https://doi.org/10.46813/2023-146-046

EXCITATION OF TM MODE BY A RELATIVISTIC ELECTRON BEAM IN AN AZIMUTHALLY CORRUGATED WAVEGUIDE

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The excitation of an electromagnetic TM mode by a relativistic electron beam in a waveguide with a sinusoidal-corrugated azimuth conducting wall in a constant uniform external guiding magnetic field is theoretically studied. We consider a thin annular electron beam moving along the waveguide axis and rotating at an equilibrium radius around its axis. In the approximation of a small corrugation depth, the analytical dependences of the growth rate of instability and resonant frequencies on the parameters of the beam and waveguide are determined.

PACS: 41.75.Ht, 52.35.Hr

INTRODUCTION

High-current relativistic electron beams are widely used to produce powerful electromagnetic radiation when interacting with waves propagating in vacuum and plasma waveguides (e.g. see [1 - 5]). This radiation can be used to heat the plasma and accelerate charged particles. Waveguides with periodically inhomogeneous in azimuth inner surface are used to generate an electromagnetic radiation in the devices with a relativistic electron beam, which rotates in an external magnetic field. In this case, an increase in the power of electromagnetic radiation during the interaction of high-current beams with electromagnetic waves requires the use of waveguides with a smooth surface change.

In [6], excitation of a TE mode by a relativistic electron beam moving in an external uniform guiding magnetic field in a waveguide with an ideally conducting inner surface sinusoidally corrugated in azimuth was considered. In this paper, we consider the excitation of a TM mode by a relativistic electron beam moving in such a structure.

1. PROBLEM STATEMENT AND DISPERSION EQUATION

Let us consider the excitation of electromagnetic waves by a relativistic electron beam in a cylindrical waveguide with an inner surface that is periodic in the azimuthal angle. The waveguide radius in a cylindrical coordinate system (r, φ, z) may be expressed as

$$R(\varphi) = R_0[1 + q\cos(N\varphi)], \qquad (1)$$

where R_0 is the mean radius of the waveguide, and N is the positive integer, q<1, $0 \le \varphi \le 2\pi$. We assume that the waveguide is homogeneous along the z axis. There is an external guiding homogeneous magnetic field $\mathbf{H}_0 = \mathbf{e}_z H_0$, where $\mathbf{e}_z$ is the unit vector along the z axis.

We assume that a thin relativistic electron beam of radius r_0 with equilibrium density $n_b = n_S\delta(r - r_0)$ moves in the waveguide, where n_S is the surface charge density, $r_0 \le R_0(1-q)$. The beam rotates around the waveguide axis and moves along z axis with equilibrium velocities $v_{\varphi 0}$ and v_{z0}, respectively.

Where $v_{\varphi 0} = r_0\omega_{H\perp}$, $\omega_{H\perp} = |e|H_0/(mc\gamma_0)$, $\gamma_0 = (1 - \beta_{\varphi 0}^2 - \beta_{z0}^2)^{-1/2}$, $\beta_{\varphi 0} = v_{\varphi 0}/c$, $\beta_{z0} = v_{z0}/c$, e is the electron charge (e<0).

The equation of beam electrons motion in the field of an electromagnetic wave and the guiding magnetic field has the form

$$\frac{d\mathbf{p}}{dt} = e\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}(\mathbf{H}_0 + \mathbf{H})]\right\}, \qquad (2)$$

where $\mathbf{p} = m\mathbf{v}\gamma$, $\mathbf{E}$, $\mathbf{H}$ are the components of the TM mode.

The electric fields due to the electrons of beam is given by

$$\Delta\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial^2 t} = 4\pi\left(\nabla\rho + \frac{1}{c^2}\frac{\partial}{\partial t}\mathbf{j}\right), \qquad (3)$$

where $\rho = en_b$, $\mathbf{j} = en_b\mathbf{v}$ are the charge density and the current density, respectively, satisfying the equation of continuity

$$\frac{\partial\rho}{\partial t} + div\mathbf{j} = 0. \qquad (4)$$

The expression for the longitudinal component of the electric field of the TM mode in the waveguide can be written as

$$E_z(r,\varphi,z,t) = \sum_{n=-\infty}^{\infty} E_{zn}(r)\exp[i(l_n\varphi + k_z z - \omega t)], \qquad (5)$$

where $l_n = s + nN$, s is an integer.

The components of the TM mode can be expressed in terms of the longitudinal electric field as follows:

$$E_r = \frac{ik_z}{k_\perp^2}\frac{\partial}{\partial r}E_z, \qquad E_\varphi = \frac{ik_z}{k_\perp^2 r}\frac{\partial}{\partial\varphi}E_z,$$

$$H_{rn} = -\frac{i\omega}{k_\perp^2 cr}\frac{\partial}{\partial\varphi}E_z, \quad H_{\varphi n} = \frac{i\omega}{k_\perp^2 c}\frac{\partial}{\partial r}E_z,$$

where $k_\perp = \sqrt{\omega^2/c^2 - k_z^2}$.

The displacement of the trajectory and the velocity of electrons relative to their equilibrium trajectory and velocity it is possible to determine from Eq. (2) in a linear approximation by the amplitude of the electromagnetic field. Substituting these values into the equation of continuity (4), we find the perturbation of the charge density and current density of the electron beam. Then, taking into account (5), we obtain from Eq. (3)

the expression for E_z, which we use in the boundary condition on the inner surface of the waveguide (1)

$$E_z[R(\varphi)] = 0 . \quad (6)$$

Considering a fixed value of s in Eq. (6), as a result of the corresponding calculations (see [6]), we obtain the following dispersion equation

$$\| a_{mn}(\omega) \| = 0 , \quad (7)$$

where

$$a_{mn} = \int_{-\pi/N}^{\pi/N} G_l[k_\perp R(\varphi)] \exp[i(n-m)N\varphi]\, d\varphi ,$$

$$G_l(x) = J_l(x) - \mu_l N_l(x),$$

$$\mu_l = \frac{\pi}{2} \frac{r_0^2}{c^2} \omega_{b\perp}^2 [B_l J_l(k_\perp r_0) + C_l J_l'(k_\perp r_0)],$$

$$B_l = \frac{J_l(k_\perp r_0)}{\bar{\omega}_l^2} \left[(k_z c - \beta_{z0}\omega)^2 \left(1 - \frac{l^2}{k_\perp^2 r_0^2} \right) - \bar{\omega}_l^2 \right],$$

$$C_l = \frac{(k_z c - \beta_{z0}\omega)^2}{\bar{\omega}_l^2 - \omega_{H\perp}^2} \left[J_l'(k_\perp r_0) - \frac{l\omega_{H\perp}}{k_\perp r_0 \bar{\omega}_l} J_l(k_\perp r_0) \right],$$

$$\omega_{b\perp} = \sqrt{4\pi e^2 n_S / (m\gamma_0 r_0)}, \qquad \bar{\omega}_l = \omega - k_z v_{z0} - l_n \omega_{H\perp},$$

$J_l(x)$, $N_l(x)$ are the Bessel functions of the first and second kind, $J_l'(x) = (d/dx) J_l(x)$.

In the limiting case of small corrugation depths (q<<1), Eq. (7) can be written as

$$\sum_{n=-\infty}^{\infty} \frac{a_{n,n+1}}{a_{n,n}} \cdot \frac{a_{n+1,n}}{a_{n+1,n+1}} = 1, \quad (8)$$

where

$$a_{mn} = \left[G_{l_n}(\alpha) + \frac{q^2\alpha^2}{4} \frac{d^2}{d\alpha^2} G_{l_n}(\alpha) \right] \delta_{n,m} +$$
$$+ \frac{q\alpha}{2} \frac{d}{d\alpha} G_{l_n}(\alpha) (\delta_{m,n+1} + \delta_{m,n-1}) +$$
$$+ \frac{q^2\alpha^2}{4} \frac{d^2}{d\alpha^2} G_{l_n}(\alpha) (\delta_{m,n+2} + \delta_{m,n-2}),$$

$\alpha = k_\perp R_0$, $\delta_{m,n}$ is the Kronecker delta.

Below we will consider the resonance of beam electrons with the main harmonic s, taking into account the first harmonics (n=±1) of an azimuthally corrugated waveguide.

2. LINEAR GROWTH OF INSTABILITY

In the absence of a beam (n_b=0), Eq. (8) describes the dispersion of the TM mode propagating in the waveguide, the inner wall of which is sinusoidally corrugated along the angle φ. In the zero approximation for the depth of the corrugation, this dispersion relation is

$$\omega_0^2 / c^2 - k_z^2 = \chi_{s,m}^2 / R_0^2 , \quad (9)$$

where $\chi_{s,m}$ is the m-th root of the s-th order Bessel function of the first kind ($J_s(\chi_{s,m}) = 0$).

Taking into account the finite depth of the corrugation leads to a shift in the frequency of the cylindrical waveguide ($\omega = \omega_0 + \Delta\omega$). From Eqs. (8) and (9) at s≠N/2, we obtain up to quadratic values of the parameter q the following expression for this displacement.

$$\Delta\omega = \frac{q^2 \chi_{s,m}^2 c^2}{4R_0^2 \omega_0} \times$$
$$\times \left[1 + \chi_{s,m} \frac{J_{s+N}'(\chi_{s,m})}{J_{s+N}(\chi_{s,m})} + \chi_{s,m} \frac{J_{s-N}'(\chi_{s,m})}{J_{s-N}(\chi_{s,m})} \right]. \quad (10)$$

This equation shows that the frequency shift in this case is proportional to the square of the corrugation depth.

For values s=N/2, when the azimuth number of the main harmonic (s) is equal to the azimuth number of the first harmonic of the corrugated waveguide, we have

$$\Delta\omega = \pm \frac{q \chi_{s,m}^2 c^2}{2R_0^2 \omega_0} . \quad (11)$$

As follows from this equation, the frequency shift in this case is proportional to the depth of the corrugation to the first degree.

Taking into account in Eq. (8) terms proportional to the density of the electron beam leads to the appearance of resonance terms at $\omega = k_z v_{z0} + s\omega_{H\perp}$ and $\omega = k_z v_{z0} + (s \pm 1)\omega_{H\perp}$.

When the TM mode is excited by an electron beam under resonance conditions $\omega = k_z v_{z0} + s\omega_{H\perp}$, at s≠N/2, the growth rate, as well as the frequency and the axial wavenumber of the wave, are of the form

$$Jm(\delta\omega) = \frac{\sqrt{3}}{2} \frac{\omega_{b\perp}^{2/3}}{\omega_0^{1/3}} \left[\frac{\xi J_s(\xi\chi_{s,m})}{J_{s+1}(\chi_{s,m})} (k_z c - \beta_{z0}\omega_q) \right]^{2/3} \times$$
$$\times \left| 1 - \frac{s^2}{\xi^2 \chi_{s,m}^2} \right|^{1/3} \left| 1 - \frac{q^2}{4} (\chi_{s,m}^2 - s^2) \right|^{1/3} , \quad (12)$$

$$\omega = \gamma_{z0}^2 (s\omega_{H\perp} - \beta_{z0}\Delta\omega) \pm$$
$$\pm \gamma_{z0}\beta_{z0} \sqrt{\gamma_{z0}^2 (s\omega_{H\perp} - \beta_{z0}\Delta\omega)^2 - \chi_{s,m}^2 c^2 / R_0} , \quad (13)$$

$$k_z = (\gamma_{z0}/c) [\gamma_{z0}\beta_{z0} (s\omega_{H\perp} - \beta_{z0}\Delta\omega) \pm$$
$$\pm \sqrt{\gamma_{z0}^2 (s\omega_{H\perp} - \beta_{z0}\Delta\omega)^2 - \chi_{s,m}^2 c^2 / R_0}] . \quad (14)$$

Here $\xi = r_0/R_0$, and $\Delta\omega$ it is determined by formula (10).

The signs (±) in Eqs. (13) and (14) correspond to two points of intersection of the frequencies of the vacuum corrugated waveguide with the beam mode. In this case, the condition must be satisfy

$$s\beta_{\varphi 0}\gamma_{z0} > \xi\chi_{s,m} (1 + \Delta\omega R_0 \gamma_{z0} / c\chi_{s,m}) .$$

Under resonance conditions $\omega = k_z v_{z0} + (s \pm 1)\omega_{H\perp}$ the growth rate takes the form

$$Jm(\delta\omega) = \frac{\omega_{b\perp}\xi}{(2\omega_0\omega_{H\perp})^{1/2}} |k_z c - \beta_{z0}\omega_q| \frac{V_{s\pm1}^{1/2}}{|J_{s+1}(\chi_{s,m})|} , \quad (15)$$

where

$$V_{s\pm1} = J_{s\pm1}(\xi\chi_{s,m}) \left[\frac{s}{\xi\chi_{s,m}} J_s(\xi\chi_{s,m}) - J_{s\pm1}(\chi_{s,m}) \right] \times$$
$$\times \left| 1 - q^2 (\chi_{s,m}^2 - s^2) / 4 \right| .$$

The necessary conditions for wave excitation in this case are $(\pm 1) V_{s\pm1} > 0$, and also $(s \pm 1)\beta_{\varphi 0}\gamma_{z0} > \xi\chi_{s,m} (1 + \Delta\omega R_0 \gamma_{z0} / c\chi_{s,m})$.

Of particular interest is the interaction of the electron beam with electromagnetic waves at s = N/2. In this case, the beam electrons can be in resonance with both the main harmonic (s-th) and the first harmonic of the azimuthally corrugated waveguide.

Calculation of the growth rate under resonance conditions $\omega = k_z v_{z0} + s\omega_{H\perp}$ at

$$(\omega_{b\perp}/\gamma_{z0})\sqrt{(s\omega_{H\perp}\gamma_{z0})^2 - \chi_{s,m}^2 c^2 / R_0^2} << \frac{q^3\chi_{s,m}^3 c^2}{8R_0^3\omega_0} Y_s$$

gives a fourth-degree equation for the correction to the frequency due to the beam, from which follows the expression for the growth rate

$$Jm(\delta\omega) = \left(\frac{q\xi c}{2R_0\omega_0}\right)^{1/2} \chi_{s.m}\omega_{b\perp}^{1/2} \times \times \left|(k_z c - \beta_{z0}\omega_q) J_s(\xi\chi_{s,m})\right|^{1/2} |Q_s|^{1/4} b_Q , \quad (16)$$

where

$$Q_s = \left[\frac{s}{\chi_{s,m}^2 [J_{s+1}(\chi_{s,m})]^2} - \frac{2}{\pi}\frac{N_{s+1}(\chi_{s,m})}{J_{s+1}(\chi_{s,m})}\right]\left(1 - \frac{s^2}{\xi^2\chi_{s,m}^2}\right),$$

$$Y_s = \left|\frac{J_{s+1}(\chi_{s,m})}{\xi J_s(\xi\chi_{s,m})}\right| \cdot \left|s - \frac{\pi\chi_{s,m}^2}{2} N_{s+1}(\chi_{s,m}) J_{s+1}(\chi_{s,m})\right|^{3/2} \times$$

$$\times \left|1 - \frac{s^2}{\xi^2\chi_{s,m}^2}\right|^{-1/2},$$

$b_Q = 1$ at $Q_s > 0$; $b_Q = \sqrt{2}/2$ at $Q_s < 0$.

CONCLUSIONS

In this work, we study the excitation of the TM mode by an annular relativistic electron beam moving in a waveguide with a sinusoidally corrugated wall in a guiding magnetic field.

The dispersion equation is obtained that describes the interaction of the electron beam with the electromagnetic wave, and analytical studies of this equation are carried out in the limiting case of a small corrugation depth. It is shown that the beam can excite both main modes of the waveguide and harmonics of cyclotron frequencies due to the corrugated wall of the waveguide. Under conditions of cyclotron resonance of beam electrons with the main mode, the instability growth rate (12) is proportional to the cube root of the beam density. It is shown that it is possible to excite a TM wave at the numbers of harmonics of cyclotron frequencies, which are shifted relative to the main by unity (15). In this case the growth rate is proportional to the square root of the beam density.

Under the conditions of cyclotron resonance of the beam with the main harmonic and the first harmonic of the corrugated waveguide, at s = N/2, there is an increase in the growth rate. In this case, the instability growth rate (16) is proportional to the fourth root of the beam density and the square root of the corrugation depth.

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Article received 29.06.2023

ЗБУДЖЕНЯ ТМ-ХВИЛІ РЕЛЯТИВІСТСЬКИМ ЕЛЕКТРОННИМ ПУЧКОМ В АЗИМУТАЛЬНО ГОФРОВАНОМУ ХВИЛЕВОДІ

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Теоретично досліджено збудження електромагнітної ТМ-хвилі релятивістським електронним пучком у хвилеводі з синусоїдально гофрованою по азимуту провідною стінкою у постійному однорідному зовнішньому ведучому магнітному полі. Розглянуто тонкий трубчатий електронний пучок, що рухається вздовж осі хвилеводу і обертається на рівноважному радіусі навколо його осі. У наближенні малої глибини гофра визначені аналітичні залежності інкрементів нестійкості і резонансних частот від параметрів пучка і хвилеводу.