

STRUCTURES OF VORTEXES NEAR THE POLES OF PLANETS OF THE SOLAR SYSTEM

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In the vicinity of the poles of the planets of the solar system, ordered crystals of vortices and intermittency are observed. The formation of vortex structures in the vicinity of the poles of the planets of the solar system is investigated. The conditions for the formation of vortexes are considered. A nonlinear equation was found describing the vortex dynamics and structures were studied.

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INTRODUCTION

It is known that in the vicinity of the poles of the planets of the solar system, ordered crystals of vortices and intermittency are observed.

Saturn auroras are bright continuous oval rings surrounding the planet's pole. The southern rings are somewhat further from the pole than the northern ones, since the magnetic field is somewhat stronger in the northern hemisphere. Sometimes the rings become spiral instead of oval. Presumably, they arise due to magnetic reconnection under the influence of the solar wind. The shape and appearance of Saturn's auroras change dramatically over time.

A plasma hurricane (vortex) has been detected in the Earth's magnetosphere. Satellites recorded a huge stream of plasma. The hurricane originated over the North Pole, its diameter reached 1000 kilometers.

They are not uncommon on other planets either: Jupiter and Saturn in particular are extremely turbulent places, not to mention seething plasma tornadoes in the Sun's atmosphere.

It developed at an altitude of 110 to 860 km and consisted of plasma with many spiral arms rotating counterclockwise at speeds up to 2100 m per second. He is a huge aurora. It lasted almost eight hours.

The rain of charged particles into the ionosphere from the solar wind is what usually causes glowing green auroras at Earth's higher latitudes, but solar conditions were relatively calm at the time.

These storms are associated with a huge amount of energy, and these space hurricanes must be caused by an unusually large and rapid transfer of energy from the solar wind and charged particles into the Earth's upper atmosphere. The reconnecting interplanetary magnetic field can create the features they observed in a cosmic

hurricane, even with a weak solar wind. In fact, a weak solar wind could be the key – it provides efficient magnetic reconnection.

This means that such storms can be fairly common. Plasma and magnetic fields in planetary atmospheres exist throughout the universe, so the results suggest that space hurricanes must be a widespread occurrence.

It also shows that even when geomagnetic conditions are relatively calm, space can cause extreme weather.

We look at the model of the plasma effect in the formation of vortexes in the vicinity of the poles of the planets of the solar system.

The solar wind is a plasma flow. It consists mainly of electrons, protons and helium ions. It is controlled by a magnetic field.

As a result of the effect of the solar wind, the stream of charged particles flies to the pole of the planet along converging magnetic field lines. Since the electrons move along the magnetic field, the electron stream compression, relative transverse polarization of the ion and electron flows occurs, and a crossed configuration of the transverse electric and longitudinal/vertical magnetic fields occurs. There is a relative drift of ions and electrons. Let us show that, depending on the degree of nonequilibrium, different numbers of vortices arise on different planets.

Accounting for collisions leads to entrainment of the gas into the vortex motion, as well as to the appearance of a threshold and a decrease in the instability growth rate. Different parameters on different planets: different magnetic field. Some planets do not have aurorae, because a very weak magnetic field, and almost no atmosphere, and therefore plasma.

Unlike polar lights of earth, the Jupiter auroras are shifted to the ultraviolet and x-ray of the spectrum. An

amazing feature of the Jupiter's aurora is that they occur not only because of the solar wind, but also because of the particle fluxes emitted by satellites of the planet: Io, Ganymede and Europe. The presence of Io is especially strongly affected, since it is volcanically active and has its own ionosphere. The solar wind together with Io has a great influence on the occurrence of auroras on Jupiter. Electron bunches with a density of $\sim 10^2 \text{ cm}^{-3}$ or more are injected into the magnetosphere of Jupiter. The density of electrons in the ionosphere of Io reaches $10^4 \dots 10^5 \text{ cm}^{-3}$, and along the orbit of Io in the gas torus to 10^3 cm^{-3} . Accelerated by the Io potential, fast electron bunches move to the magnetic poles of Jupiter along the "Io magnetic tube," which narrows to Jupiter. It is important that the bunch electron density n_b increases from $10^2 \dots 10^3$ near Io to $10^4 \dots 10^5 \text{ cm}^{-3}$ near Jupiter. Thus there are powerful continuous auroras at both poles in a wide frequency range, including X-ray. Electron bunches are formed and move to Jupiter. The electron beam at its moving to Jupiter is focused, and plasma density increases near Jupiter.

The bunches of fast electrons of density $10^4 \dots 10^5 \text{ cm}^{-3}$ are injected into the ionosphere at those heights where the ionospheric density is approximately compared with the density of the bunch $n_b \approx n_{pe}$. Largest electron density 10^5 cm^{-3} belongs to a height 1000 km.

Thus since the magnetic lines of Jupiter are directed to its poles, the flow is focused to Jupiter, and the flow density increases.

Passage of Juno over the polar region has shown that electrons are accelerated towards Jupiter to energies of 0.4 MeV. High-energy particles directed by a magnetic field enter the atmosphere in the region of the poles. Due to the more powerful magnetic field of Jupiter than that of the Earth, the auroras on Jupiter are more intense in hundreds times than on Earth.

In this paper the vortex dynamics of plasma electrons are considered in the near the poles of planets of the solar system. The possibility of vortex structures formation due to the interaction of charged particle flow with the planet's ionosphere plasma is discussed.

The influence of Io on Jupiter's magnetosphere has been studied for a long time [1, 2]. This active satellite is constantly erupting particles into the ionosphere of Jupiter. The interaction between Io and Jupiter occurs in the form of electric currents that moves from Io along the Jupiter magnetic field lines and close through the ionosphere near the planet poles, forming Io magnetic flux tube. Besides the projections of other satellites are also visible, but of much lower brightness. Io effect is more intense in northern areas [3, 4], where the magnetic field is stronger than in the southern hemisphere. Jupiter's equatorial field strength is 4.3 gauss, and ranging from 10 gauss at the south pole to 14 gauss at the north pole.

Since 2016 NASA's Juno spacecraft has been orbiting Jupiter, equipped with devices to compile detailed information about the planet. Based on these data, new effects were discovered, for example, the observations report of distinct, high-energy, discrete electron acceleration in Jupiter's auroral polar regions and also about upward magnetic-field-aligned electric potentials of up to 400 keV, an order of magnitude larger than the largest potentials observed at Earth. Also Juno's Energetic

particle Detector Instrument detected intense electron beams moving away from Jupiter's polar regions. It has found a correlation between the swirl emergence from Ultraviolet Spectrograph and the very intense beams from Io [5 - 7].

1. VORTEX IN THE PLANE (r, θ) IN A MAGNETIZED PLASMA

So, there is a nonequilibrium: the azimuthal drift of electrons relative to ions in a crossed transverse polarization electric field and a longitudinal/vertical magnetic field. Polarization is created by a stream of charged particles in a converging magnetic field. This nonequilibrium leads to instability in the formation of ordered vortices.

Now we consider the vortex dynamics [8 - 18] of a plasma electrons near poles of planets in a plane orthogonal to the magnetic field. Since at some height as a result of the relative polarization of electrons and ions and the focusing of the electron beam in a converging magnetic field there is a crossed configuration of the radial electric E_{or} and longitudinal/vertical magnetic field H_θ , then vortices can also be formed in the plane (r, θ), because the nonequilibrium state is maintained due to the drift of electrons along the angle θ with a velocity

$$V_{\theta 0} = -\frac{eE_{or}}{m_e \omega_{He}} = \left(\frac{\omega_{pe}^2}{2\omega_{He}} \right) \left(\frac{\Delta n}{n_{oe}} \right) r \equiv r\omega_{\theta 0}, \quad (1)$$

$$\Delta n \equiv n_{oe} - q_i n_{oi} / e, \quad (2)$$

where q_i, n_{oi} – ion charge and density.

Consider the vorticity α , electron vortex characteristic

$$\alpha \equiv \vec{e}_z \text{rot} \vec{V} = \frac{1}{r} \partial_r r V_\theta - \frac{1}{r} \partial_\theta V_r. \quad (3)$$

The physical sense of α becomes obvious if we introduce the angular velocity of electron rotation in a vortex $\Omega \equiv \frac{V_\theta}{r}$, then

$$\alpha = 2\Omega + r \partial_r \Omega - \frac{1}{r} \partial_\theta V_r. \quad (4)$$

If $\Omega \neq \Omega(r)$ and $V_r = 0$, then the vorticity is equal to the double angular velocity of electron rotation, $\alpha = 2\Omega$.

2. NONLINEAR EQUATION DESCRIBING THE EXCITATION OF VORTEXES NEAR POLES OF PLANETS

Let us obtain a nonlinear equation that describes the excitation and properties of vortexes near poles of planets. We use the hydrodynamic equations for electrons at times shorter than the trap with taking into account electron collisions

$$\begin{aligned} \frac{\partial \vec{V}}{\partial t} + v_e \vec{V} + (\vec{V} \nabla) \vec{V} &= \left(\frac{e}{m_e} \right) \vec{\nabla} \varphi + [\vec{\omega}_{He}, \vec{V}] - \left(\frac{V_{th}^2}{n_e} \right) \vec{\nabla} n_e, \\ \frac{\partial n_e}{\partial t} + \vec{\nabla} (n_e \vec{V}) &= 0 \end{aligned} \quad (5)$$

and the Poisson equation for the electric potential φ

$$\Delta\varphi = 4\pi(en_e - q_i n_i). \quad (6)$$

Here \vec{V} , n_e – electron velocity and density, V_{th} – thermal electron velocity, \vec{V}_i , n_i , q_i – velocity, density, charge of the ions. As we will see below the dimensions of vortex disturbances are much larger than the Debye electron radius $r_{de} \equiv \frac{V_{th}}{\omega_{pe}}$, then we can neglect the last term in (5).

We obtain a unified nonlinear equation describing the vortex dynamics of electrons. For this we use rot for (5), i.e. we act vectorally by the operator $\vec{\nabla} \times$ on (5). Then we get

$$\frac{\partial \vec{\alpha}}{\partial t} + [\vec{\nabla} \times (\vec{V} \vec{\nabla}) \vec{V}] = [\vec{\nabla} \times [\vec{\omega}_{He} \times \vec{V}]]. \quad (7)$$

Here $\vec{\alpha} = [\vec{\nabla} \times \vec{V}]$. To transform the last equation we use the expression

$$\begin{aligned} [\vec{\nabla} \times [\vec{\omega}_{He} \times \vec{V}]] &= (\vec{\nabla} \vec{V}) \vec{\omega}_{He} - (\vec{\nabla} \vec{\omega}_{He}) \vec{V} = \\ &= \vec{\omega}_{He} (\vec{\nabla} \vec{V}) + (\vec{\nabla} \vec{\omega}_{He}) \vec{V} - (\vec{\omega}_{He} \vec{\nabla}) \vec{V} \end{aligned} \quad (8)$$

at $\vec{\nabla} \vec{\omega}_{He} = 0$ and

$$[\vec{\nabla} \times [\vec{\nabla} \times \vec{V}]] = [\vec{\nabla} \times \vec{\alpha}] = 0.5 \vec{\nabla} V^2 - (\vec{V} \vec{\nabla}) \vec{V}. \quad (9)$$

From (9) we get the expression

$$\begin{aligned} [\vec{\nabla} \times (\vec{V} \vec{\nabla}) \vec{V}] &= -[\vec{\nabla} \times [\vec{V} \times \vec{\alpha}]] = -(\vec{\nabla} \vec{\alpha}) \vec{V} + (\vec{\nabla} \vec{V}) \vec{\alpha} = \\ &= (\vec{V} \vec{\nabla}) \vec{\alpha} + \vec{\alpha} (\vec{\nabla} \vec{V}) - (\vec{\alpha} \vec{\nabla}) \vec{V} - \vec{V} (\vec{\nabla} \vec{\alpha}). \end{aligned} \quad (10)$$

From (7), (8), (10) and $\vec{\nabla} \vec{\alpha} = 0$ we find

$$\begin{aligned} \partial_t \vec{\alpha} + (\vec{V} \vec{\nabla}) \vec{\alpha} + \vec{\alpha} (\vec{\nabla} \vec{V}) - (\vec{\alpha} \vec{\nabla}) \vec{V} &= \\ = \vec{\omega}_{He} (\vec{\nabla} \vec{V}) + (\vec{\nabla} \vec{\omega}_{He}) \vec{V} - (\vec{\omega}_{He} \vec{\nabla}) \vec{V}. \end{aligned} \quad (11)$$

Hence we have

$$\begin{aligned} d_t (\vec{\alpha} - \vec{\omega}_{He}) + v_e \vec{\alpha} + (\vec{\alpha} - \vec{\omega}_{He}) (\vec{\nabla} \vec{V}) &= \\ = ((\vec{\alpha} - \vec{\omega}_{He}) \vec{\nabla}) \vec{V}, \end{aligned} \quad (12)$$

$$d_t \equiv \partial_t + (\vec{V} \vec{\nabla}).$$

We transform the third term of the left side of (12) as follows

$$\begin{aligned} (\vec{\alpha} - \vec{\omega}_{He}) (\vec{\nabla} \vec{V}) &= -(\vec{\alpha} - \vec{\omega}_{He}) \frac{1}{n_e} [\partial_t + (\vec{V} \vec{\nabla})] n_e = \\ &= -(\vec{\alpha} - \vec{\omega}_{He}) \frac{1}{n_e} d_t n_e. \end{aligned} \quad (13)$$

From (12), (13) we find

$$d_t \vec{W} + v_e \frac{\vec{\alpha}}{n_e} = (\vec{W} \vec{\nabla}) \vec{V}, \quad (14)$$

where $\vec{W} \equiv \frac{\vec{\alpha} - \vec{\omega}_{He}}{n_e}$. Thus we have obtained a nonlinear vector equation describing the vortex dynamics of electrons.

From (14) one can derive the linear dispersion relation, describing the vortex instability development. From this dispersion relation one can show that at

$$\left(\frac{\Delta n}{n_{oe}} \right) \left(\frac{r}{\omega_{He}} \right) \omega_{pe}^2 \left| \partial_r \left(\frac{1}{\omega_{He}} \right) \right| \ll \frac{m_e}{m_i} \quad (15)$$

the number

$$\ell_\theta = \frac{\omega_{pi}}{\omega_\theta} = 2 \sqrt{\frac{m_e}{m_i}} \left(\frac{\omega_{He}}{\omega_{pe}} \right) \left(\frac{n_{oe}}{\Delta n} \right) \gg 1 \quad (16)$$

of quick vortexes are formed

$$\begin{aligned} V_{ph} &\approx V_\theta, \quad \omega = \omega^{(o)} + \delta\omega, \quad \omega^{(o)} = \omega_{pi} = \ell_\theta \omega_\theta, \\ \omega_\theta &= \left(\frac{\omega_{pe}^2}{2\omega_{He}} \right) \left(\frac{\Delta n}{n_{oe}} \right), \quad \Delta n \equiv n_{oe} - \frac{q_i n_{oi}}{e}, \quad \delta\omega = i\gamma_q, \\ \gamma_q &\approx \left(\frac{\omega_{pe}}{k} \right) \left[\left(\frac{\omega_{pi}}{2} \right) \left(\frac{\ell_\theta}{r} \right) \left| \partial_r \left(\frac{1}{\omega_{He}} \right) \right| \right]^{1/2}. \end{aligned} \quad (17)$$

From dispersion relation one can show that if the condition (15) is not satisfied the number

$$\ell_\theta \gg 2 \left(\frac{\omega_{pi} \omega_{He}}{\omega_{pe}^2} \right) \left(\frac{n_{oe}}{\Delta n} \right) \quad (18)$$

of slow vortexes are formed

$$\begin{aligned} V_{ph} &\ll V_\theta, \\ k^2 &= - \left(\frac{1}{V_\theta} \right) \omega_{pe}^2 \partial_r \left(\frac{1}{\omega_{He}} \right), \quad Re \omega_s = \frac{\gamma_s}{\sqrt{3}}, \\ \gamma_s &\approx \left(\frac{\sqrt{3}}{2^{4/3}} \right) \omega_{pi}^2 \left[-V_\theta \omega_{pe}^2 \partial_r \left(\frac{1}{\omega_{He}} \right) \right]^{1/6}. \end{aligned} \quad (19)$$

From (5) one can derive the equation,

$$\vec{V}_\perp = - \left(\frac{e}{m_e \omega_{He}} \right) [\vec{e}_z, E_{ro}] + \left(\frac{e}{m_e \omega_{He}} \right) [\vec{e}_z, \vec{\nabla}_\perp \phi], \quad (20)$$

describing

$$\begin{aligned} \frac{d\theta_l}{dt} &= \left(\frac{\Delta n}{n_{oe}} \right) \left(\frac{\omega_{pe}^2}{2} \right) \left[\frac{1}{\omega_{He}(r)} - \frac{1}{\omega_{He}(r_q)} \right] + \left(\frac{e}{rm_e \omega_{He}} \right) \partial_r \phi, \\ \frac{d\theta}{dt} &= \frac{d\theta_l}{dt} + \omega_{ph}, \quad \delta r \equiv r - r_q \end{aligned} \quad (21)$$

oscillatory dynamics of electrons in the quick vortexes in the case of small amplitude of vortex-hole

$$(\delta r)^2 - \frac{2\omega_{He}(r_q)\phi}{\pi e \Delta n r_q (\partial_r \omega_{He})|_{r=r_q}} = const \quad (22)$$

and the radial size of the vortex-hole

$$\delta r_h \approx 2 \left[\frac{\phi_o \omega_{He}(r_q)}{\pi e \Delta n r_q (\partial_r \omega_{He})|_{r=r_q}} \right]^{1/2} \quad (23)$$

and in the case of large amplitude of vortex

$$\delta r = \pm \left[\frac{2(\phi + \phi_o)\omega_{He}(r_q)}{\pi e \Delta n r_q (\partial_r \omega_{He})|_{r=r_q}} + (\delta r_{cl})^2 \right]^{1/2}, \quad (24)$$

$$\delta r_q = \left[\frac{4\phi_o \omega_{He}(r_q)}{\pi e \Delta n r_q (\partial_r \omega_{He})|_{r=r_q}} + (\delta r_{cl})^2 \right]^{1/2}, \quad (25)$$

δr_{cl} is the radial width of the vortex-bunch of the electrons.

CONCLUSIONS

The Juno measurements of the aurora reveal valuable information about the precipitating particle population, which interact with the Jovian ionosphere plasma at varying altitudes. Fine structure is observed on scale of approximately tens of kilometers. The research of vortex structure formation mechanism can give an understanding of the reasons for these morphologies and of Jovian auroral processes.

In this paper the vortex electron dynamics is considered, in particular, in the framework of Io – Jupiter interaction. Currents flow from Io along the magnetic field lines and closed in the ionosphere near the poles. Particles impact Jovian plasma and atmosphere.

The conditions of vortex formation, their properties and dynamics in the crossed configuration of the radial electric and longitudinal magnetic fields are described. The nonlinear vector equation is used that describes the vortex dynamics of electrons. It is also analyzed how the vortex motion depends on the occurrence of electron density perturbations.

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СТРУКТУРИ ВИХОРІВ БЛЯ ПОЛЮСІВ ПЛАНЕТ СОНЯЧНОЇ СИСТЕМИ

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В околицях полюсів планет Сонячної системи спостерігаються впорядковані кристали вихрів і перемежування. Досліджено утворення вихрових структур поблизу полюсів планет Сонячної системи. Розглянуто умови утворення вихорів. Знайдено нелінійне рівняння, що описує динаміку вихорів, досліджено їх структури.