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NECESSARY CONDITIONS FOR CONTROL OF OBJECTS WITH DISTRIBUTED CONSTANTS

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Problems of optimal control over objects with distributed constants described by nonlinear differential equations with partial derivatives of elliptic, parabolic and hyperbolic types have been considered.

INTRODUCTION

The significant progress in the development of non-linear functional analysis methods [1, 2, 3], which have become widely adopted in different sections of mathematics, favours the research of applied non-linear tasks, which are in natural environment and used for many industrial technologies. Bringing them to corresponding operators or differential-operator equations in functional spaces allows to reveal general regularities and connections for entire tasks classes, which are different according to their specific content [4].

With the help of some methods given by the non-linear analyses, we can research on the question of extreme of functionals with restrictions, which appear during solution of great number of important manufacturing and technical tasks. The restrictions in the kind of functional equations and inequalities allow to form up mathematical models of objects functioning, considering the physical essence of the task.

The present work considers the task of optimal controlling for objects with distributed parameters, which are described by non-linear differential equations with partial derivatives of elliptical parabolic and hyperbolic types.

TASK SETTING

Let X, Y, U be Banach spaces, the functional J be determined in $X \times U$ and the operator G reflect the space $X \times U$ on Y, that is $J : X \times U \to R$, $G : X \times U \to Y$.

Let us consider an extremal task:

$$J(x,u) \to \inf , \tag{1}$$

$$G(x,u) = y,\ x \in X,\ u \in U,\ y \in Y , \tag{2}$$

where the functional J and the operator G are non-linear.

Let us mark through X^*, Y^*, U^* the conjugated spaces to X, Y, U respectively;

$L(X;Y)$, $L(U,Y)$ are spaces of non-linear continuous operators, which act on X and U on Y respectively;

D_xJ, D_uJ, D_xG, D_uG are partial derivatives according to Gato [1] in the point $(x;u)\in X\times U$ of the reflection J and G, that is

$$D_xJ=\frac{\partial J(x,u)}{\partial x},\ D_uJ=\frac{\partial J(x,u)}{\partial u},\ D_xJ=\frac{\partial G(x,u)}{\partial x},\ D_uJ=\frac{\partial G(x,u)}{\partial u}.$$

Theorem. Let us consider

1) the functional J and the operator G have partial derivatives according to Gato in some interval $W_{(x_0,u_0)}\subset X\times U$ of the element $(x_0;u_0)\in X\times U$ and the reflection

$$D_{x_0}J:W_{(x_0;u_0)}\to X^*,\ D_{u_0}J:W_{(x_0;u_0)}\to U^*,\ D_{x_0}G:W_{(x_0;u_0)}\to L(X;Y)$$

and

$$D_{u_0}G:W_{(x_0;u_0)}\to L(U;Y) \text{ are continuous;}$$

2) the space patterns X, U with the reflections $D_{x_0}G$ and $D_{u_0}G$ are closed in Y.

At the same time, if the element (pair) (x_0,u_0) is the solution of the tasks (1), (2), then such correlations occur:

$$G(x_0;u_0)=y\,, \tag{3}$$

$$\lambda_1\langle D_{x_0}J(x_0;u_0),x\rangle_X+\langle[D_{x_0}G(x_0;u_0)]^*y_1^*,x\rangle_X=0\quad\forall(x,y)\in X\times U\,, \tag{4}$$

$$\lambda_2\langle D_{x_0}J(x_0;u_0),u\rangle_U+\langle[D_{u_0}G(x_0;u_0)]^*y_2^*,u\rangle_U=0\,, \tag{5}$$

where $\lambda_1,\lambda_2\in R$, $y_1^*,y_2^*\in Y^*$ and $\left|\lambda_1\right|+\left|\lambda_2\right|+\left\|y_1^*\right\|_{Y^*}+\left\|y_2^*\right\|_{Y^*}\neq 0$.

Proof. According to the conditions of the multitude theorem

$$P_1=\left\{D_{x_0}G(x_0;u_0)x;\ \forall x\in X\right\}\subseteq Y,\ P_2=\left\{D_{x_0}G(x_0;u_0)u;\ \forall u\in U\right\}\subseteq Y$$

form closed spaces in Y, that is subspaces P_1 and P_2 hold all their border points. If $P_1\neq Y$ and $P_2\neq Y$, that is P_1 and P_2 are proper subspaces of the Banach space Y, then according to lemma about annihilator [5], non-zero functionals $y_1^*,y_2^*\in Y^*$ can be found; they are equals to zero at P_1 and P_2 correspondingly. For linear continuous functionals y_1^* and y_2^* with $\forall x\in X$ and $\forall u\in U$ we obtain

$$\left\langle[D_{x_0}G(x_0;u_0)]^*y_1^*,x\right\rangle_X=\left\langle y_1^*,D_{x_0}G(x_0;u_0)x\right\rangle_Y=0$$

and

$$\left\langle [D_{u_0} G(x_0;u_0)]^* y_2^*, u\rangle_U = \langle y_2^*, D_{u_0} G(x_0;u_0) u \right\rangle_Y = 0,$$

as elements $D_{x_0} G(x_0;u_0)x$ and $D_{u_0} G(x_0;u_0)u$ belongs correspondingly to subspaces $P_1 \subset Y$, $P_2 \subset Y$.

Let us assume that $\lambda_1 = 0$ and $\lambda_2 = 0$ if we take into account the last correlations, we obtain equations (4), (5).

Let us now consider the case $P_1 = Y$ and $P_2 = Y$. If we apply to the reflection G the theorem of Lustenik [5, 6], we will have that $\forall h \in X$ and $\forall u \in U$, which satisfy the conditions

$$\langle D_{x_0} G(x_0;u_0), h\rangle_X = 0 \text{ and } \langle D_{u_0} G(x_0;u_0), \nu\rangle_U = 0, \tag{6}$$

at rather small numbers t and τ there exist such elements

$$x(t,h) = x_0 + th + r_1(t) \text{ and } u(\tau,\nu) = u_0 + \tau\nu + r_1(\tau),$$

that $G(x(t,h), u(\tau,\nu)) - y = 0,\ \dfrac{\|r_1(t)\|_X}{t} \to 0,\ \dfrac{\|r_2(\tau)\|_U}{\tau} \to 0$ with $t \to 0$ and $\tau \to 0$.

Let us consider the function $\varphi(t,\tau) = L(x(t,h), u(\tau,\nu))$. Its partial derivatives according to Gato become

$$\left.\frac{\partial\varphi}{\partial t}\right|_{\substack{t=0\\ \tau=0}} = \langle D_{x_0} J(x_0;u_0), h\rangle_X = C_1,\quad \left.\frac{\partial\varphi}{\partial \tau}\right|_{\substack{t=0\\ \tau=0}} = \langle D_{u_0} J(x_0;u_0), \nu\rangle_U = C_2$$

and should be equal to zero. Indeed, if

$$\langle D_{x_0} J(x_0;u_0), h\rangle_X = C_1 \neq 0 \text{ and } \langle D_{u_0} J(x_0;u_0), \nu\rangle_U = C_2 \neq 0,$$

then the signs of expressions

$$J(x(t,h), u_0) - J(x_0;u_0) = \langle D_{x_0} J(x_0;u_0), x(t,h) - x_0\rangle_X + o(t) =$$

$$= \langle D_{x_0} J(x_0;u_0), th + r_1(t)\rangle_X + o(t) = t\langle D_{x_0} J(x_0;u_0), h\rangle_X +$$

$$+ \langle D_{x_0} J(x_0;u_0), r_1(t)\rangle_X + o(t) = C_1 t + \langle D_{x_0} J(x_0;u_0), r_1(t)\rangle_X + o(t)$$

and

$$J(x_0, u(\tau,\nu)) - J(x_0;u_0) = \langle D_{u_0} J(x_0;u_0), u(\tau,\nu) - u_0\rangle_U + o(t) =$$

$$= C_2\tau + \langle D_{u_0} J(x_0;u_0), r_2(\tau)\rangle_U + o(t),$$

taking into account that $\dfrac{\|r_1(t)\|_X}{t} \to 0,\ \dfrac{\|r_2(\tau)\|_U}{\tau} \to 0$ with $t \to 0$ and $\tau \to 0$, are determined in terms of $C_1 t$ and $C_2\tau$ and, as a result, they change when substitute t and τ for $-t$ and $-\tau$ accordingly.

At the same time there should not be an extreme at the point (x_0, u_0). Exactly this contradiction proves our statement. Consequently, taking into the account (6), we have

$$\langle D_{x_0} J(x_0;u_0), h\rangle_X = 0 \quad \forall h \in \operatorname{Ker} D_{x_0} G(x_0;u_0) \tag{7}$$

and

$$\langle D_{u_0} J(x_0;u_0), v\rangle_U = 0 \quad \forall v \in \operatorname{Ker} D_{u_0} G(x_0;u_0)\,. \tag{8}$$

In other words, $D_{x_0} J(x_0;u_0)$ is the element within X^*, which is orthogonal to subspace $\operatorname{Ker} D_{x_0} G(x_0;u_0) \subset X$, that is

$$D_{x_0} J(x_0;u_0) \in \left[\operatorname{Ker} D_{x_0} G(x_0;u_0)\right]^{\perp} \cap X^*\,.$$

Similarly, $D_{u_0} J(x_0;u_0)$ is the element within V^*, which is orthogonal to $\operatorname{Ker} D_{x_0} G(x_0;u_0) \subset U$, that is $D_{u_0} J(x_0;u_0) \in \left[\operatorname{Ker} D_{u_0} G(x_0;u_0)\right]^{\perp} \cap U^*$. According to the lemma about annihilator [5] we obtain

$$\left[\operatorname{Ker} D_{x_0} G(x_0;u_0)\right]^{\perp} = \operatorname{Im}\left[D_{x_0} G(x_0;u_0)\right]^* \tag{9}$$

and

$$\left[\operatorname{Ker} D_{u_0} G(x_0;u_0)\right]^{\perp} = \operatorname{Im}\left[D_{u_0} G(x_0;u_0)\right]^*. \tag{10}$$

Consequently, if

$$D_{x_0} J(x_0;u_0) \in \left[\operatorname{Ker} D_{x_0} G(x_0;u_0)\right]^{\perp} \cap X^*,$$

$$D_{u_0} J(x_0;u_0) \in \left[\operatorname{Ker} D_{u_0} G(x_0;u_0)\right]^{\perp} \cap U^*,$$

then it can be found such functionals $y_1^*, y_2^* \in Y^*$, such that

$$\left\langle D_{x_0} J(x_0;u_0), x\right\rangle_X = -\left\langle \left[D_{x_0} G(x_0;u_0)\right]^* y_1^*, x\right\rangle_X \tag{11}$$

and

$$\left\langle D_{u_0} J(x_0;u_0), u\right\rangle_U = -\left\langle \left[D_{u_0} G(x_0;u_0)\right]^* y_2^*, x\right\rangle_U\,. \tag{12}$$

Assuming $\lambda_1 = \lambda_2 = 1$ and taking into consideration that $(x,u) \in X \times U$, we obtain the expressions (4) and (5), which prove the theorem.

This theorem is an infinitely measurable generalization of Lagrangian coefficients rule, which is known from classical analysis and necessary conditions for extremal tasks with restrictions.

Let us mention, that the system of equations (3)–(5), which presents necessary conditions for functional optimum (1) with restrictions (2), can be written (for $\lambda_1 = \lambda_2 = 1$) in the operator form:

$$G(x_0;u_0) = y\,, \tag{13}$$

$$D_{x_0} J(x_0;u_0) + \left[D_{x_0} G(x_0;u_0)\right]^* y_1^* = 0\,, \tag{14}$$

$$D_{u_0} J(x_0;u_0) + \left[D_{u_0} G(x_0;u_0)\right]^* y_2^* = 0\,, \tag{15}$$

where

$$\left[D_{x_0} G(x_0;u_0)\right]^* : Y^* \to X^*,\ \left[D_{u_0} G(x_0;u_0)\right]^* : Y^* \to U^*.$$

Hence, the solution (x_0, u_0, y_1^*, y_2^*) of system (3)–(5) or (13)–(15) can be interpreted as a generalized solution.

TASK OF OPTIMAL CONTROL FOR THE OBJECTS WITH DISTRIBUTED CONSTANTS, WHICH ARE DESCRIBED BY NON-LINEAR DIFFERENTIAL EQUATIONS — ELLIPTIC TYPE

Let us assume that functions, which determine the state $x(\omega)$ of an object and the control parameter $u(\omega)$ are defined in the restricted area $\Omega \subset R^N$ with the limit $\partial\Omega$.

We get necessary optimal conditions as solution of functional equation system.

Let us consider such optimization task:

$$I(x,u) = \int_\Omega J(x(\omega), u(\omega))d\omega \to \inf , \qquad (16)$$

$$\sum_{i=1}^{N} \frac{\partial^2 x}{\partial \omega_i^2} + G(\omega, x, \frac{\partial x}{\partial \omega_1}, ..., \frac{\partial x}{\partial \omega_n}, u) = f , \qquad (17)$$

$$x\big|_{\partial\Omega} = 0 . \qquad (18)$$

We make the following assumptions:

1. Let us assume that

$$x, \frac{\partial x}{\partial \omega_i}, \frac{\partial^2 x}{\partial \omega_i^2} \in L_p(\Omega),\ \ i = 1,...,N;\ \ \ p \ge 2\ ;\ \ u \in L_r(\Omega),\ \ r > 1 .$$

2. Let function $G : \Omega \times \mathbb{R}^{n+2} \to \mathbb{R}$ belong to the class CAR $(G \in \text{CAR})$, that is if $\forall \xi \in \mathbb{R}^{n+2}$ the function $\Omega \ni \omega \mapsto G(\omega; \xi) m$ is measurable, and for almost all $\omega \in \Omega$ the function $\mathbb{R}^{n+2} \ni \xi \mapsto G(\omega, \xi)$ is continuous.

Let us also assume, that

$$\left| G(\omega; \xi) \right| \le a(\omega) + c\left(\sum_{i=1}^{N+1} \left|\xi_i\right|^{p-1} + \left|\eta\right|^{r/q} \right),$$

where $a(\omega) \in L_q(\Omega),\ \xi = (\xi_1, ..., \xi_{N+2}) \in R^{N+1}, \eta \in \mathbb{R},\ c > 0$.

3. Functional f allows the representation in the form $f = \sum_{|\alpha| \le 2} D^\alpha v_\alpha$, $v_\alpha \in L_q(\Omega)$.

Taking into the boundary conditions (18), we obtain

$$x \in \overset{\circ}{W}{}_p^1(\Omega) \cap W_p^2(\Omega) = X .$$

Marking

$$Lx = \sum_{i=1}^{N} \frac{\partial^2 x}{\partial \omega_i^2} \text{ and } G(x,u) = G(\omega; x; \frac{\partial x}{\partial \omega_1}, ..., \frac{\partial x}{\partial \omega_N}, u),$$

we obtain [1, 4, 7, 8] linear operator $L : X \to L_q(\Omega)$, non-linear operator $G(X) \times \times L_r(\Omega) \to L_q(\Omega)$ and non-linear functional $I : L_p(\Omega) \times L_r(\Omega) \to R$.

According to the Lagrange principle, if the pair (x_0, u_0) is the solution of the task (16), (17), then it gives the inferior extreme to the functional

$$\Phi(x,u) = I(x,u) + \langle y^*, G(x,u) - y \rangle_Y, \text{ where } y^* \in Y^*, \ Y = L_q(\Omega), \quad (19)$$

which is called Langrangian of the task (16), (17).

We obtain necessary conditions of extreme, when calculating functional variations (19) and partial variations (partial derivatives according to Gato) and then separately putting them to zero.

Indeed, for $\forall h \in X$ (it could be assumed $h = x + \delta x - x$)

$$\Phi_\alpha = \Phi(x + \alpha h, u) = I(x + \alpha h, u) + \langle y^*, G(x + \alpha h, u) - y \rangle_Y .$$

Let us find the derivative from $\Phi(x + \alpha h, u)$ by the parameter α:

$$\frac{\partial \Phi_\alpha}{\partial \alpha} = \left\langle \frac{\partial I_\alpha}{\partial x}, h \right\rangle_X + \left\langle y^*, \frac{\partial G_\alpha}{\partial x} h \right\rangle_Y = \left\langle \frac{\partial I_\alpha}{\partial x}, h \right\rangle_X + \left\langle \left[\frac{\partial G_\alpha}{\partial x} \right]^* y^*, h \right\rangle_X =$$

$$= \left\langle \frac{\partial I_\alpha}{\partial x} + \left[\frac{\partial G_\alpha}{\partial x} \right]^* y^*, h \right\rangle_X .$$

Hence, when passing on to the limit under $\alpha \to 0$, we obtain the functional Φ variation, that is

$$\delta_x \Phi = \left\langle \frac{\partial I}{\partial x} + \left[\frac{\partial G}{\partial x} \right]^* y^*, \ h \right\rangle_X . \quad (20)$$

Similarly we obtain the variation Φ by u:

$$\delta_u \Phi = \left\langle \frac{\partial I}{\partial u} + \left[\frac{\partial G}{\partial u} \right]^* y^*, \ v \right\rangle_U, \ \forall \ v \in U = L_r(\Omega). \quad (21)$$

From the correlations (20) and (21) we get the necessary conditions for the task (16), (17), which are similar to conditions (4), (5) of extremal task (1), (2). At the same time there is an element $y^* \in Y^*$ that satisfies equations (4), (5), that is

$$y^* = y_1^* = y_2^* .$$

Let us write down appropriate Langrangian similarly to the task (16)–(18).

$$F(x,y) = I(x,y) + \left\langle \Psi, \sum_{i=1}^{N} \frac{\partial^2 x}{\partial \omega_i^2} + G(\omega, x, \frac{\partial x}{\partial \omega_1}, ..., \frac{\partial x}{\partial \omega_N}, u) - f \right\rangle_{L_q(\Omega)} =$$

$$= I(x,u) + \int_\Omega \Psi(\omega) \sum_{i=1}^N \frac{\partial^2 x}{\partial \omega_i^2} \partial\omega + \int_\Omega \Psi(\omega) G(\omega; x(\omega,) \frac{\partial x}{\partial \omega_i}, ..., \frac{\partial x}{\partial \omega_N}, u(\omega)) d\omega -$$

$$- \int_\Omega \Psi(\omega) f(\omega) d\omega.$$

Taking into account (16)–(18)

$$F(x,u) = \int_\Omega J(x,u) d\omega + \int_\Omega \sum_{i=1}^N \frac{\partial^2 \Psi}{\partial \omega_i^2} x(\omega) d\omega +$$

$$+ \int_\Omega \Psi(\omega) G(\omega; x(\omega), \frac{\partial x}{\partial \omega_1}, ..., \frac{\partial x}{\partial \omega_N}, u(\omega)) d\omega - \int_\Omega \Psi(\omega) f(\omega) d\omega .$$

Let us find partial variations of functional F .

$$\delta_x F = \int_\Omega \frac{\partial J}{\partial x} \delta x d\omega + \int_\Omega \sum_{i=1}^N \frac{\partial^2 \Psi}{\partial \omega_i^2} \delta x d\omega + \int_\Omega \Psi(\omega) \left[\frac{\partial G}{\partial x} \delta x + \sum_{i=1}^N \frac{\partial G}{\partial x_{\omega_i}} \delta x_{\omega_i} \right] d\omega , \quad (22)$$

where

$$x_{\omega_i} = \frac{\partial x}{\partial \omega_i}, \; i = 1, 2, 3, ..., N ,$$

$$\delta_u F = \int_\Omega \frac{\partial J}{\partial u} \delta u d\omega + \int_\Omega \Psi \frac{\partial G}{\partial u} \delta u = \int_\Omega \left(\frac{\partial J}{\partial u} + \Psi \frac{\partial G}{\partial u} \right) \delta u d\omega . \quad (23)$$

Taking into that

$$\frac{\partial \delta x}{\partial \omega_i} = \frac{\partial x}{\partial \omega_i} - \frac{\partial x_0}{\partial \omega_i} = \delta \frac{\partial x}{\partial \omega_i}$$

and

$$\frac{\partial G}{\partial x_{\omega_i}} \delta \left(\frac{\partial x}{\partial \omega_i} \cdot \frac{\partial}{\partial \omega_i} (\delta x) \right) = \frac{\partial}{\partial \omega_i} \left(\delta x \frac{G}{\partial x_{\omega_i}} \right) - \delta x \frac{\partial^2 G}{\partial x_{\omega_i} \partial \omega_i} ,$$

and using the integration by pats rule, let us write variation (22) in form:

$$\delta_x F = \int_\Omega \left(\frac{\partial J}{\partial x} + \sum_{i=1}^N \frac{\partial^2 \Psi}{\partial \omega_i^2} + \Psi \frac{\partial G}{\partial x} \right) \delta x d\omega +$$

$$+ \int_\Omega \Psi \sum_{i=1}^N \left[\frac{\partial}{\partial \omega_i} \left(\delta x \frac{\partial G}{\partial x_{\omega_i}} \right) - \delta x \frac{\partial^2 G}{\partial x_{\omega_i} \delta \omega_i} \right] d\omega =$$

$$= \int_\Omega \left\{ \frac{\partial J}{\partial x} + \sum_{i=1}^N \left[\frac{\partial^2 \Psi}{\partial \omega_i^2} + \Psi \left(\frac{\partial G}{\partial x} - \frac{\partial^2 G}{\partial x_{\omega_i} \partial \omega_i} \right) \right] \right\} \delta x d\omega +$$

$$+ \int_\Omega \Psi \sum_{i=1}^N \frac{\partial}{\partial \omega_i} \left(\frac{\partial G}{\delta x_{\omega_i}} \right) d\omega = \quad (24)$$

$$= \int_{\Omega} \left\{ \frac{\partial J}{\partial x} + \sum_{i=1}^{N} \left[\frac{\partial^2 \Psi}{\partial \omega_i^2} - \frac{\partial \ \Psi}{\partial \omega_i} \frac{\partial G}{\partial \omega_i} + \Psi \left(\frac{\partial G}{\partial x} - \frac{\partial^2 G}{\partial \omega_i^2 \partial \omega_i} \right) \right] \right\} \delta x d\omega +$$

$$+ \sum_{i=1}^{N} \left[\Psi \frac{\partial G}{\partial x_{\omega_i^2}} \delta x \right] \Bigg|_{\partial \Omega} .$$

Then, putting partial variations (23), (24) to zero, we get necessary optimal conditions:

$$\frac{\partial J}{\partial u} + \Psi \frac{\partial G}{\partial u} = 0 , \tag{25}$$

$$\frac{\partial J}{\partial x} + \sum_{i=1}^{N} \left[\frac{\partial^2 \Psi}{\partial \omega_i^2} - \frac{\partial G}{\partial x_{\omega_i}} \cdot \frac{\partial \Psi}{\partial \omega_i} + \left(\frac{\partial G}{\partial x} - \frac{\partial^2 G}{\partial x_{\omega_i} \partial \omega_i} \right) \right] \Psi = 0 , \tag{26}$$

$$\left[\Psi \frac{\partial G}{\partial x_{\omega_i}} \right] \Bigg|_{\partial \Omega} = 0 \ \text{(or } \Psi\big|_{\partial \Omega} = 0 \text{)}. \tag{27}$$

TASK OF OPTIMAL CONTROL FOR THE OBJECTS WITH DISTRIBUTED CONSTANTS, WHICH ARE DESCRIBED BY NON-LINEAR DIFFERENTIAL EQUATIONS - PARABOLIC TYPE

Let us assume, that functions, which determine object's state $x(t,\omega)$ and the control parameter $u(t,\omega)$, are determined in the restricted domain $\Omega \subset R^N$ with the limit $\partial\Omega$ at time interval $[0,T] = S$.

Time-dependent tasks of optimal control for the objects with distributed constants, which are described by non-linear differential equations with particle derivatives of parabolic type, look like:

$$\int_0^T \int_{\Omega} J[x(t,\omega), u(t,\omega)] d\omega dt = I(x,u) \to \inf , \tag{28}$$

$$\frac{\partial x}{\partial t} - \sum_{i=1}^{N} \frac{\partial^2 x}{\partial \omega_i^2} + Q(t,\omega; x(t,\omega), \frac{\partial x}{\partial \omega_1}, ..., \frac{\partial x}{\partial \omega_N}, u(t,\omega)) = f(t,\omega) , \tag{29}$$

$$x(0,\cdot) = 0 , \quad x(t,\omega)\big|_{\partial \Omega} = 0 . \tag{30}$$

In this case initial boundary conditions are put to zero. Such assumption does not affect the general task setting, as non-zero conditions can be put to zero [5, 6].

Considering such tasks we have to deal with functions $x(t,\omega)$, which are irrespective of time and position, which associate each a pair $(t,\omega) \in S \times \Omega$ with real number or vector $x(t,\omega)$. At the same time, variables t and ω are presented as independent. There had been used time functions for the convenience of mathematical description of time dependent processes, associate each a time t with function $x(t,\cdot)$ of position. Consequently, there had been considered functions, designated on S which have values in some spaces X, that is $x \in (S \to X)$ [4].

Let us introduce

$$y(t) = x(t,\cdot),\ v(t) = u(t,\cdot),\ y'(t) = \frac{\partial x}{\partial t},\ y(0) = x(0,\cdot) = 0,\ g(t) = f(t,\cdot),$$

$$Ly(t) = \sum_{i=1}^{N} \frac{\partial^2 y(t)}{\partial w_i^2} = \sum_{i=1}^{N} \frac{\partial^2 x(t,w)}{\partial w_i^2},$$

$$G(y(t),v(t)) = Q\left(t, y(t), \frac{\partial y}{\partial \omega_1},\dots,\frac{\partial y}{\partial \omega_N}, v(t)\right). \tag{31}$$

Let us present the tasks (28)–(30) in the operator form, taking into account (31):

$$I(y,v) \to \inf, \tag{32}$$

$$y'(t) - Ly(t) + G(y(t),v(t)) = g(t), \tag{33}$$

$$y(0) = 0. \tag{34}$$

Here the function $l(t;\xi_1,\dots,\xi_N) \in \mathrm{CAR}$, which corresponds to the linear reflection

$$Ly(t) = \sum_{i=1}^{N} \frac{\partial^2 y(t)}{\partial \omega_i^2},$$

satisfies the condition

$$\left|l(t;\xi_1,\dots,\xi_N)\right| = \left|\xi_1 + \xi_2 + \dots + \xi_N\right| \le \left|\xi_1\right| + \left|\xi_2\right| + \dots + \left|\xi_N\right| \le \sum_{i=1}^{N} \left|\xi_i\right|^{p-1},\ p \ge 2.$$

For the function $Q(t;\xi_1,\dots,\xi_{N+2}) \in \mathrm{CAR}$, which corresponds to the non-linear reflection

$$G(y(t),v(t)) = Q\left(t; y(t), \frac{\partial y}{\partial \omega_1},\dots,\frac{\partial y}{\partial \omega_N}, v(t)\right),$$

we demand the fulfilment of condition

$$Q(t;\xi_1,\dots,\xi_{N+2}) \le a(t) + c\sum_{i=1}^{N+1} \left|\xi_i\right|^{p-1} + \eta^{r/q},\quad a \in (S \to L_u(\Omega)),\ c > 0,\ p \ge 2.$$

On the assumption of boundary conditions and taking into account, that $X = \overset{\circ}{W}{}_p^2(\Omega) \cap W_p^2(\Omega) \subset L_p(\Omega)$ i $X^* \subset L_q(\Omega)$, we assume, that

$$y \in (S \to X),\ y' \in (S \to X^*),$$

$$\frac{\partial y}{\partial \omega_i} \in (S \to L_p(\Omega)),\ v \in (S \to L_p(\Omega)),\ g(t) \in L_q(\Omega).$$

Then

$$I : L_p(S;X) \times L_r(S;L_r(\Omega)) \to \mathbb{R},$$

$$L : L_p(S;X) \to L_q(Q),\ Q = S \times \Omega\,,$$

$$G : L_p(S;X) \to L_q(Q)\,.$$

We form appropriate Lagrange function for getting necessary conditions of optimal task (32)–(34), which is equivalent to the task (28)–(30), by introducing conjugated function $\psi \in (S \to L_p(\Omega))$. Then,

$$F(y,v) = I(y,v) + \int_S \left\langle \psi(t), \frac{dy(t)}{dt} - Ly(t) + G(y(t),v(t)) - g(t) \right\rangle_{L_q(\Omega)} dt =$$

$$= I(y(t),v(t)) + \int_0^T\int_\Omega \psi(t)\frac{dy(t)}{dt}d\omega dt - \int_0^T\int_\Omega \psi(t)\sum_{i=1}^N \frac{\partial^2 y(t)}{\partial\omega_i^2}d\omega dt +$$

$$+ \int_0^T\int_\Omega \psi(t)Q(t,y(t),\frac{\partial y(t)}{\partial\omega_1},...,\frac{\partial y(t)}{\partial\omega_N},v(t)d\omega dt - \int_0^T\int_\Omega \psi(t)g(t)d\omega dt\,. \quad (35)$$

Using the integration by parts rule about first and second integral and changing integration sequence in first integral, we obtain:

$$F(y,v) = I(y,v) + \int_\Omega \left\{ [\psi(0)y(0) - \psi(T)y(T)] - \int_0^T y(t)\frac{d\psi(t)}{dt}dt \right\} d\omega -$$

$$- \left[\int_0^T \left\{ \left[\psi(t)\sum_{i=1}^N \frac{\partial\psi(t)}{\partial\omega_i} \right]\Bigg|_{\partial\Omega} - \int_\Omega \sum_{i=1}^N \frac{\partial\psi(t)}{\partial\omega_i}\cdot\frac{\partial y(t)}{\partial\omega_i}d\omega \right\} dt \right] +$$

$$+ \int_0^T\int_\Omega \psi(t)Q(t,y(t),\frac{\partial y(t)}{\partial\omega_1},...,\frac{\partial y(t)}{\partial\omega_N},v(t))d\omega dt - \int_0^T\int_\Omega \psi(t)g(t)d\omega dt =$$

$$= I(y,v) + \int_\Omega \left\{ [\Psi(0)y(0) - \Psi(T)y(T)] - \int_0^T y(t)\frac{d\Psi(t)}{dt}dt \right\} d\omega -$$

$$- \left[\int_0^T \left\{ \left[\psi(t)\sum_{i=1}^N \frac{\partial y(t)}{\partial\omega_i} \right]\Bigg|_{\partial\Omega} - \left[\sum_{i=1}^N \frac{\partial\psi(t)}{\partial\omega_i}y(t) \right]\Bigg|_{\partial\Omega} + \int_\Omega y(t)\sum_{i=1}^N \frac{\partial^2\psi(t)}{\partial\omega_i^2}d\omega \right\} dt \right] +$$

$$+ \int_0^T\int_\Omega \psi(t)Q(t,y(t),\frac{\partial y(t)}{\partial\omega_1},...,\frac{\partial y(t)}{\partial\omega_N},v(t))d\omega dt - \int_0^T\int_\Omega \psi(t)g(t)d\omega dt\,. \quad (36)$$

Taking into account that $y(t)\big|_{\partial\Omega} = 0 \quad \forall t \in S$, it follows

$$F(y,v) = I(y,v) + \int_\Omega \left\{ [\psi(0)y(0) - \psi(T)y(T)] - \int_0^T y(t)\frac{d\psi(t)}{dt}dt \right\} d\omega -$$

$$-\int_0^T\int_\Omega y(t)\sum_{i=1}^{N}\frac{\partial^2\psi(t)}{\partial\omega_i^2}d\omega dt+\int_0^T\int_\Omega \psi(t)Q\left(t,y(t),\frac{\partial y(t)}{\partial\omega_1},...,\frac{\partial y(t)}{\partial\omega_N},\nu(t)\right)d\omega dt-$$

$$-\int_0^T\int_\Omega \psi(t)g(t)d\omega dt\,. \qquad (37)$$

Under the condition that $y(0)=0$ and for conjugated task $\psi(T)=0$, we obtain

$$F(y,\nu)=I(y,\nu)-\int_0^T\int_\Omega y(t)\frac{d\psi(t)}{dt}d\omega dt-$$

$$-\int_0^T\int_\Omega y(t)\sum_{i=1}^{N}\frac{\partial^2\psi(t)}{\partial\omega_i^2}d\omega dt+\int_0^T\int_\Omega \psi(t)Q(t,y(t),\frac{\partial y(t)}{\partial\omega_1},...,\frac{\partial y(t)}{\partial\omega_N},\nu(t))d\omega dt-$$

$$-\int_0^T\int_\Omega \psi(t)\,g(t)d\omega dt\,. \qquad (38)$$

Then, we find partial variations of the functional $F(y(t),\nu(t))$

$$\delta_y F=\int_0^T\int_\Omega \frac{\partial J}{\partial y(t)}\delta y(t)d\omega dt-\int_0^T\int_\Omega \frac{d\psi(t)}{dt}\delta y(t)dtd\omega-\int_0^T\int_\Omega\sum_{i=1}^{N}\frac{\partial^2\psi(t)}{\partial\omega_i^2}\delta_{y(t)}d\omega dt+$$

$$+\int_0^T\int_\Omega \psi(t)\left[\frac{\partial Q}{\partial y(t)}\delta y(t)+\sum_{i=1}^{N}\frac{\partial Q}{\partial y_{\omega_i}(t)}\delta y_{\omega_i}(t)\right]d\omega dt=$$

$$=\int_0^T\int_\Omega \frac{\partial \tau}{\partial y(t)}\delta y(t)dtd\omega-\int_0^T\int_\Omega \frac{d\psi(t)}{dt}\delta y(t)dtd\omega-\int_0^T\int_\Omega\sum_{i=1}^{N}\frac{\partial^2\psi(t)}{\partial\omega_i^2}\delta y(t)d\omega dt+$$

$$+\int_0^T\int_\Omega \psi(t)\frac{\partial Q}{\partial y(t)}\delta y(t)d\omega dt+$$

$$+\int_0^T\int_\Omega \psi(t)\sum_{i=1}^{N}\left[\frac{\partial}{\partial\omega_i}\left(\frac{\partial Q}{\partial y_{\omega_i}(t)}\delta y(t)\right)-\delta y(t)\frac{\partial^2 Q}{\partial y_{\omega_i}(t)\partial\omega_i}\right]d\omega dt=$$

$$=\int_0^T\int_\Omega \frac{\partial \tau}{\partial y(t)}\delta y(t)dtd\omega-\int_0^T\int_\Omega \frac{d\psi(t)}{dt}\delta y(t)dtd\omega-\int_0^T\int_\Omega\sum_{i=1}^{N}\frac{\partial^2\psi(t)}{\partial\omega_i^2}\delta y(t)d\omega dt+$$

$$+\int_0^T\int_\Omega \psi(t)\frac{\partial Q}{\partial y(t)}\delta y(t)d\omega dt+\int_0^T\int_\Omega \psi(t)\sum_{i=1}^{N}\delta y(t)\frac{\partial^2 Q}{\partial y_{\omega_i}(t)\partial\omega_i}d\omega dt+$$

$$+\int_0^T\int_\Omega \psi(t)\sum_{i=1}^{N}\frac{\partial}{\partial\omega_i}\left(\frac{\partial Q}{\partial y_{\omega_i}(t)}\delta y(t)\right)d\omega dt\,. \qquad (39)$$

Let us mark the first five integrals through Σ. Using formula of Green, it follows

$$\delta_y F = \Sigma + \int_0^T \int_\Omega \psi(t) \sum_{i=1}^N \frac{\partial}{\partial \omega_i} \left(\frac{\partial Q}{\partial y_{\omega_i}(t)} \delta y(t) \right) d\omega dt =$$

$$= \Sigma + \int_0^T \psi(t) \sum_{i=1}^N \delta y(t) \frac{\partial^2 Q}{\partial y_{\omega_i}(t) \partial \omega} \Bigg|_{\partial\Omega} dt - \int_0^T \int_\Omega \sum_{i=1}^N \frac{\partial \psi(t)}{\partial \omega_i} \cdot \frac{\partial Q}{\partial y_{\omega_i}(t)} \delta y(t) d\omega dt =$$

$$= \int_0^T \int_\Omega \frac{\partial J}{\partial y(t)} \delta y(t) dt d\omega - \int_0^T \int_\Omega \frac{d\psi(t)}{dt} \delta y(t) dt d\omega - \int_0^T \int_\Omega \sum_{i=1}^N \frac{\partial^2 \psi(t)}{\partial \omega_i^2} \delta y(t) d\omega dt +$$

$$+ \int_0^T \int_\Omega \psi(t) \frac{\partial Q}{\partial y(t)} \delta y(t) d\omega dt - \int_0^T \int_\Omega \psi(t) \sum_{i=1}^N \frac{\partial^2 Q}{\partial y_{\omega_i}(t) \partial \omega_i} \delta y(t) d\omega dt +$$

$$+ \int_0^T \left[\psi(t) \sum_{i=1}^N \frac{\partial Q}{\partial y_{\omega_i}(t)} \delta y(t) \right] \Bigg|_{\partial\Omega} dt - \int_0^T \int_\Omega \sum_{i=1}^N \frac{\partial \psi(t)}{\partial \omega_i} \cdot \frac{\partial Q}{\partial y_{\omega_i}(t)} \delta y(t) d\omega dt =$$

$$= \int_0^T \int_\Omega \left(\frac{\partial J}{\partial y(t)} - \sum_{i=1}^N \left\{ \begin{aligned} & \frac{\partial^2 \psi(t)}{\partial \omega_i^2} + \frac{\partial \psi(t)}{\partial \omega_i} \cdot \frac{\partial Q}{\partial y_{\omega_i}(t)} + \\ & + \left[\frac{\partial^2 Q}{\partial y_{\omega_i}(t) \partial \omega} - \frac{\partial Q}{\partial y(t)} \right] \psi(t) + \frac{d\psi(t)}{dt} \end{aligned} \right\} \right) \delta y(t) d\omega dt +$$

$$+ \int_0^T \left[\psi(t) \sum_{i=1}^N \frac{\partial Q}{\partial y_{\omega_i}(t)} \delta y(t) \right] \Bigg|_{\partial\Omega} dt. \qquad (40)$$

$$\delta_u F = \int_\Omega \frac{\partial J}{\partial u(t)} \delta u(t) d\omega + \int_0^T \int_\Omega \psi(t) \sum_{i=1}^N \frac{\partial^2 Q}{\partial u_{\omega_i} \partial \omega_i} \delta u(t) d\omega dt =$$

$$= \int_0^T \int_\Omega \frac{\partial J}{\partial u(t)} \delta u(t) dt d\omega + \int_0^T \int_\Omega \psi(t) \frac{\partial Q}{\partial u(t)} \delta u(t) d\omega dt =$$

$$= \int_0^T \int_\Omega \left[\frac{\partial J}{\partial u(t)} + \frac{\partial Q}{\partial u(t)} \psi(t) \right] \delta u(t) dt d\omega. \qquad (41)$$

Having put partial variations (40), (41) to zero, we obtain the necessary optimal conditions:

$$\frac{\partial J}{\partial y} - \frac{\partial \psi}{\partial t} - \sum_{i=1}^N \left\{ \frac{\partial^2 \psi(t)}{\partial \omega_i^2} + \frac{\partial \psi(t)}{\partial \omega_i} \frac{\partial Q}{\partial y_{\omega_i}} + \left[\frac{\partial^2 Q}{\partial y_{\omega_i} \partial \omega_i} - \frac{\partial Q}{\partial y} \right] \psi(t) \right\} = 0, \quad (42)$$

$$\psi\langle T\rangle=0,\ \left.\left[\psi(t)\sum_{i=1}^{N}\frac{\partial Q}{\partial y_{\omega_i}}\right]\right|_{\partial\Omega}=0\ \text{(or } \psi(t)|_{\partial\Omega}=0\text{)}, \tag{43}$$

$$\frac{\partial J}{\partial u}+\frac{\partial Q}{\partial u}\psi(t)=0\,. \tag{44}$$

TASK OF OPTIMAL CONTROL FOR THE OBJECTS WITH DISTRIBUTED CONSTANTS, WHICH ARE DESCRIBED BY NON-LINEAR DIFFERENTIAL EQUATIONS - HYPERBOLIC TYPE

Let the state of an object $x(t,\omega)$ and control parameter $u(t,\omega)$ be determined in the restricted domain $\Omega\subset R^N$ with the boundary $\partial\Omega$ and time interval $[0,T]=S$.

Such task will take place:

$$\int_0^T\int_\Omega J[x(t,\omega),u(t,\omega)]d\omega dt=I(x,u)\to\inf\,, \tag{45}$$

$$\frac{\partial^2 x}{\partial t^2}-\sum_{i=1}^{N}\frac{\partial^2 x}{\partial\omega_i^2}+Q\left(t,\omega,x(t,\omega),\frac{\partial x}{\partial\omega_1},\dots,\frac{\partial x}{\partial\omega_N},u(t,\omega)\right)=f(t,\omega)\,, \tag{46}$$

$$x(0,\cdot)=0,\ x'(t,\cdot)|_{t=0}=\frac{\partial x(t,\cdot)}{\partial t}|_{t=0}=x'(0,\cdot)=0,\ \ x(t,\omega)|_{\partial\Omega}=0,\ \forall t\in S\,. \tag{47}$$

Let us introduce the corresponding markings according to the assumption as for Q and J, as in the task (28)–(30):

$$y(t)=x(t,\cdot),\ u(t,\cdot)=v(t),\ y'(t)=\frac{\partial x}{\partial t},\ y''(t)=\frac{\partial^2 x}{\partial t^2},\ y(0)=x(0,\cdot)=0\,,$$

$$y'(0)=\frac{\partial x(t,\cdot)}{\partial t},\ g(0)=f(t,\cdot),\ Ly(t)=\sum_{i=1}^{N}\frac{\partial^2 y(t)}{\partial\omega_i^2}=\sum_{i=1}^{N}\frac{\partial^2 x(t,\omega)}{\partial\omega_i^2}\,,$$

$$G(y(t),v(t))=Q\left(t,y(t),\frac{\partial y}{\partial\omega_1},\dots,\frac{\partial y}{\partial\omega_N},v(t)\right). \tag{48}$$

Then, the expressions (45), (46) will look like:

$$\int_0^T\int_\Omega J(y(t),v(t))d\omega dt=I(y,v)\to\inf\,, \tag{49}$$

$$y''(t)-Ly(t)+G(y(t),v(t))=g(t), \tag{50}$$

$$y(0)=y'(0)=0. \tag{51}$$

On the assumption of boundary conditions, we assume that

$$y \in (S \to \overset{\circ}{W}{}_p^1(\Omega) \cap W_p^2(\Omega)) = X$$

$$y', y'' \in X^*;\ \frac{\partial y}{\partial \omega_i} \in (S \to L_p(\Omega)),\ v \in (S \to L_r(\Omega)),\ g \in (S \to L_p(\Omega)).$$

Then

$$I : L_p(S; X) \times L_r(S; L_r(\Omega)) \to \mathbb{R},$$

$$L : L_p(S; X) \to L_q(Q),\ Q = S \times \Omega,$$

$$G : L_p(S; X) \to L_q(Q).$$

Let us form appropriate Lagrange function, introducing conjugated function:

$$F(y,v) = I(y,v) + \int_0^T \int_\Omega \psi(t) \frac{\partial^2 y(t)}{\partial t^2} d\omega\, dt - \int_0^T \int_\Omega \psi(t) \sum_{i=1}^N \frac{\partial^2 y(t)}{\partial \omega_i^2} d\omega\, dt -$$

$$- \int_0^T \int_\Omega \psi(t) Q\left(t, \omega, y(t), \frac{\partial y}{\partial \omega_1}, \ldots, \frac{\partial y}{\partial \omega_N},\ v(t) \right) d\omega\, dt - \int_0^T \int_\Omega \psi(t) g(t)\, d\omega\, dt, \quad (52)$$

where $\psi \in X$.

Acquired Langrangian differs from (28)–(30) only for second member $\int_0^T \int_\Omega \psi(t) \frac{\partial^2 y(t)}{\partial t^2} d\omega\, dt$.

Assuming that the conditions of Fubini's theorem are fulfilled [4], and substituting integration sequence and using the integration by parts rule, we obtain

$$\int_0^T \int_\Omega \psi(t) \frac{\partial^2 y(t)}{\partial t^2} d\omega\, dt = \int_0^T \int_\Omega \psi(t) \frac{\partial^2 y}{\partial t^2} d\omega\, dt =$$

$$= \int \left[\left(\psi(t) \frac{\partial y}{\partial t} \right) \Big|_0^T - \int_0^T \frac{\partial \psi(t)}{\partial t} \frac{\partial y(t)}{\partial t} dt \right] d\omega =$$

$$= \int_\Omega \left\{ \left[\psi(t) \frac{\partial y(t)}{\partial t} - \psi(0) \frac{\partial y(0)}{\partial t} \right] - \left[\frac{\partial \psi(t)}{\partial t} y(t) \right] \Big|_0^T + \int_0^T y(t) \frac{\partial^2 \psi}{\partial t^2} dt \right\} d\omega =$$

$$= \int_\Omega \left\{ \left\{ \psi(t) \frac{\partial y(t)}{\partial t} - \psi(0) \frac{\partial y(0)}{\partial t} \right\} - \left\{ \frac{\partial \psi(T)}{\partial t} y(T) - \frac{\partial \psi(0)}{\partial t} y(0) \right\} + \right.$$

$$\left. + \int_0^T y(t) \frac{\partial^2 \psi}{\partial t^2} dt \right\} d\omega. \quad (53)$$

Taking into account the initial conditions $y(0) = y'(0) = 0$ and assuming for conjugated task $\psi(t) = \psi'(t) = 0$, we obtain

$$\int_0^T \int_\Omega \psi(t) \frac{\partial^2 y(t)}{\partial t^2} d\omega\, dt = \int_0^T \int_\Omega y(t) \frac{\partial^2 \psi}{\partial t^2} d\omega\, dt \,. \tag{54}$$

Then, determining partial variations and putting them to zero, the necessary optimal conditions are obtained

$$\frac{\partial J}{\partial y} + \frac{\partial^2 \psi}{\partial t^2} - \sum_{i=1}^{N} \left[\frac{\partial^2 \psi(t)}{\partial \omega_i^2} + \frac{\partial Q}{\partial y_{\omega_i}} \cdot \frac{\partial \psi(t)}{\partial \psi_i} + \left(\frac{\partial^2 Q}{\partial y_{\omega_i} \partial \omega_i} - \frac{\partial Q}{\partial y} \right) \psi(t) \right] = 0 \,, \tag{55}$$

$$\psi(T) = \psi'(T) = 0,\ \psi(t) = \psi'(t) = 0|_{\Omega} \quad \forall t \in S, \tag{56}$$

$$\frac{\partial J}{\partial y} + \frac{\partial Q}{\partial u} \psi(t) = 0. \tag{57}$$

CONCLUSION

In this work, we have considered how to find possible optimal solution of task of control (16)−(18) for the objects with distributed constants, which are described by non-linear differential equations (elliptic type): we have the system of 3 equations (17), (25), (26) with boundary conditions (18), (27) for unknown quantities (x_0, u_0, Ψ_0). The given system of equations is non-linear as for unknown quantities (x,u) and linear as for ψ.

First, from (17), (18) we find $x_0 \ \forall u \in U$. Then x_0, which depends on *u*, is substituted in (25)−(27); finally we find appropriate value (ψ_0, u_0) to x_0.

Then, finding second variation of functional $F(x,u)$ within the interval of the element (x_0, u_0) and checking its sign, we get final answer on optimal element $(x_0, u_0) \in X \times U$.

To find possible optimal solution of task (28)−(30) of control for the objects with distributed constants, which are described by non-linear differential equations (parabolic type), we have to solve system of 3 functional equations (29), (42), (44) with boundary conditions (30), (43) for 3 unknown quantities $x_0 = y_0$, u_0, ψ_0.

For the objects with distributed constants, which are described by non-linear differential equations (hyperbolic type) we have considered the possibility of using the operator scheme in the absence of restrictions for phase variables and control functions.

At the same time at first $\forall u$ the non-linear task (45)−(47) is solved, then (54)−(57). Consequently, a possible optimal solution of task is found by solving a linear equation system for Ψ, u.

On the other hand, the task of optimal control (45)−(47) with obvious restrictions of a kind of integral inequalities generates interest, namely:

let $J_i : S \times \Omega \times \mathbb{R} \times \mathbb{R} \to \mathbb{R},\ i = 1,...,m$ the inequalities

$$\int_0^T \int_\Omega J_i(t,\omega,x(t,\omega),u(t,\omega))d\omega\, dt = I_i(x,u) \geq 0,\ \ i=1,...,m \qquad (58)$$

takes place.

Let us assume, that the functions J_i satisfy the following conditions:

1) the function $J_i : S\times\Omega\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is a Borel one for the multitude of variables;

2) the function $(x;u)\mapsto J_i(t,\omega,x,u)$ of class C^1 at $\mathbb{R}^2$ $\forall(t;\omega)\in S\times\Omega$;

3) $\exists a_i \in X,\ c_i > 0$, то $\|J_i'(t,\omega,x,u)\| \leq a_i(t,\omega) + c_i(|x|^{p-1} + |u|^{1/q})$,

where $J_i' = (J_{ix}';J_{iu}')$ is a gradient of function J_i.

At that conditions the functionals $I_i : L_p(S;X)\times L_r(Q)\to\mathbb{R},\ i=1,...,m$ are definitely differential and that is why the conditions of the theorem about Lagrange coefficients are satisfied. Then the Lagrange function has the form:

$$F(\lambda,\psi,y,v) = \lambda_0 \int_0^T \int_\Omega J_0(t,\omega,y(t,\omega),v(t,\omega))d\omega dt +$$

$$+\sum_{i=1}^m \lambda_i \int_0^T \int_\Omega J_i(t,\omega,y(t,\omega),\vartheta(t,\omega))d\omega dt +$$

$$+\int_0^T \int_\Omega \psi(t)\frac{\partial^2 y(t)}{\partial t^2}d\omega\, dt - \int_0^T \int_\Omega \psi(t)\sum_{i=1}^N \frac{\partial^2 y}{\partial t^2}d\omega\, dt -$$

$$-\int_0^T \int_\Omega \psi(t) Q\left(t,\omega,y(t),\frac{\partial y}{\partial\omega_1},...,\frac{\partial y}{\partial\omega_N},\ \vartheta(t)\right)d\omega\, dt - \int_0^T \int_\Omega \psi(t)g(t)\,d\omega\, dt. \qquad (59)$$

It can be found such multitudes $\hat\lambda,\ \hat\psi$, for which the conditions of Lagrange function stationary state will be satisfied

$$F_y'(\hat\lambda,\hat\psi,y,u) + F_u'(\hat\lambda,\ \hat\psi,y,u) = 0, \qquad (60)$$

as well as the conditions of sign accordance $\hat\lambda_i \geq 0$ and the conditions of completable slackness

$$\hat\lambda_i\, I_i(y,u) = 0,\ \ i=1,...,m.$$

Then, having determined partial variations of function (59), with the help of (60) an appropriate modification of necessary optimal conditions (55)–(57) can be obtained.

The obtained results enable to conduct analysis of tasks of optimal control for the objects with distributed constants, which are described by non-linear differential equations of various types, using modern methods of non-linear functional analysis, which imply significant calculations simplification.

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