УДК 338.1:51-7

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# APPLICATION OF NUMBER THEORY METHODS FOR TASK SOLUTION OF INTERTEMPORAL BALANCE IN ECONOMY

In this article an intertemporal balance is being researched in one-commodity model of economy, which consists of two aggregated agents: *consumer-owner* and *manufacturer*. The peculiarity of this model is that the balance is researched not only at goods market, but also at stock market. To research the model till the very end we don't consider market resources, other words we ignore the necessity to use labour and natural resources in manufacturing.

Also, building models, which allow to reveal causal relations between variables, is a very important problem of economics modeling. Such models are assigned for definition of quantitative effect of independent variables on dependent one [1]. The solution of this task is rather difficult. Discrepancy between the results of modeling to correlations, which are taking place in reality, appears due to the number of reasons, as a matter of fact because of breaking the main prerequisites of regression analysis. Most often they are broken because of multicollinearity [2]. This phenomenon brings to estimation of parameters with great dispersion, which, in most of cases, doesn't allow to interpret them intensionally because of wrong signs, for instance. The usage of prior information is a powerful means of building regression models mentioned above. As a matter of fact it decreases the influence of multicollinearity on the estimation of parameters. But formalization of the prior information is not very easy because of its ambiguity. One of the possible approaches to the solution of this task is being researched in this article.

## 1. Necessary and sufficient conditions of regular optimality.

In conditions of regular balance, in Lagrange functional it is possible to make the integration by parts, as a result, the functional will take the form:

$$\begin{aligned}&£_{\psi,\ddot{\varphi},\xi,\Phi}[Q(t),\mathrm{X}(t),P(t),\mathrm{Y}(t)] = \psi(0)X(0)+\xi(0)Q(0)+\\&+\int_0^T\left(f(P(t))e^{-\chi t}+\xi(t)p(t)P(t)\right)dt+\int_0^T\left(\frac{\partial}{\partial t}\xi(t)+\widetilde{\rho}(t)\right)Q(t)dt+\\&+\int_0^T\left(\xi(t)r(t)+\frac{\partial}{\partial t}\widetilde{\psi}(t)+\widetilde{\varphi}(t)\right)X(t)dt+\int_0^T\left(\psi(t)-\xi(t)s(t)\right)Y(t)dt+\\&+Q(T)\left(\Phi-\xi(T)\right)+\left(\widetilde{\Phi}a-\psi(T)\right)X(T)\end{aligned} \qquad (1)$$

It is clear that this expression reaches the maximum in accord with piecewise differentiable functions $X(t)$, $Q(t)$ with specified initial conditions and in accord with piecewise continuous functions $P(t)\geq 0$, $Y(t)$ only that time when almost everywhere at $[0,T]$ derivatives turn to zero by $Q(t),X(t),P(t),Y(t)$ of integrand in (2.1), and also derivatives (2.1) by $Q(T),X(T)$

$$f'(P(t))e^{-\chi t} = \xi(t)p(t), \qquad (2)$$

$$\frac{\partial}{\partial t}\xi(t)+\widetilde{\rho}(t)=0, \qquad (3)$$

$$\xi(t)r(t)+\frac{\partial}{\partial t}\widetilde{\psi}(t)+\widetilde{\varphi}(t)=0, \qquad (4)$$

$$\psi(t)=\xi(t)s(t), \qquad (5)$$

$$\widetilde{\Phi}=\xi(T) \qquad (6)$$

$$\widetilde{\Phi}a=\psi(T) \qquad (7)$$

[3] contains the rigorous proof of this fact.

Thereby, to make the regular solution of agent's task [4] by the collection of straight lines $[\widetilde{Q}(t),\mathrm{X}(t),\widetilde{P}(t),Y(t)]$ and dual ones $[\psi(t),\widetilde{\varphi}(t),\xi(t),\widetilde{\rho}(t),\widetilde{\Phi}]$, it is *necessary* that at $[0,T]$ equalities (2) — (7) and slackness conditions must be accomplished [4].

Derived conditions can be considerably simplified.

Notice, firstly, because of the non-negativeness of dual variables $p(t),\Phi$ from (3), (6) appears from the above that the continuous function $\xi(t)$ is non-negative when all $t\in[0,T]$.

Let us consider condition (2). As far as function $P(t)$ — is a piecewise continuous one, then it is limited per $[0,T]$, and then, under the function's properties of usefulness $f(.)$, inequality is accomplished $P\{t\}>0$, and the left part of the equation (2) is positive and is separated from 0 per $[0,T]$. It means, that $\xi(t)$ is also separated from 0, and, under (6),

$$\Phi>0. \qquad (8)$$

From (8), because of the last condition in [4], it follows that terminal condition in regular balance *is being done like equation:*

$$Q(T) + aX(T) = 0. \qquad (9)$$

As far as functions $\xi(t) > 0$ and $\psi(t)$ — are absolutely continuous, from (5) it follows that informational variables *s(t)* at regular balance must be almost everywhere equal to absolutely continuous functions. It is obvious, that overdetermination *s(t)* at the set of measure 0 doesn't break the conditions of balance, it can be considered that the functions *s(t)* at regular balance are *absolutely continuous*. It means that the path of stock price *s(t)* and price *p(t)*, which compose vector-functions *s(t)*, must be also absolutely continuous. But then from (2) it follows that the price *p(t)* is positive and is separated from 0 per *[0,T]*.

When *s(t)* are absolutely continuous the equation (5) is done for all $t \in [0,T]$. When *t = T* it gives the relation $\psi(T) = \xi(T)s(T)$, from which because of (6), (7), (8) it follows that the agent's task has a regular solution, *if only the coefficients of the terminal condition are coordinated with the prices*:

*a=s(T)*.

This result saves us from the necessity to discuss the question about the way agents evaluate their funds at the end of the process. In the bounds of the researched model they must evaluate them just in accord with existing in the final courses.

Notice, if we didn't set the terminal limit, then we would have conditions of optimality $\xi(T) = 0$ and $\psi(T) = 0$, to provide which, it is easy to check, we would have to allow unlimited threads into the momentт *T*: either real *P(t)*, or nominal *p(t)P(t)*.

After we have received these conclusions, it is possible to exclude the dual variable $\psi(T)$, and also the auxiliary straight variable Y(t) from the system of conditions of optimality. It is also possible to exclude the dual variables $\xi(t)$ and Φ. In order to do this instead of $\tilde{\varphi}(t), \tilde{\rho}(t)$ we will introduce normalized quantity (см. (3)):

$$\varphi(t) = \frac{\tilde{\varphi}(t)}{\xi(t)}, \qquad \rho(t) = \frac{\tilde{\rho}(t)}{\xi(t)} = -\frac{1}{\xi(t)}\frac{\partial}{\partial t}\xi(t).$$

The equation (4) in these variables after exclusion of $\psi(T)$ will look like $r(t) - \rho(t)s(t) + \frac{\partial}{\partial t}s(t) + \varphi(t) = 0$, and as long as $\xi(t) > 0$, in inequations in a set of conditions of complementary slackness [4] one can substitute $\tilde{\varphi}(t), \tilde{\rho}(t)$ for $\varphi(t), \rho(t)$. Finally, due to the fact that the first part (2) is absolutely continuous, the function *P(t)* can also be considered absolutely continuous. Taking logarithmic derivative from both parts of the equation (2), we will have:

$$-\frac{\nu}{P(t)}\frac{\partial}{\partial t}P(t) - \chi = -\rho(t) + \iota(t), \qquad (10)$$

Where $\nu = \frac{f''(P(t))}{f'(P(t))}P(t) = const > 0$ — is the agent's odium on risk ($\nu = B$ for an enterprise and $\nu = \beta$ for an owner), and $\iota(t) = \frac{1}{p(t)}\frac{\partial}{\partial t}P(t)$ — the rate of inflation. Constant of integration in equation (10) ultimately will be defined from the terminal condition (9), so there is no need to identify $\xi(t)$ and Φ for the agent's task solution.

Thus, we arrive at the following statement.

**Statement 2.**

1) If a regular balance exists, the path of price *p(t)* and the stock price *s(t)* are absolutely continuous, and $p(t) > 0$ where $t \in [0,T]$.

2) The agents' conduct in a regular balance is described with the following conditions:

$$\frac{\partial}{\partial t}Q(t) = r(t)X(t) - s(t)\frac{\partial}{\partial t}X(t) - p(t)P(t); \quad (11)$$

$$\frac{\partial}{\partial t}P(t) = \frac{(\rho(t) - \iota(t) - \chi)}{\nu}P(t) \;; \qquad (12)$$

$$\varphi(t) = \rho(t)s(t) - r(t) - \frac{\partial}{\partial t}s(t) \;; \qquad (13)$$

$$\varphi(t)X(t) = 0,\ \varphi(t) \ge 0,\ X(t) \ge 0; \qquad (14)$$

$$\rho(t)Q(t) = 0,\ \rho(t) \ge 0,\ Q(t) \ge 0; \qquad (15)$$

$$Q(T) + s(T)X(T) = 0 \;; \qquad (16)$$

where the initial values $X(0) \ge 0$, $Q(0) = 0$ are given.

2. The integral of the owned capital.

The equation of financial balance (11) can be rewritten the following way:

$$\frac{\partial}{\partial t}\big(Q(t) + s(t)X(t)\big) = \left(r(t) + \frac{\partial}{\partial t}s(t)\right)X(t) - p(t)P(t) \quad (17)$$

On the agent's optimal path, due to (13), (14), (15):

$$\left(r(t) + \frac{\partial}{\partial t}s(t)\right)X(t) = \rho(t)s(t)X(t) + \varphi(t)X(t) =$$

$$= \rho(t)s(t)X(t) + \rho(t)Q(t) + \varphi(t)X(t) = \rho(t)\big(Q(t) + s(t)X(t)\big).$$

Thus, for the value

$$\Omega(t) = Q(t) + s(t)X(t) \qquad (18)$$

From (17), (16) we have

$$\frac{\partial}{\partial t}\Omega(t) = \rho(t)\Omega(t) + p(t)P(t),\ \Omega(T) = 0 \qquad (19)$$

In terms of the theory of optimal management, the value $\Omega(T)$ is a counterpart of the integral of motion of Hamiltonian system of optimum condition. According to Nether theory, the most interesting integrals of motion are connected with the task's symmetry. As it is shown in [3], integral $\Omega(T)$ has the same origin — it emerges as a result of the fact the agent's task is homogeneous in relation to the funds $Q(t), X(t)$.

From the economic point of view, as we will see below, and as in a more common case it is shown in [3], the valuey $\Omega(T)$ can be interpreted as the agent's *owned capital.* The correlation (18) is in fact a report balance in remains — it presents the owned capital through pecuniary valuation of funds (assets and liabilities).

The correlations (19) show that on the optimum path the owned capital must coincide with the given sum of the planned "useful expenses" *p(t)P{t)* (dividends for the enterprise and consumer expenses for the owner):

$$\Omega(t) = \int_t^T p(u)P(u)e^{-\int_t^u \rho(v)dv} du. \qquad (20)$$

Other words, as it will be shown below, the term $\rho(t)\Omega(t)$ corresponds to the agent's *balance profit* — the difference of the income and expenses taking into account the profit on reevaluation of the funds. Then the dual variable $\rho(t)$ expresses the agent's *capital profitability*, and the right part of the equation in (19) shows that the capital is derived from the *undistributed profit.*

Finally, notice, that the formula (20) is positive when $t \neq T$. It is known, that the negativity of the owned capital is a characteristic feature of the "financial pyramid". Thus, the terminal limits applied above, which result in the formula (20), in fact, play the role of condition of pyramid's absence (no ponzi game condition), which is usually applied in tasks of financial planning. Besides, the next statement emerges from (20), (18).

**Statement 3.**

Regular balance exists only if the initial conditions for both agents meet the inequations:

$\Omega(0) = Q(0) + s(0)X(0)$.

However, one must bear in mind that in these inequations $X(0) \geq 0$ are given, and *S(0)* are defined from the conditions of balance, where some components of these vectors are knowingly negative *(which follows from the definition of the agent's task regular solution).*

3. Regimes realized in a regular balance.

Hereon we conclude the general overview and move on to the construction of a regular balance [4]. Conditions of complementary slackness (14), (15) describe alternative *regimes* of optimum agents' functioning. Provided to be in a regular balance at all moments of time a definite set of regimes can be realized.

First of all, it is worth mentioning that if the money funds of agents are zero in the beginning, they will remain the same in a balance [4]. That is why, in a regular balance the condition (15) comes to the condition:

$$\rho(t) \geq 0,\ Q(t) = 0 \qquad (21)$$

As it was shown above, in a balance*p(t) > O*, *C(t)>O* так, что потребительские расходы собственника положительны. such a way, that the consumer expenses of the owner are positive. When the fund of money is absent, [4], the positive expenses require positive share stock (see [4]), and the positive consumption requires the positive issue stock in such a way, that in a balance

$S(t) = A(t) > 0, \qquad Y(t) > 0$ where $t \in [0,T)$.

Due to [4] these inequations mean that in the balance conditions of complementary slackness (14) for both agents are realized a

$$\varphi(t) = 0,\ X(t) > 0. \qquad (22)$$

4. Solution of the enterprise task.

According to statement 2, the solution of enterprise task is described with system (11) — (16), for sets:

$Q(t) = W(t), X(t) = \langle A(t), Y(t) \rangle,\ Y(t) = \left\langle \frac{\partial}{\partial t}A(t), \frac{\partial}{\partial t}Y(t) \right\rangle$;

$\varphi(t) = \langle \varphi_A(t), \varphi_Y(t) \rangle, \rho(t) = \rho_W(t)$ where $r(t) = \langle 0, p(t) \rangle$,

$s(t) = \langle -s(t), bp(t) \rangle$.

Taking into account (21), (22), this system transforms into:

$$p(t)Y(t) + s(t)\frac{\partial}{\partial t}A(t) - bp(t)\frac{\partial}{\partial t}Y(t) - Z(t) = 0; \qquad (23)$$

$$\frac{\partial}{\partial t}\left(\frac{Z(t)}{p(t)}\right) = \frac{(\rho_W(t) - \iota(t) - \Delta)}{B}\left(\frac{Z(t)}{p(t)}\right); \qquad (24)$$

$$-\rho_W(t)s(t) + \frac{\partial}{\partial t}s(t) = 0; \qquad (25)$$

$$\rho_W(t)bp(t) - p(t) - b\frac{\partial}{\partial t}p(t) = 0; \qquad (26)$$

$$-s(T)A(T) + bp(T)Y(T) = 0. \qquad (27)$$

Excluding (25), (26) $\rho_W(t)$, we get the connection between informational variables: stock rate *s(t)* and the price *p(t)*:

$$s(t) = s(0)e^{\frac{t}{b}}e^{\left(\int_0^t \iota(u)du\right)}, \iota(t) = \frac{1}{p(t)}\frac{\partial}{\partial t}p(t). \quad (28)$$

Thus, the system (24) — (26) is degenerated: when the correlation (28) is done, it has multiple solutions, and when it is broken, it has none. The ambiguity of optimum enterprise conduct can be interpreted as continuous flexibility of stock offer function *A(t)*. When the connection (28) is broken between *s(t)* and *p(t)* the enterprise either doesn't issue safety stock or tries to place them as much as it is possible. When the connection (28) is stable the volume of issue is indifferent for the enterprise.

Due to the dependence of (24) — (26) system, the equation (23) is better to change with the following from (24) — (26) equation (19) for the enterprise's capital:

$$\Omega(t) = p(t)bY(t) - s(t)A(t), \quad (29)$$

and the terminal condition (27) — with condition $\Omega(T) = 0$. Differential equations (19), (24) can be solved easily and with (26), give the terms for *Z(t)* and $\Omega(t)$:

$$Z(T) = \frac{\Omega(0)\left(1 - B - \Delta b\right)e^{\left(\frac{(1-\Delta b)t}{Bb}\right)}e^{\left(\int_0^t \iota(u)du\right)}}{B\left(e^{\frac{((1-B-\Delta b)T}{Bb}} - 1\right)b}; \quad (30)$$

$$\Omega(t) = \Omega(0)e^{\frac{t}{b}}e^{\left(\int_0^t \iota(u)du\right)}\left(1 - \frac{e^{\frac{((1-B-\Delta b)t}{Bb}} - 1}{e^{\frac{((1-B-\Delta b)T}{Bb}} - 1}\right). \quad (31)$$

From the terms for the capital (31), (29), with (28), we have the connection between the stock offer *A(t)* and the product offer *Y{t)*:

$$\Omega(0)e^{\frac{t}{b}}\left(1 - \frac{e^{\frac{((1-B-\Delta b)t}{Bb}} - 1}{e^{\frac{((1-B-\Delta b)T}{Bb}} - 1}\right) = -s(0)e^{\frac{t}{b}}A(t) + bp(0)Y(t). \quad (32)$$

Individually values $A(t)$ and $Y(t)$ cannot be defined from the task solution, but the requirement of the task solution of the enterprise fixes the connection (28) between the price of the product and the stock price.

From the conditions of optimal behavior of the enterprise one more condition for the price condition appears. As far as $\rho_W(t) \geq 0$, from (26) it is an inequation:

$$\iota(t) = \frac{1}{p(t)}\frac{\partial}{\partial t}p(t) > -\frac{1}{b}, \quad (33)$$

which means that there can't be a very strong deflation on the equilibrium path.

5. The owner's task solution.

According to the statement 2, the owner's task solution is described by the system (11) — (16):

$$Q(t) = M(t), X(t) = \langle S(t)\rangle, P(t) = C(t), Y(t) = \left\langle \frac{\partial}{\partial t}S(t)\right\rangle;$$

$$\varphi(t) = \langle\tilde{\varphi}_S(t)\rangle, \rho(t) = \rho_M(t)$$

where $r(t) = \langle r(t)\rangle$, $s(t) = \langle s(t)\rangle$.

With (21), (22), this system starts to look like:

$$r(t)S(t) - s(t)\frac{\partial}{\partial t}S(t) - p(t)C(t) = 0; \quad (34)$$

$$\frac{\partial}{\partial t}C(t) = \frac{\left(\rho_M(t) - \iota(t) - \Delta\right)}{\beta}C(t); \quad (35)$$

$$\rho_M(t)s(t) - r(t) - \frac{\partial}{\partial t}s(t) = 0 \quad (36)$$

$$s(T)S(T) = 0. \quad (37)$$

This system solution asks the question on the consumer market *C(t)* and on the equity market *S(t)*. Let us remind that due to (21) the return on capital of the owner $\rho_M(t)$ is nonnegative in the balance. From (36) it follows that even in case when the entitlement payment is zero *r(t)* = 0 the capital grows due to the growth of the stock course price *s{t)*.

6. The description of the balance.

The price path *p(t)*, the stock course *s(t)* and the dividend rate *r(t)* must be formally defined from the balances [4]. Substituting the owner's demand into the stock *S(t)*, with (28), and the stream of dividends *Z(t)* (30) in the partition condition and supposing

$$G(t) = e^{\int_0^t \frac{r(u)}{s(u)}du}, \ \omega = \frac{\Omega(0)}{s(0)A(0)} = \frac{p(0)bY(0)}{s(0)A(0)} - 1 > 0, \quad (38)$$

we will get for *G(t)* a nonlinear integral differential equation:

$$\frac{\partial G(t)}{\partial t}\left(\int_t^T G^{\frac{1-\beta}{\beta}}(u)e^{\frac{1-\beta-\delta b}{\beta b}u}du\right) = \frac{\omega(1 - B - \Delta b)e^{\frac{1-B-\Delta b}{Bb}t}}{B\left(e^{\frac{1-B-\Delta b}{Bb}T} - 1\right)b}\left(\int_0^T G^{\frac{1-\beta}{\beta}}(u)e^{\frac{1-\beta-\delta b}{\beta b}u}du\right) \quad (39)$$

and the original condition $G(0) = 1$. (40)

**Statement 4.** On the initial conditions that

$A(0) = S(0) > 0, Y(0) > 0, M(0) = W(0)$,

the regular balance could exist, it is necessary and enough, that at some $\omega > 0$ the equation (39) had a positive and absolutely continuous solution *G(t)*, which satisfies (40) conditions.

*The proof.* The necessity has been already proved. Let's try to prove the sufficiency, other words let's build the regular balance, starting from the positive and absolutely continuous function *G(t)*, which satisfies (39), (40) when some $\omega > 0$. First of all, when *G(t)* > 0 from (39), (40) it is the positivity of the second factor in the left part of (39), and from $\omega > 0$ the positivity of the right part of (39) is obvious. Such way, $\frac{\partial}{\partial t} G(t) > 0$, and from the definition *G{t)* (38) we can find the real rate of paying dividends:

$$\frac{r(t)}{s(t)} = \frac{1}{G(t)} \frac{\partial}{\partial t} G(t) > 0 \tag{41}$$

Let's initialize now the absolutely continuous price path randomly *p(t)* > 0 in such a way to satisfy the condition (33). Then, from (28) it is possible to identify the nominal stock rate *s(t)* accurate within the initial value. The value *s(0)* can be defined due to and the initial conditions from (38). Meanwhile the nominal value of the capital of the enterprise will be positive.

$$s(t) = \frac{p(t) e^{\frac{t}{b}} b Y(0)}{(\omega + 1) A(0)} > 0,$$
$$\Omega(0) = p(0) b Y(0) - s(0) A(0) = \frac{\omega}{\omega + 1} p(0) b Y(0) > 0. \tag{42}$$

Thus, the informational variables are identified in the balance. They define the positive profitability (see (25), (36)):

$$\rho_M(t) = \iota(t) + \frac{1}{b} > 0, \rho_W(t) = \frac{\frac{\partial}{\partial t} G(t)}{G(t)} \iota(t) + \frac{1}{b} > 0.$$

Submitting the term for $p_w(t)$ in (35), we will have the term for consumption accurate within the factor:

$$C(t) = C(0)(G(t))^{\frac{1}{\beta}} e^{\frac{1-\delta b}{b} t}, \tag{43}$$

and from (34), (37) with (41), (43), (42) — the term for the volume of outstanding stock:

$$A(t) = S(t) = C(0) \frac{p(0)}{s(0)} G(t) \int_t^T (G(u))^{\frac{1-\beta}{\beta}} e^{-\delta u} du > 0. \tag{44}$$

The initial value *p(0)* we have assigned above, and *s(0)* we have found in a such way that the initial consumption *A(0) > 0* is being defined categorically from (44) in accord with assigned *C(0)> 0*.

Now, from (32) the issue *Y{t)*, will be defined, which will be positive due to the positivity of the capital of the enterprise along all the path.

7. The existence and the uniqueness of the balance: logarithmic utility.

The question about the solvalibility of system (39), (40) and also about the existence of the system is still open. The equation (39) is reduced to a differential equation of the form with standard methods:

$$\left(\frac{\partial}{\partial x} y(x)\right) y(x) = a y^2(x) + 2b y(x) + c x^2 + f y(x) + g(x),$$

which, as far as we know, cannot be solved in quadratures. To achieve function *y(x)* from function *G(t)*, we have to execute two quadratures and to solve two final transcendental equations.

However, it is possible to pay attention to the case when the system (39), (40) is solved easily — this is the case of the logarithmic utility function of the consumer $\beta = 1$ (see (1.13)). When $\beta = 1$ the solution of the equation (39) reduces to quadratures, and for the real rate of dividend payment (41) we have the term:

$$\frac{s(t)}{r(t)} = \int_0^t \frac{\left(e^{-\delta t} - e^{-\delta T}\right) e^{\frac{1-B-\Delta b}{Bb} u}}{\left(e^{-\delta u} - e^{-\delta T}\right) e^{\frac{1-B-\Delta b}{Bb} t}} du + \frac{Bb\left(e^{\frac{1-B-\Delta b}{Bb} T} - 1\right)}{\omega(1 - B - \Delta b)\left(e^{-\delta T} - 1\right)}, \tag{45}$$

which in fact shows that if $\beta = 1$, then the balance exists with all positive $B, \Delta, T, b, \delta$ for any $\omega > 0$.

Statement 4 shows, that regular balances in the model are not singular, it being known that this non-uniqueness has the dual character. For one thing, if the equation exists under some path of price change *p(t)*, so it will exist and under another function *p(t)*, which satisfies (33). It is not surprising. As far as agents don't keep money, inflation in the model is almost equivalent to the denomination, which should not change the substantive behavior of economic agents. When we change *p(t)* the equilibrium paths of real variables *Y(t)* and *C(t)*, it is easy to check, do not change. Restriction (33) on deflation appears because in the model agents can want to stash money and it brings a profit, when the rate of price declination becomes higher than the profitability of the enterprise, which in our model is value $b^{-1}$. The non-uniqueness of the balance, connected with the possibility to size the prices, con be considered as immaterial.

More important ambiguity $\omega > 0$ в (39). As follows from general considerations, and as it is evident from the logarithmic utility (45), system (39), (40), if it has any solution, then a whole range of values $\omega$.

At the same time, the trajectory of real values for different $\omega$ are different.

From the statement 4 it can be seen that the choice of the value $\omega$ actually equal to the choice of the initial value of the real stock price $\frac{s(0)}{p(0)}$.

If we assume that this value is inherited from the history of the economic system, then the resulting equilibrium trajectory can be regarded as an idealized description of the transition process.

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**Zhykhareva Y. I. Application of number theory methods for task solution of intertemporal balance in economy**

Economic model is presented in which two agents are taking place, which are producer-company and owner-user. They are aggregated presenting productive and non-productive economic fields. Mission is to learn a principal of exploitations availability of intertemporal balance for explanations of change processes in economics. The problem was fully solved at any initial conditions.

*Key words:* agent, property asset, intertemporal balance, balance income, profitableness.

**Жихарєва Ю. І. Застосування методів теорії чисел для вирішення завдання міжтимчасової рівноваги в економіці**

У статті розглядається модель економіки, в якій діють два агенти: фірма-виготовник та власник-споживач, які агрегійовано представляють виробничу та невиробничу сфери економіки. Мета — вивчення питання про принципову можливість використання моделі міжчасової рівноваги для описування перехідних процесів в економіці. Отримано повне розв'язання задачі при будь-яких початкових умовах.

*Ключові слова:* агент, власний капітал, міжчасова рівновага, балансовий прибуток, доходність.

**Жихарева Ю. И. Применение методов теории чисел для решения задачи межвременного равновесия в экономике**

Рассматривается модель экономики, в которой действуют два агента: фирма-производитель и собственник-потребитель, которые агрегировано представляют производственную и непроизводственную сферы экономики. Цель — изучение вопроса о принципиальной возможности использования модели межвременного равновесия для описания переходных процессов в экономике. Получено полное решение задачи при любых начальных условиях.

*Ключевые слова:* агент, собственный капитал, межвременное равновесие, балансовая прибыль, доходность.

Received by the editors: 24.11.2010
and final form in 01.12.2010