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Fatigue Life of Welded Joints According to Energy Criteria in the Critical Plane

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Долговечность сварных соединений согласно энергетическому критерию в критической плоскости

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Рассчитаны теоретические коэффициенты концентрации напряжений с использованием понятия "фиктивный радиус" для трубных и трубно-фланцевых сварных соединений. Предложено для оценки усталостной долговечности материалов в условиях циклического чистого изгиба, чистого кручения и комбинации пропорционального изгиба с кручением использовать параметры плотности энергии нормальных и сдвиговых деформаций в критических плоскостях. Критические плоскости определяли методами, базирующимися на максимальных параметрах плотности энергии нормальных деформаций и сдвиговых деформаций.

Ключевые слова: сварное соединение, плотность энергии деформации, теоретический коэффициент концентрации напряжений.

Introduction. In the case of local approach to the welded joints subjected to multiaxial loadings, it is necessary to know stress concentration at the weld edge [1]. However, usually it is not possible to measure a real radius of the toe of weld, so a suitable method of measurements is necessary. This problem was successfully solved for welded joints subjected to uniaxial loading where the fictitious radius method was applied [1]. This method was based on the Neuber theory [2]. This paper presents a model of fatigue life estimation based on the parameter of strain energy density under the complex stress state. In this paper, the results obtained for the tube–tube and flange–tube welded joints under pure bending, torsion, in- and out-of-phase combination [1] are estimated with use of some chosen criteria based on the energy parameter [3, 4].

1. **Theoretical Notch Coefficient**. According to the Neuber proposal [2], stress averaging is based on the microstructure hypothesis and it is assumed that crack initiation is controlled by stress in the notch bottom averaged in a small volume of the material in the place where the maximum stress occurs. The suitable material parameter is a substitute microstructural length ρ^*. Stresses in the notch bottom should be averaged along this length normal to the notch surface. The calculated fictitious radius is expressed by the following equation:

$$\rho_f = \rho + s\rho^* = s\rho^* . \tag{1}$$

When the radius is known, we can calculate the notch coefficient. The fictitious radius also depends on geometry of specimens and a loading type [1, 2]. The fictitious notch radius ρ_f depends on the actual notch radius ρ, the substitute microstructural length ρ^* and the multiaxiality coefficient s, according to the Neuber proposal. In many cases, the zero notch radius is assumed, $\rho = 0$, with $\rho^* = 0.4$ mm for welded steels [2, 5]. For round specimens subjected to bending (for $\nu = 0.3$), we obtain $s = (5 - 2\nu + 2\nu^2)/(2 - 2\nu + 2\nu^2)$ and $\rho_{fb} =$ 1.16 mm for bending and $s = 1$ and $\rho_{ft} = 0.4$ mm for torsion.

2. **Energy Model for Fatigue Life Assessment**. The strain energy density parameter is defined as [2]

$$W(t) = \frac{1}{2}\sigma(t)\varepsilon(t)\frac{\text{sign}[\varepsilon(t) + \text{sign}[\sigma(t)]}{2} = \frac{1}{2}\sigma(t)\varepsilon(t)\text{sign}[\varepsilon(t), \sigma(t)]. \qquad (2)$$

Generalized criterion of the normal and shear strain energy density parameter in the critical plane may be written as [4]

$$W_{eq}(t) = \beta W_{\eta s}(t) + \kappa W_{\eta}(t), \qquad (3)$$

where

$$W_{\eta s}(t) = 0.5\tau_{\eta s}(t)\varepsilon_{\eta s}(t)\text{sign}[\tau_{\eta s}(t), \varepsilon_{\eta s}(t)], \qquad (4)$$

$$W_{\eta}(t) = 0.5\sigma_{\eta}(t)\varepsilon_{\eta}(t)\text{sign}[\sigma_{\eta}(t), \varepsilon_{\eta}(t)], \qquad (5)$$

and the coefficients β and κ are in order to chose the criterion.

Criterion of the Maximum Parameter of Shear and Normal Strain Energy Density in the Plane Determined by the Normal Strain Energy Density Parameter. The critical plane is determined by the normal strain energy density parameter and criterion (3) may be written as

$$W_{eq}(t) = \beta W_{\eta s}(t) + W_{\eta}(t), \qquad (6)$$

where coefficient β is selected depending on the material according to the results of nonproportional tests.

Criterion of the Maximum Parameter of Shear and Normal Strain Energy Density in the Plane Determined by the Shear Strain Energy Parameter. In this case, the critical plane is determined by the shear strain energy density parameter and criterion (3) may be written as

$$W_{eq}(t) = k(1-\nu)W_{\eta s}(t) + \frac{4 - k(1-\nu)^2}{1+\nu}, \qquad (7)$$

where $k = (\sigma_{af}/\tau_{af})$.

An algorithm for determination of fatigue life may be written in the following steps. For local strains we obtain the following values of strains (stage 1)

$$\varepsilon_{xx}(t)=(1-\nu^2)K_{tb}\frac{\sigma_{xx,n}(t)}{E}, \tag{8}$$

$$\varepsilon_{zz}(t)=-2\nu K_{tb}\frac{\sigma_{xx,n}(t)}{E}, \tag{9}$$

$$\varepsilon_{xy}(t)=\frac{\gamma_{xy}(t)}{2}=\frac{K_{tt}\gamma_{xy,n}(t)}{2}=K_{tt}\frac{\tau_{xy,n}(t)}{2G}. \tag{10}$$

Owing to the biaxial stress state on the surface and the edges of the notch (or weld), the stresses $\sigma_\eta(t)$ and $\tau_{\eta s}(t)$ and strains $\varepsilon_\eta(t)$ and $\varepsilon_{\eta s}(t)$ are

$$\sigma_\eta(t)=\hat{l}_\eta^2\sigma_{xx}(t)+\hat{m}_\eta^2\sigma_{yy}(t)+2\hat{l}_\eta\hat{m}_\eta\sigma_{xy}(t), \tag{11}$$

$$\tau_{\eta s}(t)=\hat{l}_\eta\hat{l}_s\sigma_{xx}(t)+\hat{m}_\eta\hat{m}_s\sigma_{yy}(t)+(\hat{l}_\eta\hat{m}_s+\hat{l}_s\hat{m}_\eta)\sigma_{xy}(t), \tag{12}$$

$$\varepsilon_\eta(t)=\hat{l}_\eta^2\varepsilon_{xx}(t)+\hat{m}_\eta^2\varepsilon_{yy}(t)+\hat{n}_\eta^2\varepsilon_{zz}(t)+2\hat{l}_\eta\hat{m}_\eta\varepsilon_{xy}(t), \tag{13}$$

$$\varepsilon_{\eta s}(t)=\hat{l}_\eta\hat{l}_s\varepsilon_{xx}(t)+\hat{m}_\eta\hat{m}_s\varepsilon_{yy}(t)+\hat{n}_\eta\hat{n}_s\varepsilon_{zz}(t)+(\hat{l}_\eta\hat{m}_s+\hat{l}_s\hat{m}_\eta)\varepsilon_{xy}(t). \tag{14}$$

If we have the stress and strains histories (stage 2) in any plane defined by the direction cosines, in this plane we can determine histories of the strain energy density parameter. For criterion (6) the critical plane is defined by the parameter of normal strain energy density according to Eqs. (5), (11), and (13). For criterion (7) the critical plane is defined by the parameter of shear strain energy density parameter – according to Eqs. (4), (12), and (14). The position of the critical plane (stage 3) is defined by the given values of direction cosines $\hat{l}_n$, $\hat{m}_n$, $\hat{n}_n$ $(n=\eta, s)$ of unit vectors $\overline{\eta}$ and $\overline{s}$ occurring in fatigue criteria. The method of fatigue damage accumulation includes fatigue damage accumulation in many planes of the given particle and selection of the plane of the maximum damage degree. In the plane stress state, the direction cosines $\hat{l}_\eta$, $\hat{m}_\eta$, $\hat{l}_s$, and $\hat{m}_s$ of the vectors $\hat{\eta}$ and $\hat{s}$, occurring in the formulas for the energy density parameter of normal strains and shear strains, are defined by one angle α as $\hat{l}_\eta=\cos\alpha$, $\hat{m}_\eta=\sin\alpha$, $\hat{l}_s=-\sin\alpha$, and $\hat{m}_s=\cos\alpha$. From the previous considerations it appears that the criterion where the critical plane is determined by the maximum parameter of normal strain energy density is valid for cast iron being a cast material. The criterion defined in the plane determined by the maximum parameter of shear strain energy density is valid for steel. Under cyclic loading it is necessary to determine history of the energy parameter only for one cycle (T).

If the normal strain energy density parameter is used for determination of the critical plane, we must calculate $W_\eta(t)$ only for one cycle. Then, amplitude of the normal strain energy density parameter $W_{\eta a}$ is

$$W_{eq,a} = W_{\eta a} = \max_{T,\alpha} W_n(t,\alpha). \qquad (15)$$

If the shear strain energy density parameter is applied for determination of the critical plane, $W_{\eta s}(t)$ is calculated only for one cycle. In such a case, amplitude of the shear strain energy density parameter $W_{\eta sa}$ is

$$W_{eq,a} = W_{\eta sa} = \max_{T,\alpha} W_{ns}(t,\alpha). \qquad (16)$$

At stage 4, it is necessary to determine the equivalent strain energy density parameter in the critical plane. In the case of the critical plane defined by the shear strain energy density parameter, we obtain the equivalent strain energy parameter using Eq. (7). If the critical plane is defined by the normal strain energy density parameter, we apply Eq. (6). At stage 5, we use the obtained extremum in a cycle for determination of amplitudes of the strain energy density parameter. At stage 6, the fatigue life is determined on the basis of the distinguished cycle amplitudes W_{aeq} directly from fatigue curves $W_a - N_f$ for energy notation according.

3. **Fatigue Life Assessment**. Experimental verification was based on the fatigue tests performed by Sonsino [1, 6]. Table 1 contains mechanical properties of the considered material (StE460 steel). Figure 1 shows geometry of the tested specimens.

The welded joints were tested under pure bending, pure torsion and combined bending with torsion in- and out-of phase (90°). Under combined bending with torsion, a ratio of nominal shearing to shear stress was determined $\tau_{an}/\sigma_{an} = 0.58$.

The further analysis of test results concerns rough specimens. In Table 2, all notch coefficients for fictitious radii are compared with coefficients of stress concentration for real radii. All the coefficients are determined by calculations using the finite element method for edges of particular welds of particular radii [1]. Now it was possible to determine coefficients of stress concentration for bending and torsion based on the fictitious radii ρ_f. All the calculated fictitious radii and coefficients of stress concentration are presented in Table 2. From the table it appears that different fictitious radii of the notch in the weld are obtained for bending and torsion.

Rescaling the values from the nominal system into the local system

$$\log N_f = A - m \log \sigma_a \qquad \text{and} \qquad \log N_f = A_\tau - m_\tau \log \tau_a \qquad (17)$$

using theoretical notch coefficients for bending and torsion respectively, we obtain new characteristics $W_a - N_f$. Their characteristics were determined from test results through analysis of regression. The determined parameters of fatigue curves are given in Table 3.

T a b l e 1

Mechanical Properties of StE460 Steel

Material	E, GPa	ν	$R_{0.2}$, MPa	R_m, MPa	A_5, %
StE460	206	0.3	520	670	25

T a b l e 2

Calculated Coefficients of Stress Concentration and Fatigue Notches

Weld joint	K_{tb}	K_{tt}	K'_{tb}	K'_{tt}
Flange–tube	3.93	1.85	3.11	1.88
Tube–tube	2.20	1.77	1.92	1.79

T a b l e 3

Parameters of $S-N$ Curves of the Tested Joints in the Local System

Welded joint	A	m	r	A_τ	m_τ	r_τ	k $(5\cdot 10^5)$
Flange–tube	17.034	4.306	0.965	25.782	8.233	0.974	1.65*
Tube–tube	16.342	4.207	0.968	–	–	–	1.65*

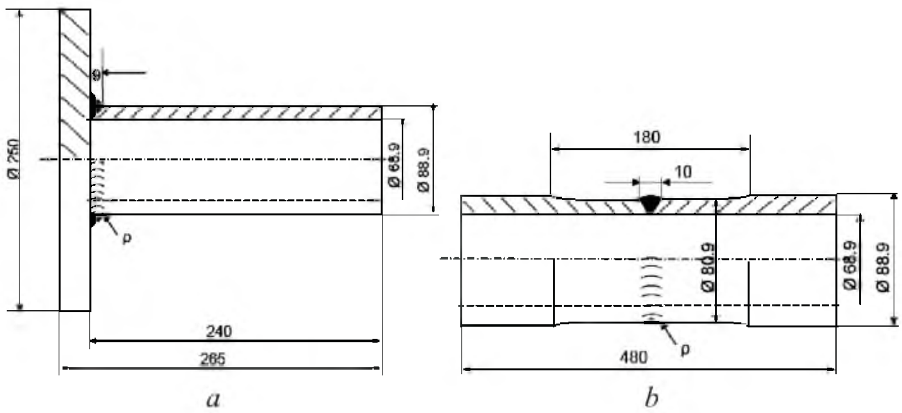

Fig. 1. Geometries of welded joints: (*a*) flange–tube and (*b*) tube–tube.

Let us verify the criterion assuming the critical plane determined by the maximum normal strain energy density parameter. In the case of flange–tube joints (Fig. 2a), the results are included into the scatter band for pure bending except for torsion. As for tube–tube joints (Fig. 2b), only one point for proportional loading and one point for non-proportional loading are located outside the scatter band for pure bending. There were no tests of tube–tube joints under pure torsion because cracks occurred outside the joint in the native material. Let us consider the criterion assuming the critical plane determined by the maximum parameter of shear strain energy density. Figure 3 shows comparisons of calculated and experimental lives for particular tests. In the case of flange–tube joints (Fig. 3a), the results are included into the scatter band for pure bending except for one point for proportional loading. As for tube–tube joints (Fig. 3b), one point for proportional loading and one point for non-proportional loading and one point for non-proportional loading are located outside the scatter band for pure bending, like in the case when the critical plane is determined by the normal strain energy density parameter (Fig. 2b).

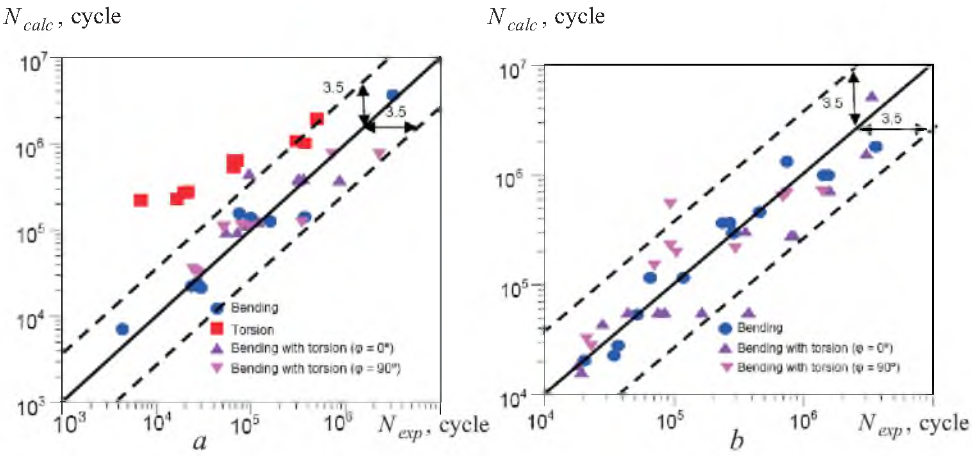

Fig. 2. Comparison of calculated and experimental lives for flange–tube (a) and tube–tube (b) welded joints according to criterion for the critical plane determined by the normal strain energy density parameter for $\beta = 10$.

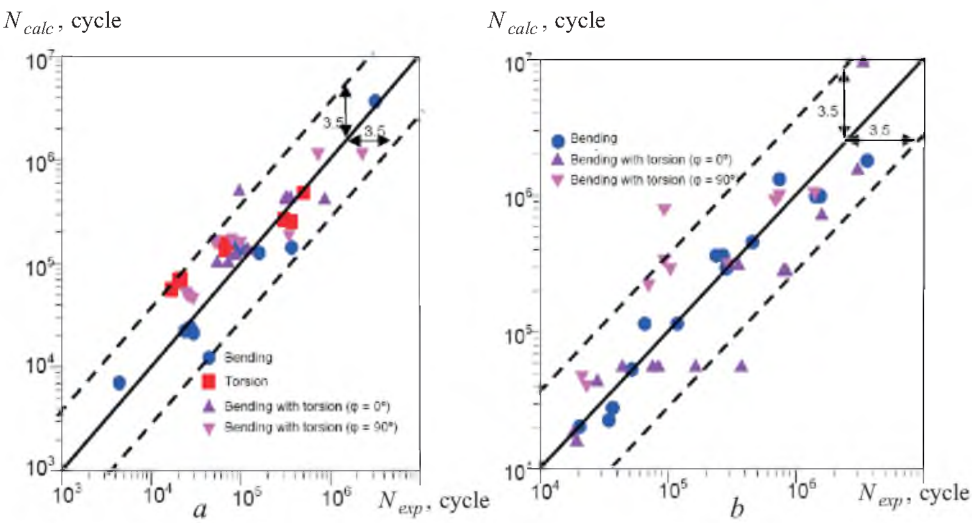

Fig. 3. Comparison of calculated and experimental lives for flange–tube (a) and tube–tube (b) welded joints according to criterion in the critical plane determined by the shear strain energy density parameter.

If the critical plane is determined by the maximum parameter of normal strain energy density, after the non-proportional tests it is necessary to determine the coefficient β including a part concerning the shear strain energy density parameter in the expression for the equivalent strain energy density. Thus, application of the energy criterion using the critical plane determined by shear strain energy density seems to be more convenient.

CONCLUSIONS

1. In order to assess history of multiaxial fatigue in welded joints by local stresses and strains, we must know a real local radius at the weld edge. Owing to application of the fictitious local radius, when in the worst case for sharp notches $\rho = 0$ means a crack, we are able to calculate notch coefficients for bending K_{fb} and for torsion K_{ft}. Thus, we should determine separate fictitious notch radii ρ_f for bending and torsion. In the case of steel welded joints, the radii are ρ_{fb} = 1.16 mm for bending and ρ_{ft} = 0.4 mm for torsion.

2. The parameter of normal and shear strain energy densities in the critical plane determined by the parameter of shear and normal strain energy density for steel welded joints gives comparable results. However, if the critical plane is determined by the normal strain energy density parameter, it is necessary to define (in experiments) the weight function including the shear strain energy density parameter in this plane. Thus, application of the energy criterion defined in the critical plane determined by the shear strain energy density parameter is recommended.

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Резюме

Розраховано теоретичні коефіцієнти концентрації напружень із використанням поняття “фіктивний радіус” для трубних і трубно-фланцевих зварних з’єднань. Запропоновано для оцінки втомної довговічності матеріалів в умовах циклічного чистого згину, чистого крутіння та комбінації пропорціонального згину з крутінням використовувати параметри густини енергії нормальних деформацій і деформації зсуву в критичних площинах. Критичні площини визначали методами, що базуються на максимальних параметрах густини енергії нормальних деформацій та деформацій зсуву.

1. C. M. Sonsino, *Fatigue Behavior of Welded Components under Complex Elasto-Plastic Multiaxial Deformations*, Eur-Report No. 16024, Luxemburg (1997).
2. H. Neuber, “Über die Berucksichtigung der Spannungskonzentration bei Festigkeitsberechnungen,” *Konstruktion*, Heft 7, 245–251 (1968).
3. T. Lagoda, “Energy models for fatigue life estimation under random loading. Pt. I. The model elaboration,” *Int. J. Fatigue*, **23**, 467–480 (2001).
4. T. Lagoda and P. Ogonovski, Fatigue life of AlCu4Mg1 aluminum alloy under constant-amplitude in- and out-of-phase bending with torsion,” in: ICF11, Turyn, CD (2005), p. 6.
5. D. Radaj and C. M. Sonsino, *Fatigue Assessment of Welded Joints by Local Approaches*, Abington Publishing, Cambridge (1998).
6. C. M. Sonsino and T. Lagoda, “Assessment of multiaxial fatigue behavior of welded joints under bending and torsion by application of a fictitious radius,” *Int. J. Fatigue*, **26**, 265–279 (2004).

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