	2.00	1337.8	1.75
PKK 0135661	0.00208	888.6	2340.3	2.63	945.7	2.47

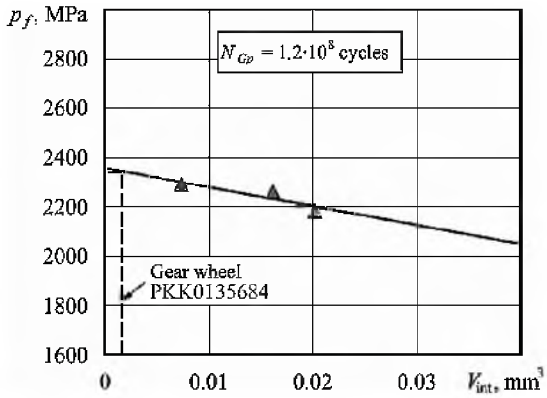

Fig. 7. Dependence of rolling fatigue limit on the critical volume value.

It is seen from Table 2 that for a gear wheel PKK 0135684 value of safety factor for the rolling endurance n_p is lower by 25% than that of PKK 0135661. This allows us to conclude that the operational durability of the PKK 0135684 gear wheel will also be lower than that of PKK 0135661. This conclusion is confirmed by data on operation of a transmission PKK 0135000 of a harvest combine "Polesie-3000" manufactured by Production Group "Gomselmash" [22].

Conclusions. Inversion of size effect in rolling fatigue is established and described for particular experimental conditions. The technique of rolling fatigue resistance assessment on the basis of statistical model of a deformable solid body with critical volume is proposed. The estimation of a rolling fatigue limit is made for particular types of gear wheels.

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Received 11. 06. 2008