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WAKE EXCITATION OF PLASMA AND ELECTROMAGNETIC OSCILLATIONS BY A RELATIVISTIC ELECTRON BUNCH IN A PLASMA RESONATOR

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The process of the excitation of plasma and electromagnetic oscillations by a relativistic electron bunch in a plasma resonator is considered. It is shown that the potential electric field of plasma oscillations contains the field of a bulk wake plasma wave and the fields of two surface plasma oscillations. Surface plasma oscillations are localized in the vicinity of the input and output ends of the plasma resonator. In a plasma resonator, as a result of the effect of transition radiation at the ends of the resonator, the electron bunch will also excite eigen electromagnetic oscillations of the resonator. The spatio-temporal structure of the electromagnetic field of these oscillations has been studied.

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INTRODUCTION

The influence of the transverse boundedness of the plasma on the process of wakefield excitation by a relativistic electron bunches has been studied for the case of plasma waveguides [1, 2]. Meanwhile, the plasma is always limited not only in the transverse direction, but also in the longitudinal one. In this work, using the example of the plasma placed in a perfectly conducting metal resonator, we study the influence of the longitudinal plasma boundaries on the pattern of the field of excited wake plasma oscillations by a relativistic electron bunch.

In case of a plasma resonator, the process of wake plasma oscillations excitation by a relativistic electron bunch will always be accompanied by the excitation of eigen electromagnetic oscillations of the resonator. The process of excitation of these oscillations is based on the effect of coherent transition radiation of a relativistic electron bunch [3], which crosses the ideally conducting input and output ends of the resonator. Attention is also paid to the transient excitation of eigen electromagnetic oscillations in the plasma resonator.

1. STATEMENT OF THE PROBLEM. BASIC EQUATIONS

The plasma resonator is made in the form of a perfectly conducting metal resonator, the volume of which is completely filled with a homogeneous isotropic cold plasma. Along the axis, an axisymmetric relativistic electron bunch moves uniformly and rectilinearly. For simplicity, we will consider a relativistic electron bunch in the form of an infinitely thin charged ring. The current density of such a bunch has the form

$$\vec{j}_b = -\vec{e}_z Q \frac{\delta(r-r_0)}{2\pi r_0}\delta\left(t - z/v_0\right), \qquad (1)$$

where $\vec{e}_z$ is unit vector in the direction of the longitudinal axis z, Q is total bunch charge. r is radial coordinate, r_0 is ring bunch radius, z is longitudinal coordinate, t is time, v_0 is electron bunch velocity.

It is convenient to reduce the original inhomogeneous system of Maxwell's equations to the following equations for determining the excited wake electric field

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{D}}{\partial^2 t^2} = \frac{4\pi}{c^2}\frac{\partial\vec{j}_b}{\partial t} + \vec{\nabla}(\vec{\nabla}\vec{E}), \qquad (2)$$

$$div\vec{D} = 4\pi\rho_b ,$$

$\vec{D} = \hat{\varepsilon}\vec{E}$, $\hat{\varepsilon}$ is integral operator of plasma permittivity. The current and charge densities of the electron bunch are in the following relation

$$\vec{j}_b = \vec{e}_z v_0 \rho_b .$$

We expand the quantities in the Maxwell equations into the Fourier integrals over frequencies

$$\left(\vec{E},\vec{H}\right) = \int_{-\infty}^{\infty}\left(\vec{E}_\omega,\vec{H}_\omega\right)e^{-i\omega\bar{t}}d\omega,$$

where $\bar{t} = t - z/v_0$,

$$\left(\vec{E}_\omega,\vec{H}_\omega\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\vec{E}(t),\vec{H}(t)\right)e^{i\omega\bar{t}}dt .$$

The system of equations (2) for the longitudinal Fourier component of the electric field can be transformed to the equation

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial E_{z\omega}}{\partial r} + \frac{\partial^2 E_{z\omega}}{\partial z^2} + k_0^2\varepsilon E_{z\omega} = \frac{i\kappa_\perp^2 Q}{\pi\varepsilon k_0 c}\frac{\delta(r-r_0)}{r_0}e^{ik_l z}, \qquad (3)$$

where $\kappa_\perp^2 = k_0^2\varepsilon(\omega) - k_l^2$, $k_l = \omega/v_0$, $k_0 = \omega/c$,

$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$ is plasma permittivity.

On the perfectly conducting side surface of the plasma resonator $r = b$, the longitudinal component of the electric field vanishes

$$E_{z\omega}(r=b) = 0 . \qquad (4)$$

The solution of the partial differential equation (3), which satisfies the boundary condition (4), is convenient to find in the form of a series in Bessel functions

$$E_{z\omega} = \sum_{n=1}^{\infty}E_{z\omega n}(z)J_0(\lambda_n r/b), \qquad (5)$$

where b is radius of the resonator, λ_n are roots of the Bessel function $J_0(x)$. Then from the partial differential equation (3) we obtain the following ordinary differential equation

$$\frac{d^2 E_{z\omega n}}{dz^2} + k_n^2 E_{z\omega n} = \frac{i\kappa_\perp^2}{\pi\varepsilon k_0 c}Q\frac{J_0(\lambda_n r_0/b)}{N_n}e^{ik_l z}, \qquad (6)$$

$k_n^2 = k_0^2\varepsilon - \lambda_n^2 / b^2$ is the square of the longitudinal wave number of the electromagnetic wave of the plasma waveguide forming the plasma resonator,

$$N_n = \frac{b^2}{2} J_1^2(\lambda_n)$$

is eigen electromagnetic wave norm. The total solution of the linear inhomogeneous equation (6) has the form

$$E_{z\omega n}(z) = E_n^{(+)} e^{ik_n z} + E_n^{(-)} e^{-ik_n z} + \\ + \frac{i\kappa_\perp^2}{\pi\varepsilon k_0 c} Q \frac{1}{k_n^2 - k_l^2} \frac{J_0(\lambda_n r_0 / b)}{N_n} e^{ik_l z} . \quad (7)$$

Here $E_n^{(\pm)}$ are constants which should be determined. Accordingly, for the expansion coefficients of the radial component of the electric field in terms of Bessel functions

$$E_{r\omega} = \sum_{n=1}^{\infty} E_{r\omega n}(z) J_1(\lambda_n r / b) \quad (8)$$

we have the following expression

$$E_{r\omega n}(z) = -i\frac{k_n b}{\lambda_n} (E_n^{(+)} e^{ik_n z} + E_n^{(-)} e^{-ik_n z}) + \Gamma_n e^{ik_n z} , \quad (9)$$

$$\Gamma_n = \frac{\lambda_n}{\pi b v_0 \varepsilon} Q \frac{1}{k_n^2 - k_l^2} \frac{J_0(\lambda_n r_0 / b)}{N_n} .$$

On the perfectly conducting ends of the plasma resonator, which have longitudinal coordinates of the input end $z = 0$ and output end $z = L$, L is the length of the resonator, the radial component of the electric field vanishes. Accordingly, all coefficients (9) of expansion (8) also vanish

$$E_{r\omega n}(z = 0) = E_{r\omega n}(z = L) = 0 .$$

From these conditions we find constants $E_n^{(\pm)}$ and, accordingly, expressions for the Fourier expansion coefficients $E_{z\omega n}(z), E_{r\omega n}(z)$. Ultimately, for the expansion coefficients of the longitudinal and radial components of the electric field, we obtain the following integral Fourier representations

$$E_{zn}(z,t) = \frac{2Q}{\pi b^2 v_0} \sigma_n \int_{-\infty}^{\infty} \frac{e^{-i\omega t} d\omega}{\varepsilon(\omega)(k_n^2 - k_l^2)} \Pi_z(\omega, z) , \quad (10)$$

$$E_{rn}(z,t) = \frac{2Q}{\pi b^2 v_0} \sigma_n \int_{-\infty}^{\infty} \frac{e^{-i\omega t} d\omega}{\varepsilon(\omega)(k_n^2 - k_l^2)} \Pi_r(\omega, z), \quad (11)$$

where

$$\sigma_n = \frac{J_0\left(\lambda_n \frac{r_0}{b}\right)}{J_1^2(\lambda_n)} ,$$

$$\Pi_z(\omega, z) = \frac{\lambda_n^2}{k_n b^2} \frac{1}{\sin k_n L} \left[\cos k_n(z - L) - e^{ik_n L} \cos k_n L\right] + \\ + \frac{i}{k_l} \left(k_0^2 \varepsilon - k_l^2\right) e^{ik_l z} ,$$

$$\Pi_r(\omega, z) = \frac{\lambda_n}{b} \left\{ \frac{1}{\sin k_n L} \left[\sin k_n(z - L) - e^{ik_n L} \sin k_n L\right] + \\ + e^{ik_l z} \right\} .$$

The integrands in expressions (10), (11) have three groups of simple poles. First of all, these are the poles

$$\omega = \pm\omega_p - i0 , \quad (12)$$

which are the roots of the plasma permittivity

$$\varepsilon(\omega) = 0$$

and correspond to the wake field of plasma oscillations, excited by a relativistic electron bunch moving in the volume of the plasma resonator. The poles (12) are located in the lower half-plane of the complex variable ω near the real axis. The poles

$$\omega = \pm\omega_{nm} - i0 , \quad (13)$$

determined from the equation

$$k_n L = \pi m ,$$

$m = 1, 2, 3, \ldots$, correspond to the frequencies of eigen electromagnetic oscillations of the plasma resonator

$$\omega_{nm} = \sqrt{\omega_p^2 + c^2 \left(\frac{\lambda_n^2}{b^2} + \frac{\pi^2 m^2}{L^2} \right)} .$$

These poles are also located in the lower half-plane near the real axis. And finally, the roots of the equation

$$k_0^2 \varepsilon - \frac{\lambda_n^2}{b^2} - k_l^2 = 0$$

determine the complex conjugated poles located on the imaginary axis

$$\omega = \pm i\omega_{nst} ,$$

where $\omega_{nst} = \beta_0 \gamma_0 \omega_{nc}$, $\omega_{nc} = \sqrt{\omega_p^2 + \lambda_n^2 c^2 / b^2}$ is the cut-off frequency of the plasma waveguide forming the resonator, $\beta_0 = v_0 / c$, γ_0 is the relativistic factor. These poles are responsible for the own quasi-static electromagnetic field of the relativistic electron bunch in the plasma resonator.

2. EXCITATION OF WAKE PLASMA OSCILLATIONS

The components of the total wakefield excited in the plasma resonator by a relativistic electron bunch are described by Fourier frequency integrals (10), (11). The calculation of the residues in the poles (12) gives analytical expressions for the expansion coefficients of the components (10), (11) of the wakefield and, ultimately, the final expressions for the components of the electric field of plasma oscillations

$$E_r^{(l)} = \frac{1}{2} E_0 k_p b G_1(r, r_0) \vartheta(t - z / v_0) \sin\omega_p (t - z / v_0) - \\ - E_0 \left[W_1(r, L - z) \vartheta(t - z / c) \sin\omega_p t + \\ + W_1(r, z) \vartheta\left(t - \frac{L}{v_0} + \frac{z - L}{c} \right) \sin\omega_p \left(t - \frac{L}{v_0} \right) \right], \quad (14)$$

$$E_z^{(l)} = \frac{1}{2} E_0 k_p b G_0(r, r_0) \vartheta(t - z / v_0) \cos\omega_p (t - z / v_0) - \\ - E_0 \left[W_0(r, L - z) \vartheta(t - z / c) \sin\omega_p t + \\ - W_0(r, z) \vartheta\left(t - \frac{L}{v_0} + \frac{z - L}{c} \right) \sin\omega_p \left(t - \frac{L}{v_0} \right) \right]. \quad (15)$$

Here $E_0 = 4Qk_p / b$, $k_p = \omega_p / v_0$, $\vartheta(x)$ is unit function

$$\vartheta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases}$$

$$W_0(r,y)=\sum_{n=1}^{\infty}S_nJ_0\left(\lambda_n\frac{r}{b}\right)\frac{ch\left(\lambda_n\frac{y}{b}\right)}{sh\left(\lambda_n\frac{L}{b}\right)},$$

$$W_1(r,y)=\sum_{n=1}^{\infty}S_nJ_1\left(\lambda_n\frac{r}{b}\right)\frac{sh\left(\lambda_n\frac{y}{b}\right)}{sh\left(\lambda_n\frac{L}{b}\right)},\quad L\ge y\ge 0,$$

$$S_n=\frac{J_0\left(\lambda_n r_0/b\right)}{J_1^2(\lambda_n)}\frac{\lambda_n}{\lambda_n^2+k_p^2b^2},$$

$$G_0(r,r_0)=\begin{cases}I_0(k_pr_0)\Delta_0(k_pb,k_pr), & r\ge r_0,\\ I_0(k_pr)\Delta_0(k_pb,k_pr_0), & r\le r_0,\end{cases}$$

$$\Delta_0(k_pb,k_pr)=I_0(k_pb)K_0(k_pr)-I_0(k_pr)K_0(k_pb),$$

$$G_1(r,r_0)=\begin{cases}I_0(k_pr_0)\Delta_1(k_pb,k_pr), & r\ge r_0,\\ -I_1(k_pr)\Delta_0(k_pb,k_pr_0), & r\le r_0,\end{cases}$$

$$\Delta_1(k_pb,k_pr)=I_0(k_pb)K_1(k_pr)+I_1(k_pr)K_0(k_pb).$$

For large values of the argument $y\ge b$, the functions $W_{0,1}(r,y)$ have asymptotic representations

$$W_\alpha(r,y)=\sum_{n=1}^{\infty}S_nJ_\alpha\left(\lambda_n\frac{r}{b}\right)e^{-\lambda_n\frac{L-y}{b}}.$$

It is easy to verify that the radial component of the electric field of the wake plasma oscillations at each moment of time satisfies the boundary conditions at the perfectly conducting ends of the plasma resonator. The unit functions included in the expressions for the components of the wakefield (14), (15) describe the motion of the wave fronts of the wakefield. Note that the total wakefield electric field is potential.

Expressions (13), (14) for the components of the wakefield of plasma oscillations excited by a relativistic electron bunch in a cylindrical plasma resonator with perfectly conducting walls contain three terms. The first term describes the volume wakefield of plasma oscillations. This field has the same spatio-temporal structure and amplitude as in a longitudinally infinite plasma waveguide. A monochromatic plasma wave propagates behind the electron bunch with a phase velocity which is equal to the bunch velocity. When the bunch leaves the resonator, the bulk wake wave exists along the entire length of the resonator. The electric field of only a traveling plasma wave cannot satisfy the boundary conditions at the perfectly conducting ends of the cavity at any time. That is why, there are two more terms in the expressions the wakefield components (14), (15) in the plasma resonator. Each of them has a simple physical meaning. The first term in each of the expressions describes the transition electromagnetic radiation that occurs when the electron bunch crosses the input end of the plasma cavity. The second term describes surface plasma oscillations located near the output end. The amplitude of these oscillations decreases with distance from the input end into the plasma volume. On the surface of the input end, the amplitude of the radial component of the electric field of surface plasma oscillations is equal to the amplitude of the bulk plasma wake wave at this end, but is always opposite in phase. Therefore, the total radial electric field on the surface of the input end at each moment of time is identically equal to zero. The leading front of the surface wave propagates in the volume of the resonator at the speed of light and slightly overtakes the relativistic electron bunch. At the moment of time, the leading front of the surface oscillations approaches to the output end and a stationary configuration of the input surface oscillations is established. At the moment of time when the electron bunch crosses the output end, the second surface plasma wave is excited, which is localized near to the output end of the plasma resonator. This surface field of plasma oscillations ensures the fulfillment of the boundary condition at the output end of the plasma resonator. The leading front of this surface wave also propagates at the speed of light from the output end to the input end. When the leading front of this wave reaches the input end at the moment of time $t=L/v_0+L/c$, in the plasma resonator the stationary structure of the wake field of plasma oscillations is established, which is described by the following expressions for the components of the electric field

$$E_r^{(l)}=\frac{1}{2}E_0k_pbG_1(r,r_0)\sin\omega_p(t-z/v_0)- \qquad (16)$$

$$-E_0\left[W_1(r,L-z)\sin\omega_pt+W_1(r,z)\sin\omega_p\left(t-\frac{L}{v_0}\right)\right],$$

$$E_z^{(l)}=\frac{1}{2}E_0k_pbG_0(r,r_0)\cos\omega_p(t-z/v_0)- \qquad (17)$$

$$-E_0\left[W_0(r,L-z)\sin\omega_pt-W_0(r,z)\sin\omega_p\left(t-\frac{L}{v_0}\right)\right].$$

Let us consider a special case when the time of flight of an electron bunch through a plasma resonator is a multiple of an even number of half-periods of plasma oscillations

$$\frac{L}{v_0}=2l\frac{\pi}{\omega_p},\quad l=0,1,2....$$

This condition is also equivalent to the requirement that an even number of spatial half-periods of the bulk wake plasma wave fit within the resonator length. In this case, from (16), (17) the following expressions follow for the components of the electric field of surface plasma oscillations

$$E_{rs}^{(l)}=-E_0U_1^{(sm)}(r,x)\sin\omega_pt,$$

$$E_{zs}^{(l)}=-E_0U_0^{(as)}(r,x)\sin\omega_pt,$$

where $x=z-L/2$,

$$U_1^{(sm)}(r,x)=W_1(r,L/2-x)+W_1(r,L/2+x)\equiv$$

$$\equiv\sum_{n=1}^{\infty}S_nJ_1\left(\lambda_n\frac{r}{b}\right)\frac{ch\left(\lambda_n\frac{x}{b}\right)}{ch\left(\lambda_n\frac{L}{2b}\right)}, \qquad (18)$$

$$U_0^{(as)}(r,x)=W_0(r,L/2-x)-W_0(r,L/2+x)\equiv$$

$$\equiv\sum_{n=1}^{\infty}S_nJ_0\left(\lambda_n\frac{r}{b}\right)\frac{sh\left(\lambda_n\frac{x}{b}\right)}{ch\left(\lambda_n\frac{L}{2b}\right)}. \qquad (19)$$

The distribution of the radial component of the electric field of surface plasma oscillations along the axis of the plasma resonator has symmetrical form $U_1^{(sm)}(r,x)=U_1^{(sm)}(r,-x)$ with respect to the central plane $z=L/2,(x=0)$. The dependence of the amplitude of each radial harmonic (18) of the surface plasma wave is described by the function

$$C_n^{(sm)}(x)=\frac{ch\left(\lambda_n x/b\right)}{ch\left(\lambda_n L/2b\right)}.$$

For the values $x=\mp L/2$ corresponding to the input and output ends of the resonator, we have $C_n^{(sm)}(x=\mp L/2)=1$, and for the central plane of the resonator $x=0$ we obtain

$$C_n^{(sm)}(x=0)=1/ch\left(\lambda_n L/2b\right).$$

It follows from this formula that the most pronounced surface character of the field is manifested in the case of a long plasma resonator $L/2b>>1$. In turn, it follows from expressions (18), (20) that the distribution of the longitudinal component of the surface field of plasma oscillations is antisymmetric $U_0^{(as)}(r,-x)=-U_0^{(as)}(r,x)$.

Let us now consider a special case when the time of flight of an electron bunch through a plasma resonator is a multiple of an odd number of half-periods of plasma oscillations

$$\frac{L}{v_0}=(2l+1)\frac{\pi}{\omega_p},\ l=0,1,2....$$

In this case, the expressions for the components of the electric field of surface plasma oscillations have the following form

$$E_{rs}^{(l)}=-E_0U_1^{(as)}(r,x)\sin\omega_p t,$$

$$E_{zs}^{(l)}=-E_0U_0^{(sm)}(r,x)\sin\omega_p t,$$

$$U_1^{(as)}(r,x)=W_1(r,L/2-x)-W_1(r,L/2+x)\equiv$$

$$\equiv\sum_{n=1}^{\infty}S_nJ_1\left(\lambda_n\frac{r}{b}\right)\frac{sh\left(\lambda_n\frac{x}{b}\right)}{ch\left(\lambda_n\frac{L}{2b}\right)}, \qquad (20)$$

$$U_0^{(sm)}(r,x)=W_0(r,L/2-x)+W_0(r,L/2+x)\equiv$$

$$\equiv\sum_{n=1}^{\infty}S_nJ_0\left(\lambda_n\frac{r}{b}\right)\frac{ch\left(\lambda_n\frac{x}{b}\right)}{ch\left(\lambda_n\frac{L}{2b}\right)}.$$

In the case under consideration, the dependence of the radial component of the surface plasma field amplitude in the resonator is antisymmetric with respect to the central plane $z=L/2,(x=0)$, $U_1^{(as)}(r,-x)=-U_1^{(as)}(r,x)$ and a similar dependence for the longitudinal component of the surface field is symmetric $U_0^{(sm)}(r,-x)=U_0^{(sm)}(r,x)$.

Thus, a relativistic electron bunch always excites in a plasma resonator a conglomerate of a bulk plasma wake wave and two surface plasma oscillations localized near the input and output ends of the plasma resonator. Only in combination these oscillations provide the fulfillment of the boundary conditions for the total wake field at the ideally conducting ends of the plasma resonator.

3. TRANSITION EXCITATION OF ELECTROMAGNETIC OSCILLATIONS

When a relativistic electron bunch crosses perfectly conducting ends of a plasma resonator, transition electromagnetic radiation arises. In the frequency Fourier integral describing the total electromagnetic field excited by a relativistic electron bunch in a plasma resonator, poles (13) are responsible for the excitation of eigen electromagnetic oscillations in the plasma resonator. The calculation of residues in these poles gives the following expressions for the components of the field of electromagnetic oscillations

$$E_z^{(em)}=\frac{8Q}{b^2\beta_0}\frac{L}{b}\sum_{n=1}^{\infty}\lambda_nR_nJ_0\left(\lambda_n\frac{r}{b}\right)\sum_{m=0}^{\infty}\frac{L_{nm}}{\varepsilon_{nm}}Z_{nm}(z,t)\times$$

$$\times\cos\left(\pi m\frac{z}{L}\right)\sin\left(\omega_{nm}t\right), \qquad (21)$$

$$E_r^{(em)}=\frac{8\pi Q}{b^2\beta_0}\sum_{n=1}^{\infty}R_nJ_1\left(\lambda_n\frac{r}{b}\right)\sum_{m=1}^{\infty}\frac{mL_{nm}}{\varepsilon_{nm}}Z_{nm}(z,t)\times$$

$$\times\sin\left(\pi m\frac{z}{L}\right)\sin\left(\omega_{nm}t\right), \qquad (22)$$

$$H_\varphi^{(em)}=\frac{8Q}{b^2\beta_0}\sum_{n=1}^{\infty}R_nJ_1\left(\lambda_n\frac{r}{b}\right)\sum_{m=0}^{\infty}\frac{\omega_{nm}L}{c}L_{nm}Z_{nm}(z,t)\times$$

$$\times\cos\left(\pi m\frac{z}{L}\right)\cos\left(\omega_{nm}t\right), \qquad (23)$$

where

$$Z_{nm}(t,z)=\vartheta\left(t-\frac{z}{c}\right)-(-1)^m e^{ik_{nm}L}\vartheta\left(t-\frac{L}{v_0}+\frac{z-L}{c}\right), \quad (24)$$

$$L_{nm}=\frac{c}{\omega_{nm}b}\frac{1}{d_{nm}},\ \varepsilon_{nm}=\varepsilon(\omega_{nm}),\ k_{nm}=\frac{\omega_{nm}}{v_0},$$

$$d_{nm}=k_{nm}^2L^2-\pi^2m^2,\ R_n=\frac{\lambda_nJ_0(\lambda_nr_0/b)}{J_1^2(\lambda_n)}.$$

The function $Z_{nm}(t,z)$ contains two terms and describes the propagation of the fronts of two pulses of transition electromagnetic radiation.

The structure of the transient electromagnetic field and the dynamics of its excitation are described by expressions (21)-(24). As follows from these expressions, at the moment when the electron bunch enters the plasma resonator $t=0$, the first pulse of transition electromagnetic radiation is formed. The leading front of this radiation propagates at the speed of light from the input end of the plasma resonator to the output one. The leading front of the transition radiation pulse is formed by the high-frequency component of the electromagnetic pulse. These frequencies are much higher than the cutoff frequencies of the eigen electromagnetic waves of the plasma waveguide forming the resonator. At the moment when the electron bunch crosses the output end, the second pulse of transition electromagnetic radiation is excited, which propagates from the output end of the plasma resonator to the input one. The phase factor in (24) $e^{i\omega_{nm}L/v_0}$ relates the phases of the transition electromagnetic oscillations to the moment when the electron bunch leaves the plasma resonator. When the leading front of the input end of the plasma resonator is reached, a stationary picture of the electromagnetic oscillations excitation is established at the moment of time $t = L/v_0 + L/c$. In this state, the field of transition electromagnetic radiation is a set of eigen electromagnetic oscillations with constant amplitudes. The factor (24) takes the form

$$Z_{nm} = 1 - (-1)^m e^{ik_{nm}L}.$$

This factor takes into account the effect of interference of electromagnetic oscillations excited by an electron bunch at the input and output ends of the plasma resonator. Oscillations with even indices m (for which the condition $k_{nm}L = 2\pi l$, where l is an integer, is not fulfilled) are not excited. For odd indices m, the total oscillations are completely extinguished at $k_{nm}L = \pi(2l+1)$. On the other hand, the maximum amplification of electromagnetic oscillations takes place for even indices m at $k_{nm}L = \pi(2l+1)$, and for odd m, when $k_{nm}L = 2\pi l$.

As is known [4], the fundamental E type oscillation of the cylindrical resonator is the oscillation with indices $m = 0$, $n = 1$. The eigen frequency of this oscillation is equal to the cutoff frequency of the cylindrical plasma waveguide.

$$\omega_{10} = \sqrt{\omega_p^2 + \lambda_1^2 c^2 / b^2}.$$

The longitudinal component of the electric field and the azimuthal component of the magnetic field are homogenious along the longitudinal axis of the plasma. There is no radial component of the electric field. For the fundamental oscillation, formulas (21)-(23) give expressions for the components of the electromagnetic field.

$$E_{z10} = E_{10} J_0\left(\lambda_1 \frac{r}{b}\right) \sin\omega_{10} t,$$

$$E_{r10} = 0,$$

$$H_{\varphi 10} = E_{10} \frac{\lambda_1}{k_{10} b} J_1\left(\lambda_1 \frac{r}{b}\right) \cos\omega_{10} t,$$

where

$$E_{10} = \frac{8Q}{b^2} \frac{v_0}{\omega_{10} L} Z_{10} \frac{J_0(\lambda_1 r_0 / b)}{J_1^2(\lambda_1)}.$$

Note that the fundamental electromagnetic oscillation of a cylindrical plasma resonator can be used to accelerate relativistic charged particles. Really, if the time of flight of the accelerated particle is equal to the half-period of the fundamental electromagnetic oscillation $\omega_{10} L / v_0 = \pi$, then the particle will be constantly in the accelerating phase during its presence in the plasma resonator. From the equations of motion of a accelerated relativistic particle

$$\frac{dp}{dt} = eE_{10} \sin\omega_{10} t$$

we find that the increasing of the relativistic factor of an accelerated particle during its flight through the plasma resonator is equal

$$\gamma - \gamma_0 = 2a_{10},$$

where

$$a_{10} = \frac{eE_{01}}{mc\omega_{10}} = \frac{16}{\pi} \frac{eQ}{b^2 mc\omega_{10}}$$

is parameter of the electromagnetic oscillation strength. Obviously, when this parameter is $a_{10} >> 1$, then there will be an effective acceleration of relativistic particles by the field of the fundamental electromagnetic oscillation in the plasma resonator.

CONCLUSIONS

The process of excitation of plasma and electromagnetic oscillations by a relativistic electron bunch in a plasma resonator is considered. First of all, the excited field includes the potential field of Langmuir oscillations. In turn, Langmuir oscillations include a bulk wake plasma wave, which has the same spatio-temporal structure as it is in a plasma waveguide that is infinite in the longitudinal direction and used for resonator formation. In addition to the bulk wake wave, two surface plasma oscillations are excited in the plasma resonator. One of them is localized in the vicinity of the input end of the resonator, and the second is localized in the vicinity of the output end. The amplitudes of surface waves decrease with distance from the ends deep into the plasma. Only a conglomerate from the field of a bulk wake wave and the fields of two surface plasma oscillations satisfies the boundary conditions at the ends of the plasma resonator.

In addition to potential plasma oscillations, the electron bunch will also excite a vortex electromagnetic field in the plasma resonator. The analytical expression for the electromagnetic field contains two terms. Each of them has a simple physical meaning. The first term describes the transition electromagnetic radiation that occurs when the electron bunch crosses the input end of the plasma resonator. The second term in the expression for the total electromagnetic field describes the transition electromagnetic radiation excited by the relativistic electron bunch when it crosses the output end of the plasma resonator with perfectly conducting walls. Finally, after the transient process, the field of electromagnetic radiation is established as a superposition of eigen electromagnetic oscillations of the plasma resonator with constant amplitudes.

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КІЛЬВАТЕРНЕ ЗБУДЖЕННЯ ПЛАЗМОВИХ ТА ЕЛЕКТРОМАГНІТНИХ КОЛИВАНЬ РЕЛЯТИВІСТСЬКИМ ЕЛЕКТРОННИМ ЗГУСТКОМ У ПЛАЗМОВОМУ РЕЗОНАТОРІ

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Досліджено процес збудження плазмових та електромагнітних коливань релятивістським електронним згустком у плазмовому резонаторі. Показано, що потенційне електричне поле плазмових коливань містить поле об'ємної кільватерної плазмової хвилі та поля двох поверхневих плазмових коливань. Поверхневі плазмові коливання локалізовані поблизу вхідного та вихідного торців плазмового резонатора. У плазмовому резонаторі в результаті ефекту перехідного випромінювання на торцях резонатора електронний згусток також збуджує власні електромагнітні коливання резонатора. Досліджено просторово-часову структуру електромагнітного поля цих коливань.