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Localization of Resonant Spherical Waves

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Локализация резонансных сферических волн

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Рассматриваются радиальные сферические резонансные волны, возбуждаемые в транс-резонансном режиме. Приближенное общее решение возмущенного волнового уравнения представляется в виде, учитывающем нелинейные, пространственные и диссипативные эффекты. Граничная задача сводится к возмущенному смешанному уравнению Бюргера–Кортевега–де Вриза, для которого построено несколько решений. Установлено, что в невязкой среде вблизи резонанса могут возникать ударные волны. Однако как вязкость, так и пространственная дисперсия вблизи резонанса предотвращают формирование ударного разрыва, в результате чего в резонаторе вместо ударных генерируются периодические локализованные волны.

One-side travelling nonlinear waves have been the subject of intense studies for the last decades [1–4]. In finite physical systems both left and right travelling waves may be excited. Near the resonant frequencies, the amplitudes of these waves increase. As a result, the balance between nonlinear, dissipative, and dispersive effects varies together with the excited frequency. Therefore, in the transresonant frequency band both shock and soliton-like waves may be excited in resonators. This dynamics was studied in [5–7] for the case of plane resonant waves in elongated resonators. Here we consider the spatial effect on the evolution of nonlinear waves in transresonant frequency bands. For simplicity, spherically symmetric pressure waves excited in a gas or liquid sphere are discussed. An oscillating monopole is located at the origin. Apparently, these types of *driven resonant-dissipative three-dimensional* systems were not considered earlier.

In accordance with [8], we write an equation of nonlinear acoustics for spherical waves taking into account only linear and quadratic terms, respectively, for the velocity potential φ:

$$a_0^2(\varphi_{rr} + 2r^{-1}\varphi_r) = \varphi_{tt} + (\gamma - 1)a_0^{-2}\varphi_t\varphi_{tt} + 2\varphi_r\varphi_{rt} - \delta a_0^{-2}\varphi_{ttt}, \qquad (1)$$

where a_0 is the speed of sound in an undisturbed medium, γ is the polytropic exponent of gas (Eq. (1) is also valid for fluid [8]), δ is the so-called "sound diffusivity." The subscripts t and r indicate the time and space derivatives, respectively. We emphasize that Eq. (1) does not take into account the third order effects and the dissipative term is of the second order [8, 9]. The solution of (1) can be presented as

$$\varphi = \varphi_1 + \varphi_2, \tag{2}$$

where φ_1 and φ_2 are the first- and the second-order values, respectively. Substituting Eq. (2) into (1) and equating the values of the same order, we obtain a system of differential equations for φ_1 and φ_2:

$$\varphi_{1rr} + 2r^{-1}\varphi_{1r} = a_0^{-2}\varphi_{1tt}, \tag{3}$$

$$a_0^2(\varphi_{2rr} + 2r^{-1}\varphi_{2r}) = \varphi_{2tt} + 2\varphi_{1r}\varphi_{1rt} + (\gamma - 1)a_0^{-2}\varphi_{1t}\varphi_{1tt} - \delta a_0^{-2}\varphi_{1ttt}. \tag{4}$$

The solution of (3) is the sum of diverging and converging waves:

$$\varphi_1 = r^{-1}(f_1 + f_2). \tag{5}$$

Here and hereinafter, $f_1 = f_1(\xi)$ and $f_2 = f_2(\eta)$, where $\xi = a_0 t - r$ and $\eta = a_0 t + r$. With allowance for (5), we rewrite Eq. (4) in the form

$$a_0^2(\varphi_{2rr} + 2r^{-1}\varphi_{2r}) - \varphi_{2tt} = 2a_0 r^{-2}(f_2' - f_1')(f_2'' - f_1'') -$$

$$-2a_0 r^{-3}[(f_2 + f_1)(f_2'' - f_1'') - (f_1')^2 + (f_2')^2] + 2a_0 r^{-4}(f_2 + f_1)(f_2' + f_1') +$$

$$+(\gamma - 1)a_0 r^{-2}(f_2' + f_1')(f_2'' + f_1'') - \delta a_0 r^{-1}(f_2''' + f_1'''), \tag{6}$$

where the primes denote a derivative with respect to the argument. The solution to (6) is

$$\varphi_2 = r^{-1}(\psi_1 + \psi_2) + 0.5\, a_0^{-1} r^{-2}[(f_1 + f_2)^2]' -$$

$$-0.25(\gamma + 1)a_0^{-1} r^{-1} \int\int r^{-1}(f_1' + f_2')(f_1'' + f_2'')d\xi d\eta +$$

$$+0.25\,\delta a_0^{-1} r^{-1}(\eta f_1'' + \xi f_2''). \tag{7}$$

Here and above, the functions $f_1, f_2, \psi_1 = \psi_1(\xi)$, and $\psi_2 = \psi_2(\eta)$ are unknown and must be found from the initial and boundary conditions of the corresponding problem [9]. However, it is complicated to solve boundary problems using (2) due to the integral in (7). To simplify solution (7), let us replace the multiplier $1/r$ under the integral by $1/R_i$. As a result, near the boundary surfaces $r = R_i$ $(i = 1, 2)$ and

$$\varphi = r^{-1}(f_1 + f_2 + \psi_1 + \psi_2) + 0.5a_0^{-1}r^{-2}[(f_1 + f_2)^2]' -$$

$$-0.25(\gamma+1)a_0^{-1}r^{-1}R_i^{-1}[0.5\eta(f_1') + 0.5\xi(f_2')^2 + f_1'f_2 + f_2'f_1] +$$

$$+0.25\delta a_0^{-1}r^{-1}(\eta f_1'' + \xi f_2''). \qquad (8)$$

Solution (8) satisfies Eq. (1) if the expression $0.5a_0r^{-2}(\gamma+1)[(f_1' + f_2')^2]\times$ $\times(1 - rR_i^{-1})$ is a value of the third order. Thus Eq. (8) is valid near the surface $r = R_i$, where $|1 - rR_i^{-1}| << 1$.

In this paper, we examine only periodical oscillations. In this case, the velocity and the pressure perturbation

$$P - P_0 = -\rho_0(\varphi_t + 0.5\varphi_r^2 - 0.5a_0^{-2}\varphi_r^2) + (\lambda + 2\nu)a_0^{-2}\varphi_{tt} \qquad (9)$$

must not contain secular terms (formula (9) defines the pressure if λ and ν are the shear and dilatational viscosities [8]). The secular terms will be eliminated if we assume in (8) that

$$\psi_1 = \Psi_1 + 0.125a_0^{-1}R_i^{-1}(\gamma+1)[\xi(f_1')^2 - 2f_1f_1'] - 0.25\delta a_0^{-1}\xi f_1'' - c\xi^2,$$

$$\psi_2 = \Psi_2 + 0.125a_0^{-1}R_i^{-1}(\gamma+1)[\eta(f_2')^2 - 2f_2f_2'] - 0.25\delta a_0^{-1}\eta f_2'' + c\eta^2,$$

where c is an arbitrary constant, f_1, f_2, $\Psi_1 = \Psi_1(\xi)$, and $\Psi_2 = \Psi_2(\eta)$ are periodic functions. As a result, near the surface $r = R_i$, the velocity potential for steady-state oscillations is given by the expression

$$\varphi = r^{-1}(f_1 + f_2 + \Psi_1 + \Psi_2 + 4ca_0rt) +$$

$$+0.5a_0^{-1}r^{-2}[1 - 0.25rR_i^{-1}(\gamma+1)][f_1 + f_2)^2]' -$$

$$-0.25(\gamma+1)a_0^{-1}R_i^{-1}[(f_1')^2 - (f_2')^2] + 0.5\delta a_0^{-1}(f_1'' - f_2''). \qquad (10)$$

This expression will be used below to solve a boundary problem. We consider the waves excited by a simple-harmonic source of pressure, which has radius R_1 and is placed in the center of a sphere. It is assumed that the pressure source region is very small relative to the excited wavelength. The other boundary of the sphere is free. Therefore, we have

$$P - P_0 = -B\cos\omega t \quad (r = R_1), \qquad (11)$$

$$P - P_0 = 0 \quad (r = R_2). \qquad (12)$$

First, resonant frequencies are determined. Then, resonant oscillations are analyzed on the basis of nonlinear relations. Using (9) and (10), we can rewrite condition (11) as

$$a_0R_1(f_1'+f_2'+\Psi_1'+\Psi_2'+4cR_1)+[1-0.25(\gamma+1)][(f_1'+f_2')^2+$$

$$+(f_1+f_2)(f_1''+f_2'')]-0.5R_1(\gamma+1)(f_1'f_1''-f_2'f_2'')+0.5\delta R_1^2(f_1'''-f_2''')+$$

$$+0.5[R_1^{-1}(f_1+f_2)+f_1'-f_2']^2-0.5(f_1'+f_2')^2-$$

$$-0.5(f_1'+f_2')^2-\rho_0^{-1}R_1(\lambda+2\nu)(f_1''+f_2'')=\rho_0^{-1}BR_1^2\cos\omega t, \qquad (13)$$

where $f_1=f_1(a_0t-R_1)$, $f_2=f_2(a_0t+R_1)$, $\Psi_1=\Psi_1(a_0t-R_1)$, and $\Psi_2=$ $=\Psi_2(a_0t+R_1)$.

Condition (12) may be presented as (13) if in (13) we substitute R_2 for R_1 and assume $B=0$. Then from (12) we can find

$$f_1(a_0t-r)=f(a_0t-r+R_2),\quad f_2(a_0t+r)=-f(a_0t+r-R_2),$$
$$\Psi_1'=-a_0^{-1}R_2^{-1}(f_1')^2=0.5\delta a_0^{-1}R_2f_1'''-2cR_2,\quad \Psi_2'=\Psi_1'. \qquad (14)$$

Let us consider Eq. (13) taking into account (14). As the first approximation, it follows from (13) that

$$f'(a_0t-R_1+R_2)-f'(a_0t+R_1-R_2)=BR_1a_0^{-1}\rho_0^{-1}\cos\omega t$$

and

$$f'(a_0t-r+R_2)=$$

$$=0.5BR_1a_0^{-1}\rho_0^{-1}\sin\omega a_0^{-1}(a_0t-r+R_2)/\sin\omega a_0^{-1}(R_2-R_1). \qquad (15)$$

From (15) we obtain resonant frequencies: $\Omega_N=\pi Na_0(R_2-R_1)^{-1}$ $(N=1,2,3,\ldots)$. The linear solution (15) is not valid near the frequencies $\omega=$ $=\Omega_N+\omega_1$, where ω_1 is a small value. We assume that $\omega_1=a_0(R_2-R_1)^{-1}\times$ $\times\sin\omega a_0^{-1}(R_2-R_1)$.

Let us consider resonant frequencies. First, the function $f'(a_0t+R_1-R_2)$ is expanded in Taylor's series at $r=R_1$:

$$f'(a_0t+R_1-R_2)=f'-2\omega^{-1}\omega_1(R_2-R_1)f''+2\omega^{-2}\omega_1^2(R_2-R_1)^2f'''-\ldots. \qquad (16)$$

It was suggested that

$$f'(a_0t-R_1+R_2-2N\pi a_0/\omega)=f'(a_0t-R_1+R_2)=f'.$$

Then using expansions (16) and (13), we obtain the following basic equation:

$$a_0 R_1 R_2 \omega^{-1} \omega_1 f'' - R_1 R_2 [a_0 \omega^{-2} \omega_1^2 (R_2 - R_1) + 0.5\delta] f''' + (f')^2 =$$

$$= \rho_0^{-1} B R_2 R_1^2 (R_2 - R_1)^{-1} (\cos^2 1/2\omega t - 0.5) + 2a_0 R_1 R_2 c. \qquad (17)$$

Equation (17) is the perturbed compound Burgers–Korteweg–de Vries equation written for a travelling wave. This equation has a nonlinear term that tends to produce "discontinuity" in this wave. The term f' dissipates through the viscous-like effect. This term disappears at resonance. The second term, which is generated due to the viscosity of the medium, disperses the wave. Due to this term, solitary waves may be excited. We write the solution of (17) for the case $c = 0.25 a_0^{-1} \rho_0^{-1} B R_1 (R_2 - R_1)^{-1}$ as $f' = \sqrt{\varepsilon}\, \Phi(\tau) \cos\tau$. Here $\Phi(\tau)$ is an unknown function and $\varepsilon = B \rho_0^{-1} R_2 R_1^2 (R_2 - R_1)^{-1}$, and $\tau = \omega t / 2$. As a result, Eq. (17) becomes

$$0.5\omega_1 R_1 R_2 (\Phi' - \Phi \tan\tau) - 0.25 a_0^{-1} \omega_1^2 R_1 R_2 (R_2 - R_1 + 0.5\delta\omega^2 a_0^1 \omega_1^{-2}) \times$$

$$\times (\Phi'' - 2\Phi' \tan\tau - \Phi) = \sqrt{\varepsilon}(1 - \Phi^2) \cos\tau. \qquad (18)$$

Here $\Phi' = d\Phi/d\tau$.

Transresonant process. Far from the resonance, when the first term in (17) is dominant, the acoustic solution (15) follows from (17). Near resonance, this term reduces together with ω_1. At the same time, the influence of the nonlinear and second terms in (17) increases. To simplify the problem, let us assume that the nonlinear term begins to distort the acoustic solution, while the dispersive effect is still small. In this case, we seek an approximate solution of (18) as the sum $\Phi = \Phi_0 + \Phi_1$, where $\Phi_0 >> \Phi_1$. The quantity Φ_0 takes into account the nonlinear and first terms in (18), while Φ_1 corrects Φ_0. We seek a solution, which is valid near the points where $|\sin\tau| << 1$. By equating the terms of the same order in (18), we obtain two differential equations:

$$\Phi_0' = 2\sqrt{q}(1 - \Phi_0^2) \cos\tau, \qquad (19)$$

$$\Phi_1' - \Phi_0 \tan\tau - \frac{1}{2} \omega_1 a_0^{-1} (R_2 - R_1 + \frac{1}{2} \delta\omega^2 a_0^{-1} \omega_1^{-2})(\Phi_0'' - \Phi_0) =$$

$$= -4\sqrt{q}\, \Phi_0 \Phi_1 \cos\tau, \qquad (20)$$

where $\sqrt{q} = \sqrt{\varepsilon}(\omega_1 R_1 R_2)^{-1}$. Equation (19) is locally satisfied if $\Phi_0 = \tanh(2\sqrt{q} \sin\tau)$ [8]. The approximate solution of (20) is

$$\Phi_1 = q_1 \sec h^2(2\sqrt{q}\sin\tau)\cos\tau,$$

where

$$q_1 = 8\omega_1 a_0^{-1} q^{1.5}(R_2 - R_1 + \frac{1}{2}\delta\omega^2 a_0^{-1}\omega_1^{-2}).$$

For the travelling waves

$$f'[a_0 t \pm (R_2 - r)] =$$

$$= \sqrt{\varepsilon}[\tanh(2\sqrt{q}\sin p)\cos p + q_1 \sec h^2(2\sqrt{q}\sin p)\cos^2 p], \qquad (21)$$

where $p = \omega t/2 \pm [\omega a_0^{-1}(R_2 - r) - \pi N]/2$. This solution indicates that the finite-amplitude travelling waves become steeper when the excitation frequency approaches the resonant frequency. According to (21), shock waves may be excited near resonance in an inviscid medium. For the latter case, if $\omega_1 = 0$, we have the solution with discontinuities [7]. However, both the viscosity and spatial dispersion begin to be important near resonance and can prevent the formation of a shock wave [5–13]. It follows from (21) that a soliton-like wave can generate near resonance. The amplitude of the soliton-like wave increases when $\omega_1 \to 0$ because Eq. (20) and solution (21) are not valid very close to resonance.

Near resonance, the influence of the first term in (18) decreases. Accordingly, the influence of the second term increases. At resonance, Eq. (18) transforms to the Korteweg–de Vries type equation

$$\Phi'' - 2\Phi'\tan\tau - \Phi = q_0^{-1}(1 - \Phi^2)\cos\tau, \qquad (22)$$

where $q_0 = -\frac{1}{8}\delta\omega^2\varepsilon^{-0.5}a_0^{-2}R_1R_2$. Let $\Phi = [A\sec h^2(\gamma\sin M^{-1}\tau) + C]\cos\tau$, where A, γ, and C are constant values. We have written the solution localized near the points where $|\sin M^{-1}\tau| << 1$ ($M = 1, 2, 3 \ldots$). This solution satisfies approximately Eq. (22) if $A = 6q_0\gamma^2 M^{-2}$, $\gamma^2 = 0.5M^2(1 - q_0^{-1}C)$, and $C_\pm = 4(q_0 \pm \sqrt{q_0^2 + 3/4})/3$. If $|q_0| << 1$, then $C_\pm \approx 1$, $\gamma^2 \approx -0.5q_0^{-1}M^2$, and $A \approx -3$. For the latter case,

$$f' = \sqrt{\varepsilon}\{1 - 3\sec h^2[M(\sin M^{-1}\tau - R)/\sqrt{-2q_0}]\}\cos^2\tau.$$

For the travelling waves,

$$f'[a_0 t \pm (R_2 - r)] = \sqrt{\varepsilon}\{1 - 3\sec h^2[M(\sin M^{-1}p)/\sqrt{-2q_0}]\}\cos^2 p. \quad (23)$$

If $M = 1$, expression (23) defines oscillations at $r - R_1$ at the frequency ωt. Solution (23) also describes subharmonic oscillations if $M = 2, 3 \ldots$. Since this solution must satisfy expansion (16), subharmonic waves corresponding to $M > 1$ may be excited only near the frequencies $\omega = M\Omega_N$. For example, the case

$M=2$ may be realized only for even resonance. Thus, solution (23) defines the spectrum ($M=2, 3 \ldots$) of subharmonic localized waves.

The case $M=1$ corresponds to solution (22). For this case, near resonance we assume $\Phi=\Phi_0+\Phi_1$, where $\Phi_0 >> \Phi_1$. The quantity Φ_0 takes into account the nonlinear and the second terms in (18), while Φ_1 corrects Φ_0. Then for travelling waves one can find

$$f'(a_0 t \pm r)=\sqrt{\varepsilon}\{[3\sec h^2(\sin p/\sqrt{2q_0})-1]\cos p+$$

$$+Q\tanh(\sin p/\sqrt{2q_0})\}\cos p. \qquad (24)$$

Here

$$Q=3\omega_1(2q_0)^{-0.5}\{4R_1R_2\varepsilon^{0.5}+0.25\delta\omega^2 a_0^{-2}q_0^{-1}[0.5\delta+a_0\omega^{-2}\omega_1^2(R_2-R_1)]\}.$$

Solution (24) is localized near the lines where $\sin p << 1$. Thus, according to (23) and (24), periodic spherical solitons may be excited in viscous media at the exact resonance. These waves contrast with the periodic spherical shock waves, which are predicted by (21) for inviscid media.

Linear (15) and nonlinear (21) and (24) solutions describe some scenarios of transresonant evolution of the waves in weakly dissipative media. Far from resonance, we have harmonic waves. These waves are distorted due to the nonlinear effect when the value of ω_1 decreases. If $\delta \approx 0$, these waves transform into the shock-like waves. However, $\delta \neq 0$ and discontinuities do not form in the system. Very close to resonance $\omega_1 \approx 0$ and spatial dispersion (the second term in (17)) begins to distort the waves. As a result, the waves may be generated which have some features of both shock and soliton-like waves. However, at the exact resonance, the first term in (17) equals zero and soliton-like waves are generated. These waves may be much localized if $\delta \approx 0$.

Now we can find pressure and velocity in the medium. However, we emphasize again that expression (10) does not take into account correctly the second-order values far from the boundaries. Therefore, we must only consider the first-order terms in expressions for velocity and pressure. For example, instead of (9) we have

$$P-P_0=r^{-1}\rho_0 a_0[f'(\eta-R_1)-f'(\xi+R_1)].$$

Thus, according to the above analysis, strongly localized waves travel inside the sphere (spherical layer). Pictures of the variation of dimensionless pressure $(P-P_0)/\rho_0 a_0\varepsilon^{0.5}$ are presented in Figs. 1, 2, and 3. There the dimensionless time τ and radius (r/R_2) are used. We calculated pressure using (21) (Fig. 1) and (23) (Figs. 2 and 3), and assuming $R_1=0.01R_2$. There is strong amplification of the waves near $r=R_1$.

In contrast to plane resonant shock waves [7, 14], resonant spherical nonlinear waves practically have not been studied [9, 13]. At the same time, the spherical model for the simulation of different physical objects is very popular. Indeed, on the one hand, the model of a pulsating sphere is widely used in astrophysics [15, 16]. On the other hand, this model is used for studying sonoluminescence in liquids when the period of oscillation and the space distances are very small [17]. The competition of nonlinear, dissipative, and dispersive effects may be important for these systems. We considered this competition in the transresonant regime. The distortion of harmonic waves into shock-like and then soliton-like waves was shown. Our considerations have been strictly limited to the aspect of nonlinear acoustics; however, the results presented may be interesting for various media and circumstances.

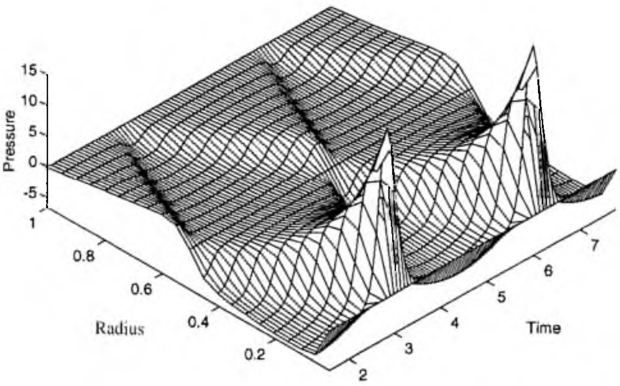

Fig. 1. Forced pressure waves inside the sphere ($q = 10$, $q_1 = 0.1$, and $N = 2$).

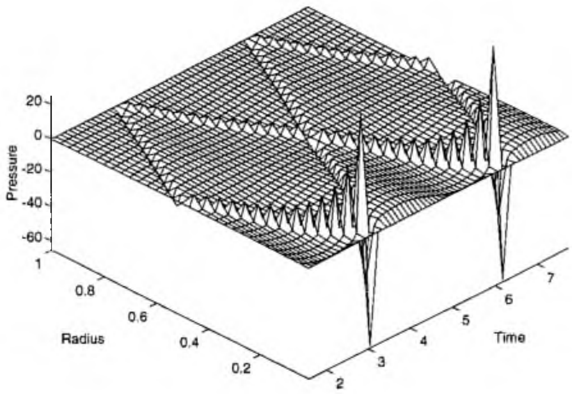

Fig. 2. Forced pressure waves inside the sphere ($q_0 = 0.001$, $N = 2$, and $M = 1$).

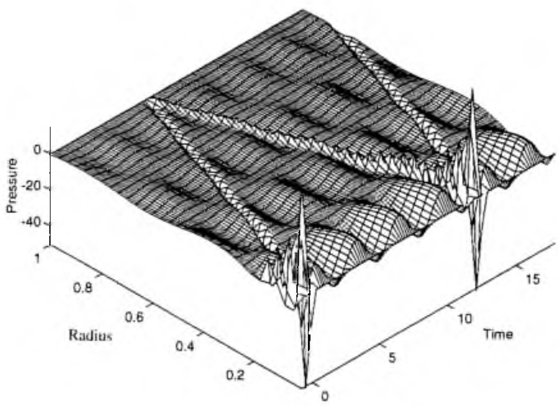

Fig. 3. Forced pressure waves inside the sphere ($q_0 = 0.01$, $N = 4$, and $M = 4$).

Резюме

Розглядаються радіальні сферичні резонансні хвилі, що збуджуються в трансрезонансному режимі. Наближений загальний розв'язок збуреного хвильового рівняння записується з урахуванням нелінійних, просторових і дисипативних ефектів. Гранична задача зводиться до збуреного змішаного рівняння Бюргера–Кортевега–де Вріза, для якого побудовано декілька розв'язків. Установлено, що в нев'язкому середовищі поблизу резонансу можуть виникати ударні хвилі. Однак як в'язкість, так і просторова дисперсія поблизу резонансу запобігають формуванню ударного розриву, в результаті чого в резонаторі замість ударних генеруються періодичні локалізовані хвилі.

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