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Two-Parametrical Fracture Criterion

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Двухпараметрический критерий разрушения

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Показано, что для полухрупких или полупластических твердых материалов требуется более сложная трактовка разрушения, чем для хрупких и пластических материалов. Применение двухпараметрических критериев разрушения позволяет решить эту задачу и более достоверно оценить механическое поведение материалов.

Ключевые слова: критерии разрушения, полухрупкие или полупластические материалы, критический коэффициент интенсивности напряжений, напряжение, деформация, трещины.

Introduction. Since 1958, when J. Irwin [1] showed the applicability of the stress intensity factor as a fracture criterion, this factor has found a wide range of applications in the engineering. Most of other criteria suggested by D. S. Dugdale, A. A. Wells, et al. [2], provided the comparable results to these of the Irwin criterion.

The universal criteria are needed for materials with brittle and plastic fracture. Such criteria must evaluate these fracture processes not in separate way but within a two-parameterical approach.

In 1970, the two-parametrical criteria were presented in Dowling and Townley study [3]. They evaluate both the fracture mechanics and the plastic collapse. The local and global criteria of strength and fracture are integrated in these works. The well known R6-approach is integrated as well [4]. This method is used for the definition of a basic route for the establishing of the integrity of structure containing crack-like defects. The R6-approach is a numerical-engineering method, and the accuracy of fracture process description depends on the amount of the experimental parameters. Classical two-parametrical criteria combine the lesser amount of parameters. This is very important for the assessment process, as the amount of data is limited. Furthermore, classical solutions of the less accurateness are critical for the further studies.

Problem Formulation. The exact expression of the evaluation of stress at the crack tip (see Fig. 1) as $|x|>0$ and $y=0$ can be written as follows [5]

$$\sigma_y = \frac{\sigma_{|x|}}{\sqrt{x^2 - l^2}}, \tag{1}$$

where $\sigma_{|x|}$ and σ_y are the normal stress in the x and y direction, respectively.

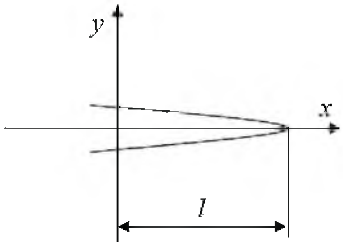

Fig. 1. Crack tip coordinates (l is length of crack).

Assume the critical stress at the crack tip σ_c and averaged distance in crack tip d it can be expressed as follows [6]

$$\frac{1}{d}\int_0^d \sigma_y dx \le \sigma_c. \tag{2}$$

After embedding values of σ_y from Eq. (1) to (2) it follows that

$$\sigma_{nom} = \frac{\sigma_c}{\sqrt{1 + \frac{2l}{d}}}, \tag{3}$$

where σ_{nom} is the nominal stress ($\sigma_{nom} < \sigma_c$).

In the case of asymptotic change of stress σ_y at the crack tip, σ_y is equal to

$$\sigma_y = \frac{K}{\sqrt{2\pi(x - l)}}, \tag{4}$$

where K is the stress intensity factor.

Then the value of σ_y from Eq. (4) can be written into (2) and it follows that

$$\frac{1}{d}\int_l^{l+d} \frac{K_c}{\sqrt{2\pi(x - l)}} dx = \sigma_c, \tag{5}$$

where K_c is the critical stress intensity factor.

From (5) it can be defined

$$\sigma_c = K_c\sqrt{\frac{2}{\pi d}}, \tag{6}$$

or

$$d = \frac{2}{\pi}\left(\frac{K_c}{\sigma_c}\right)^2. \tag{7}$$

After embedding values σ_c from (6) to (3) it follows that

$$\sigma_{nom} = K_c\sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{d+2l}}, \tag{8}$$

and after embedding values d from Eq. (7) and assessed that $K_{nom} = \sigma_c\sqrt{\pi l}$, it follows that

$$\left(\frac{\sigma_{nom}}{\sigma_c}\right)^2 + \left(\frac{K_{nom}}{K_c}\right)^2 = 1. \tag{9}$$

The same equation as (9) was obtained by Griffits [7]. Thus in the case of the brittle fracture and if the material plastic strain in the crack tip is not assessed the fracture process could be defined by two-parametrical criterion according to Eq. (9).

The material resistance to fracture in the tip of crack under plastic deformation depends upon the stress value.

According to the Irwin opening model of plane stressed state

$$K_{\mathrm{I}} = \sigma\sqrt{\pi(a + r_{0.2})} = \sigma\sqrt{\pi a}\sqrt{1 + \left(\frac{\sigma\sqrt{\pi a}}{\sigma_{0.2}}\right)^2}, \tag{10}$$

or

$$K_{\mathrm{I}cp} = K_{\mathrm{I}e}\sqrt{1 + \frac{1}{2}\left(\frac{\sigma}{\sigma_{0.2}}\right)^2}, \tag{11}$$

where K_{I} is the stress intensity factor in the case of opening model, $K_{\mathrm{I}cp}$ and $K_{\mathrm{I}e}$ are the appropriate stress intensity factors after plastic correction and this in the case of elastic fracture, respectively, and $\sigma_{0.2}$ is the yield stress.

According to the Dugdale model [2], in the case of plane stressed state, the crack opening δ can be expressed as follows

$$\delta = \frac{K_{\mathrm{I}cp}^2}{E\sigma_{0.2}} = \frac{8\sigma_{0.2}a}{\pi E}\ln\left[\cos\left(\frac{\pi\sigma}{2\sigma_{0.2}}\right)\right]^{-1} = \frac{8\sigma_{0.2}(\sigma\sqrt{\pi a})}{2\pi\sigma^2 E}\ln\left[\cos\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{0.2}}\right)\right]^{-1}. \tag{12}$$

It follows that

$$K_{\mathrm{I}cp} = K_{\mathrm{I}e}\sqrt{\frac{8}{\pi^2}\left(\frac{\sigma_{0.2}}{\sigma}\right)^2\ln\left[\cos\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{0.2}}\right)\right]^{-1}}. \tag{13}$$

The equations (11) and (13) have a similar character

$$\frac{K_{\mathrm{I}e}}{K_{\mathrm{I}cp}} = f\left(\frac{\sigma}{\sigma_{0.2}}\right) = f\left(\frac{F}{F_L}\right), \tag{14}$$

where F is the load and F_L is the yield load.

As material behavior is elastic, the elastic fracture condition is as follows

$$K_{\mathrm{I}cp} = K_{\mathrm{I}c}, \tag{15}$$

where $K_{\mathrm{I}c}$ is the material property in the elastic state.

The fracture of plastic material is specified by the load

$$F = F_L. \tag{16}$$

So, the relation of plastic and brittle criteria is the function of loads specifying elastic and plastic fracture, i.e.,

$$\frac{K_{\mathrm{I}e}}{K_{\mathrm{I}c}} = f\left(\frac{F}{F_L}\right). \tag{17}$$

The stress intensity factor K_{I} and the deformation zone at the crack tip r are related by dependences as the strain is ε [5]

$$K_{\mathrm{I}} = \sigma_{nom}\sqrt{\pi r}, \tag{18}$$

and

$$K_{\mathrm{I}c} = \sigma_{nom}\sqrt{\pi r_0}, \tag{19}$$

where r_0 is the length of zone as the elastic-plastic strain has a maximal value $\varepsilon_{\max}$.

It follows that

$$\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{I}c}}\right)^2 = \frac{r}{r_0}. \tag{20}$$

The relation of deformation zones r/r_0 is proportional to the relation of strains $\varepsilon/\varepsilon_{\max}$ and the followings can be written

$$\frac{r}{r_0} = \frac{\varepsilon}{\varepsilon_{\max}} = \left(\frac{K_{\mathrm{I}}}{K_{\mathrm{I}c}}\right)^2. \tag{21}$$

The strain fracture criterion consists of two strain components that are responsible for fracture [6] is obtained the equation

$$\left(\frac{\varepsilon}{\varepsilon_{\max}}\right)^{s}+\left(\frac{\varepsilon_{nom}}{\varepsilon_{0.2}}\right)^{s}=1, \tag{22}$$

where s is the interpolation parameter and $\varepsilon_{0.2}$ is the yielding strain.

Considering that s parameter is coherent [7], in the case of $s=2$, Eq. (22) can be written as

$$\left(\frac{\varepsilon}{\varepsilon_{\max}}\right)^{2}+\left(\frac{\varepsilon_{nom}}{\varepsilon_{0.2}}\right)^{2}=1. \tag{23}$$

After embedding value of $\varepsilon/\varepsilon_{\max}$ expressed by $(K_{\mathrm{I}}/K_{\mathrm{Ic}})^2$ to Eq. (23), it can be defined as

$$\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{Ic}}}\right)^{4}+\left(\frac{\varepsilon_{nom}}{\varepsilon_{0.2}}\right)^{2}=1. \tag{24}$$

If the deformation low assume to be $\varepsilon=(\sigma/C)^n$, where C is the material constant and n is the hardening exponent, then Eq. (24) can be written as follows

$$\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{Ic}}}\right)^{4}+\left(\frac{\sigma_{nom}}{\sigma_{0.2}}\right)^{2/n}=1. \tag{25}$$

A. G. Miller [8] proposed an expression without material hardening

$$\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{Ic}}}\right)^{4}+\left(\frac{\sigma_{nom}}{\sigma_{0.2}}\right)^{2}=1. \tag{26}$$

These three expressions could be used as express information in comparing with the so-called R6-approach in the UK or European flaw assessment method SINTAP. The early version of the R6-approach recognized that, at one extreme, linear elastic fracture mechanics was applicable and fracture occurred when the stress intensity factor, in this case, K_{I} became equal to the fracture toughness K_{Ic}. At the other extreme-failure occurred, when the load σ_{nom} reached its value, $\sigma_{0.2}$, at plastic collapse. The R6-approach recognized that the use of K beyond the elastic regime underestimated the crack tip loading and, therefore, some plasticity correction was required. [9, 10] In general, this correction is a function of the material and the component, the crack geometry and the type of loading. However, in the early R6-approach, it was recognized that by using the two normalizing parameters [9, 10]

$$K_r=\frac{K_{\mathrm{I}}}{K_{\mathrm{Ic}}}, \tag{27}$$

and

$$S_r=\frac{\sigma_{nom}}{\sigma_{0.2}}, \tag{28}$$

where K_r is the fracture-related stress intensity ratio and S_r is the plastic-collapse-related stress ratio. The plasticity correction could be converted to a general purpose failure avoidance curve

$$K_r = f(S_r). \tag{29}$$

In terms of Eq. (29), the Dugdale plasticity correction corresponds to

$$f(S_r) = S_r\left[\frac{8}{\pi^2}\ln\sec\left(\frac{\pi}{2}S_r\right)\right]^{-1/2}. \tag{30}$$

The failure assessment diagram (FAD) of Eq. (30) could then be interpreted simply as an interpolation between the two limiting failure states – brittle fracture and plastic collapse – of a cracked component (Fig. 2).

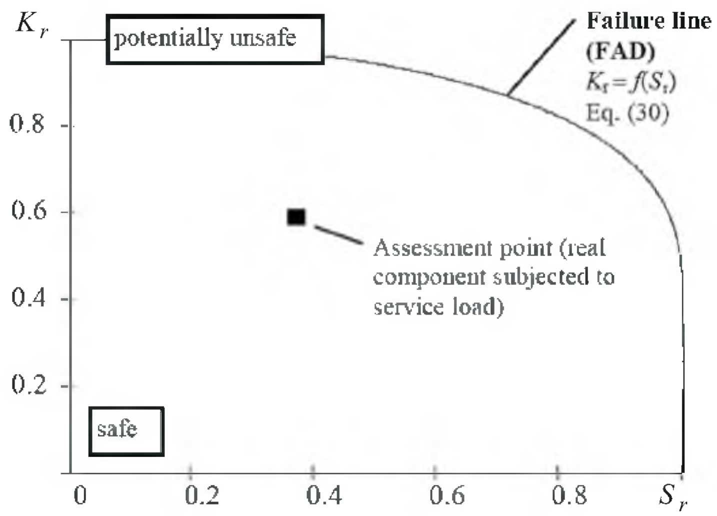

Fig. 2. FAD according to R6-Rev. 1, schematically [9].

Results and Discussion. The agreement between Eqs. (9), (24), (25) and experimental results was verified for to types of austenitic steel: 14Ch17N2 and 08Ch10N10T. Herewith, standard specimens (ASME E 8M) were used for standard compact specimens (ASTM E 1921-97) for failure in tension (Fig. 3).

Basic crack growth data were obtained on standard compact specimens with a thickness $B = 25$ mm and width of the sample $W = 50$ mm ($l_{\max} = 27.5$ mm, $N_{\max} = 3.1$ mm, and $G_{\min} = 27.5$ mm). The K_{Ic} value was computed using the following formula:

$$K_{Ic} = \frac{F_c Y}{BW^{1/2}},$$

$$Y = 29.6(l/W)^{1/2} - 185.5(l/W)^{3/2} + 665.7(l/W)^{5/2} -$$

$$-1017(l/W)^{7/2}+638.9(l/W)^{9/2},$$

where F_c is the critical load.

Investigations were performed by tensile testing machine SDMPu-10 (Germany). For steel 14Ch17N2 it was obtained that $\sigma_{0.2}=560$ MPa, $n=0.7$, and $K_{Ic}=84$ MPa $\cdot$ m$^{1/2}$, while for steel 08Ch10N10T: $\sigma_{0.2}=640$ MPa, $n=0.65$, and $K_{Ic}=78$ MPa $\cdot$ m$^{1/2}$, respectively.

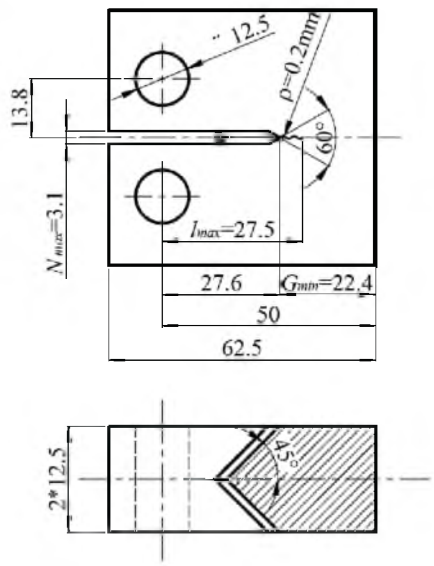

Fig. 3. The geometry of the compact test specimen.

The ratios in Eqs. (9), (25), and (26) were changed by parameters K_r and S_r. Obtained equations were compared with Dugdale model. The results are shown as FAD in Fig. 4.

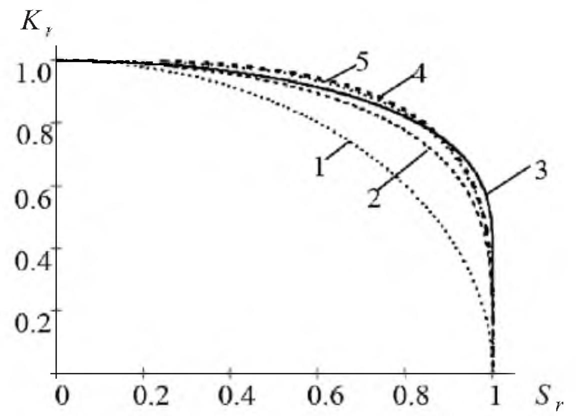

Fig. 4. FAD's for various models: (*1*) Griffits model; (*2*) Miller model; (*3*) Dugdale or R6-Rev. 1 model; (*4*) new introduced model for austenitic steel 14CH17N2; (*5*) new introduced model for austenitic steel 08Ch10N10T.

Conclusions. The brittle fracture of materials is defined precisely enough by second order two-parametrical fracture criteria, the main parameters of which are the stresses and stress intensity factors.

The fracture of semi-brittle or semi-plastic materials is evaluated according to equation suggested by author, in which the hardening exponent is evaluated.

Two-parametrical fracture criteria suggested by author and R6-approach provide the best result of the fracture assessment.

Р е з ю м е

Показано, що напівкрихкі чи напівпластичні тверді матеріали потребують більш складного трактування руйнування, аніж крихкі і пластичні матеріали. Використання двопараметричних критеріїв руйнування дозволяє розв'язати цю задачу та більш вірогідно оцінити механічну поведінку матеріалів.

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