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ELECTROMAGNETIC DIPOLAR MODES IN MAGNETIZED NONUNIFORM PLASMA-VACUUM-METAL WAVEGUIDE

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This article presents the results of theoretical study of the phase and attenuation properties of the dipolar electromagnetic modes in cylindrical plasma-vacuum-metal waveguide structure. Plasma is described in hydrodynamic approach and is supposed to be slightly non-uniform in axial direction and strongly non-uniform in radial direction. It was studied the influence of the external magnetic field value, the electron effective collision frequency and other waveguide parameters on the phase and attenuation properties of the considered waves.

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INTRODUCTION

At present time the intensive both experimental and theoretical studies of the different kinds of discharges are carried out. One of the directions of the research is the studying of creation and sustaining of rather long plasma columns by travelling surface waves. The travelling wave that sustains the discharge propagates along plasma column that usually placed in metal waveguide structure and is the eigen wave of it. The properties of such surface wave sustained discharges strongly depend as on the value of external magnetic field value and waveguide parameters (the plasma column radius, the dielectric wall thickness and the electric permittivity of discharge tube or a presence of cooling and/or screening systems) and also discharge parameters (plasma density value and spatial distribution) depends on the properties of the wave [1, 2].

Among other approaches to model such surface wave sustained discharges one can use electrodynamics approach. According this approach plasma is modeled by the simplified model equations and the main attention is paid to the description of the wave that sustains the discharge. Such approach can be used to model stable state of gas discharges that are sustained by the electromagnetic waves of the surface type.

In the framework of such approach it was carried out the theoretical studies of influence of plasma density radial profile on the properties of the discharge that is sustained by the symmetric wave [3]. In the number of papers [1, 2, 4] it was declared that the dipolar waves with azimuth wavenumber ($m = \pm 1$) can be successfully used for plasma column sustaining. The aim of this article is to study phase and attenuation properties of the dipolar waves for different plasma density radial profiles, external magnetic field value, plasma column radius, the electron effective collision frequency and other waveguide parameters.

1. BASIC EQUATIONS

Let us study the propagation of the electromagnetic dipolar with azimuth wave number $m = \pm l$ waves in the three component cylindrical waveguide structure. The considered waveguide structure is composed of magnetized plasma column with radius R_p , vacuum gap between plasma column and waveguide metal wall with radius R_m . External constant magnetic field B_0 is directed along the axis of this structure.

Plasma is considered as cold and weakly absorbed media that is characterized by the effective electron collision frequency ν that is constant in the plasma volume and is small as compared with the wave frequency ω . The radial plasma density distribution n(r)modeled in the Bessel-like form was as: $n(r) = n(0) J_0(\delta r)$, were J_0 is the Bessel function of the first kind and δ is plasma density non-uniformity parameter. This parameter can varies from $\delta = 0$ (radially uniform plasma) up to $\delta = 2.405$ (strong radially non-uniform plasma that corresponds to ambipolar diffusion regime) [3]. Also, plasma density is considered to be slightly varying in the axial direction on the distances of wavelength order. So, the wave field change slightly along the waveguide structure and the solutions of the system of Maxwell equations that govern wave propagation can be found in WKB approach:

$$E, H_{r,\phi,z}(r,\phi,z) = E, H_{r,\phi,z}(r,z) e^{-i\omega t + im\phi + \int_{z_0}^{z_0} k_3(z')dz'}, (1)$$

where m and k_3 are azimuth and axial wavenumber, *E*, *H* – amplitude of electric and magnetic wave field components, respectively. Changing the A value along the discharge at the distances of the wavelength order is small compared to the magnitude of this quantity $((A^{-1}(\partial A/\partial z) \le k_3))$, where symbol A denotes E, H, k_3 , or n. So, all terms of order $O(k_3^{-1}(\partial \ln(A)/\partial z)))$ are neglected in further expressions [6]. Taking in account the expression (1) the equations for radial wave components in plasma region can be written as:

$$\begin{cases} H_r^p(r) = \frac{m}{kr} E_z^p(r) - \frac{k_3}{k} E_{\varphi}^p(r) \frac{i\varepsilon_2(r) E_{\varphi}^p(r)}{\varepsilon_l(r)}, \\ E_r^p(r) = \frac{k_3 H_{\varphi}^p(r)}{k\varepsilon_l(r)} - \frac{m H_z^p(r)}{k r \varepsilon_l(r)} - \frac{i\varepsilon_2(r) E_{\varphi}^p(r)}{\varepsilon_l(r)}, \end{cases}$$
(2)

where $\varepsilon_{1,2,3}(r)$ are the components of permittivity tensor of magnetized collisional plasma [5] that depends on radial coordinate *r* throw the dependence of n(r). The ordinary differential equations for tangential wave field components in plasma region can be written as:

$$\begin{aligned} \left\{ \frac{dE_z^p}{dr} = \frac{k_3 \varepsilon_2(r)}{\varepsilon_1(r)} E_\varphi^p(r) + i \frac{k_3^2 - k^2 \varepsilon_1(r)}{k \varepsilon_1(r)} H_\varphi^p(r) - \frac{i m k_3}{k r \varepsilon_1(r)} H_z^p(r), \\ \frac{dE_\varphi^p}{d r} = \left(\frac{m \varepsilon_2(r)}{r \varepsilon_1(r)} - \frac{1}{r} \right) E_\varphi^p(r) + \frac{i k_3 m}{k r \varepsilon_1(r)} H_\varphi^p(r), \\ + i k \left(1 - \frac{m^2}{k^2 r^2 \varepsilon_1(r)} \right) H_z^p(r) + \frac{i k_3 m}{k r \varepsilon_1(r)} H_\varphi^p(r), \\ \frac{dH_z^p}{d r} = \frac{k_3 \varepsilon_2(r)}{\varepsilon_1(r)} H_\varphi^p(r) - i \frac{p(r)}{k \varepsilon_1(r)} E_\varphi^p(r) + \frac{i k_3 m}{k r \varepsilon_1(r)} H_z^p(r), \\ \frac{dH_\varphi^p}{d r} = -\frac{1}{r} H_\varphi^p(r) - \frac{i k_3 m}{k r} E_\varphi^p(r) + i k \left(\frac{m^2}{k^2 r^2} - \varepsilon_3(r) \right) E_z^p(r), \end{aligned}$$
(3)

where $p(r) = \varepsilon_l(r)(k^2_3 - k^2 \varepsilon_l(r)) + k^2 \varepsilon_2(r)$, $k = \omega/c - is$ the vacuum wavenumber.

It is possible to obtain analytic solutions for wave field components in vacuum region:

$$\begin{cases} E_{z}(r) = A_{l}I_{m}(\kappa r) + A_{2}K_{m}(\kappa r), \\ E_{\varphi}(r) = \frac{mk_{3}A_{l}I_{m}(\kappa r)}{\kappa^{2}r} + \frac{mk_{3}A_{2}K_{m}(\kappa r)}{\kappa^{2}r} + \\ + \frac{ikA_{3}I_{m}(\kappa r)}{\kappa} + \frac{ikA_{4}K_{m}(\kappa r)}{\kappa}, \\ E_{r}(r) = -\frac{ik_{3}A_{1}I_{m}(\kappa r)}{\kappa} - \frac{ik_{3}A_{2}K_{m}(\kappa r)}{\kappa} + \\ + \frac{mkA_{3}I_{m}(\kappa r)}{\kappa^{2}r} + \frac{mkA_{4}K_{m}(\kappa r)}{\kappa^{2}r}, \\ H_{z}(r) = A_{3}I_{m}(\kappa r) + A_{4}K_{m}(\kappa r), \\ H_{\varphi}(r) = -\frac{ikA_{1}I_{m}(\kappa r)}{\kappa} - \frac{ikA_{2}K_{m}(\kappa r)}{\kappa} + \\ + \frac{mk_{3}A_{3}I_{m}(\kappa r)}{\kappa^{2}r} + \frac{mkA_{4}A_{m}(\kappa r)}{\kappa^{2}r}, \\ H_{r}(r) = -\frac{mkA_{1}I_{m}(\kappa r)}{\kappa^{2}r} - \frac{mkA_{2}K_{m}(\kappa r)}{\kappa^{2}r} - \\ - \frac{ik_{3}A_{3}I_{m}(\kappa r)}{\kappa} - \frac{ikA_{3}A_{4}K_{m}(\kappa r)}{\kappa}, \end{cases}$$

where $k^2 = k^2_3 - k^2$ is the transverse wave number in vacuum, I_m , K_m – modified Bessel function of the first and second kind, respectively and A_{1-4} are field

constants and a stroke denotes derivative with respect to argument.

The expressions for A_{1-4} can be obtained from the boundary conditions on the plasma – vacuum interface at $r = R_p$ [3] as (5).

$$\begin{split} A_{l} &= -\kappa R_{p} K_{m}^{\prime}(\kappa R_{p}) E_{z}^{P}\left(R_{p}\right) - i \frac{m k_{3} K_{m}(\kappa R_{p})}{k} H_{z}^{P}(R_{p}) + \\ &+ i \frac{\kappa^{2} R_{p} K_{m}(\kappa R_{p})}{k} H_{\varphi}^{P}(R_{p}), \\ A_{2} &= \kappa R_{p} I_{m}^{\prime}(\kappa R_{p}) E_{z}^{P}\left(R_{p}\right) + i \frac{m k_{3} I_{m}(\kappa R_{p})}{k} H_{z}^{P}(R_{p}) - \\ &- i \frac{\kappa^{2} R_{p} I_{m}(\kappa R_{p})}{k} H_{\varphi}^{P}(R_{p}), \\ A_{3} &= -\kappa R_{p} K_{m}^{\prime}(\kappa R_{p}) H_{z}^{P}\left(R_{p}\right) + i \frac{m k_{3} K_{m}(\kappa R_{p})}{k} E_{z}^{P}(R_{p}) - \\ &- i \frac{\kappa^{2} R_{p} K_{m}(\kappa R_{p})}{k} E_{\varphi}^{P}(R_{p}), \\ A_{4} &= \kappa R_{p} I_{m}^{\prime}(\kappa R_{p}) H_{z}^{P}\left(R_{p}\right) - i \frac{m k_{3} I_{m}(\kappa R_{p})}{k} E_{z}^{P}(R_{p}) + \\ &+ i \frac{\kappa^{2} R_{p} I_{m}(\kappa R_{p})}{k} E_{\varphi}^{P}(R_{p}). \end{split}$$

$$(5)$$

The wave field components at plasma-vacuum interface $E^{p}(R_{p})$, $H^{p}(R_{p})$ that are present in formula (5) must be obtained by numerical integration of the system of ordinary differential equations (3).

The analogue of the dispersion equation (local dispersion equation) can be obtained from the boundary conditions at vacuum-metal interface (the vanishing of the electric tangential components of the wave at the waveguide metal wall $r = R_m$):

$$\begin{cases} A_{1}I_{m}(\kappa R_{m}) + A_{2}K_{m}(\kappa R_{m}) &= 0, \\ A_{3}I'_{m}(\kappa R_{m}) + A_{4}K'_{m}(\kappa R_{m}) &= 0. \end{cases}$$
(6)

Let us note, that the solution of the local dispersion equation (6) connects the values of local density n and complex axial wave vector k_3 at the given wave frequency value ω .

2. MAIN RESULTS

Let us firstly study the phase properties and the spatial attenuation of the dipolar electromagnetic wave that propagates along the plasma column and sustains it. This is the first step in the studying of axial structure of the discharge in the framework of the electrodynamic approach [1-4]. The basic idea of this approach is that the wave that sustains the discharge is the eigenwave of this discharge structure on the whole length of the plasma column. Therewidth there is a mutual influence of the wave considered and sustained plasma density value. So, due to such sircumastances the axial plasma source parameters variation in such approach is determined by the phase properties and the spatial attenuation of the wave sustaining the discharge. Also, according to the Zakrzhevsky stability criterium on the basis of studied attenuation properties [7] it is possible to estimate the region, where the eigen wave can sustaine the discharge.

Let us present the results of the phase and attenuation properties of the eigen dipolar $(m = \pm 1)$ waves of the studied waveguide structure. The considered wave possesses all six components of electric and magnetic wave field, so the solution of the analogue of the dispersion equation for arbitrary problem parameters is possible only with the help of numerical methods. To carry out the numerical study of the dispersion equation (6) the following dimensionless variables and parameters are introduced: wave frequency $\varpi = \omega/\omega_p$, axial wave number $x = \text{Re}(k_3)R_p$, attenuation coefficient $\alpha = \text{Im}(k_3)R_p$, effective collision frequency $\tilde{\nu} = \nu/\omega$, external magnetic field value $\Omega = \omega_{Ce}/\omega$, radius of plasma column $\sigma = R_p \omega/c$ and radius of metal enclosure $\eta = R_m/R_p$.

The dependence of the wave phase properties for the dipolar modes $m = \pm I$ (the dependence of normalized frequency ϖ versus x) in the approach of radially uniform plasma ($\delta = 0$) is presented in the Fig. 1. It is necessary to point that under the fixed generator frequency ω the value of ϖ varies due to the changing of plasma density and consequently ω_p value along the plasma column. The calculations are made for the following normalized parameters: $\Omega = 0.2$, $\sigma = 0.3$, $\eta = 1.1$ and $\tilde{\gamma} = 0.001$.



Fig. 1. The eigen wave normalized frequency ϖ *via axial wavenumber* x *for the dipolar modes* $m = \pm 1$



Fig. 2. The eigen wave normalized attenuation coefficient α via axial wavenumber x for the dipolar modes $m = \pm 1$

The carried out research has shown that for the fixed normalized frequency ϖ value the dipolar wave with m = +1 has the solution with more wave length that the wave with m = -1. The appropriative results for the attenuation coefficient α study for the same parameter

set are presented in the Fig. 2. The parameters set is the same as in the Fig. 1. It is shown that these waves possesses the similar $\alpha(x)$ dependence, but for attenuation coefficient value is somewhat bigger than for the m = +1 mode. Due to this circumstances one can expect that the discharges, sustained by the m = +1 mode possesses somewhat bigger maximum possible density and shorter length that the discharges sustained by the m=-1 mode. It is necessary to mention the existence of the region with extremely rapid growth of spatial attenuation coefficient when normalized wavenumber is small ($x \approx 0$) for the both modes. The existence of such region leads to the limitation of the area, where waves considered can maintain stable long discharge.

The carried out analysis have shown that this modes possesses the wave field structure analogous to the wave of surface type in the regions where wavenumber is rather small (see blue and red points in Figs. 1, 2). When normalized wavenumber is rather big (see green points in Figs. 1, 2) the radial structure of the dipolar eigen wave becomes of more complex. The radial wave field structure for this case is presented in the Fig. 3 for m = -1 mode and in the Fig. 4 for m = +1mode. The wave field components in the Figs. 3, 4 are normalized by the $E_{\varphi}(r=0)$. The score parameters are the same as for the Fig. 1. For both dipolar modes the calculations are made for the same value of the dimensionless parameter $\varpi = 0.485$. The displayed solution of the system (6) for m = -1 mode (see Fig. 3) is the following: x = 3.987, $\alpha = 0.0049$. Corresponding eigen values for m = +1 (see Fig. 4) are: x = 3.398, $\alpha = 0.0052.$



Fig. 3. The wave field radial distribution for the dipolar mode m = -1 for green point from Figs. 1, 2

Red line on the Figs. 3, 4 corresponds to the wave field component in plasma column and blue line – in the region of vacuum gap. The abscissa axis corresponds to the normalized radial coordinate r / R_p .

According to the classification presented in [2] this wave is of the pseudo-surface one. The H_r wave field component possesses the maximum absolute value in plasma column far from the plasma–vacuum interface. Other wave field components have its maximum absolute value exactly at plasma-vacuum interface.



Fig. 4. The wave field radial distribution for the dipolar mode m = +1 for green point from Figs. 1, 2

It was carried out the study of the influence of the vacuum gap size and the plasma column radius on the phase properties and spatial attenuation of dipolar modes for different values of the problem parameters. The studies have shown that for both dipole modes $m = \pm l$ the increase of the vacuum gap size leads to an increase in the normalized wave frequency ϖ , especially in the region of small x values. The influence of the waveguide metal wall on ϖ becomes practically insignificant when the skin depth of the wave field into the vacuum becomes much smaller than the size of vacuum gap $(\eta > 1.8)$. Also, the increase of the parameter η leads to the decrease of the eigen wave group velocity and to the corresponding increase of the spatial attenuation coefficient α in the whole x range for both dipolar modes.

Carried out studies have also shown that in the case of radially uniform plasma the increase of the plasma column normalized radius σ leads to the decrease of the normalized wave frequency ϖ of the dipole waves with $m = \pm 1$ in the whole range of axial wave numbers x. The strongest influence of the parameter σ value on the dipolar wave phase properties occurs in the region of small and moderate normalized axial numbers x < 1.5.

Besides, the increase of the plasma column normalized radius σ value leads to the decrease of the spatial attenuation coefficient α . It is necessary to mention that when the parameter σ increases in the range $0.3 < \sigma < 0.6$, the strongest decrease of coefficient α occurs in the region x < 1.5. At the same time, for large σ values ($\sigma > 1$), the strongest decrease of α occurs in the region x > 4.

It was also carried out the study of phase and attenuation properties of eigen modes the waveguide structure upon external magnetic field value. The results of this study are presented in Figs. 5-8. The numerical calculations were carried out for the case of radially uniform plasma. Other score parameters are the same: $\sigma = 0.3$, $\eta = 1.1$, $\tilde{\nu} = 0.001$. Numbers on the figures just near the curves correspond to the different Ω values.



Fig. 5. The eigen wave normalized frequency ϖ via axial wavenumber x for the dipolar mode m=+1for different Ω values

The carried out study has shown that the dependence of phase and attenuation properties of dipolar modes with m = +1 and m = -1 is substantially different. Thus, the increase of the parameter Ω value from 0.2 up to 0.85 leads to the uniform growth of the normalized wave frequency ϖ for the m = +1 mode in the whole range of axial wave numbers x (see Fig. 5). At the same time frequency ϖ for the m = -1 mode when external magnetic field increases and parameter Ω growth from 0.2 up to 0.9 decreases in the region x < 3 and increases in the region x > 4 (see Fig. 7). It is necessary to mention that normalized frequency of the eigen dipolar mode m = +1 are somewhat greater than the normalized frequency of the m = -1 mode for the same parameters of waveguide structure. Besides, external magnetic field influences much greatly on the phase characteristics of the m = +1 mode than on the m = -1 mode.

External magnetic field strongly affects the normalized attenuation coefficient α . Thus, for eigen mode with m = +1 the strengthening of the external magnetic field leads to the increase of spatial attenuation coefficient α , mainly in the regions of small x < 1 and large x > 3 axial wave numbers values (see Fig. 6). The influence of external magnetic field value Ω on the attenuation coefficient α for the m = -1 mode is qualitatively similar to that of m = +1 mode, but quantitatively is in almost one order smaller (see Fig. 8).



Fig. 6. The eigen wave normalized attenuation coefficient α via axial wavenumber x for the dipolar mode m=+1 for different Ω values



Fig. 7. The eigen wave normalized frequency ϖ via axial wavenumber x for the dipolar modes m=-1 for different Ω values



Fig. 8. The eigen wave normalized attenuation coefficient α via axial wav enumber x for the dipolar mode m=-1 for different Ω values



Fig. 9. The eigen wave normalized frequency ϖ via axial wave number x for the dipolar modes m=+1 for different non-uniformity parameter δ values

It was also studied the influence of effective collisional frequency value \tilde{v} on the phase and attenuation properties of the waves considered. The increase of the parameter \tilde{v} value leads to the slight decrease of the frequency ϖ for m = +1 mode in the region x < 1 and to its slight increase in the region x > 1. At the same time the influence of \tilde{v} value on the ϖ eigen frequency ϖ of the m = -1 dipolar is practically absent. The spatial attenuation coefficient α for both dipolar modes increases with the increase of \tilde{v} value, especially in the region x > 4. The attenuation coefficient α value of the mode with m = +1 is in one order greater than attenuation coefficient of the m = -1 mode.

It was also studied the influence of phase and attenuation properties of the eigen dipolar modes of the

waveguide structure upon non-uniformity parameter δ value. The results of this study are presented in Figs. 9-12. The numerical calculations were carried out for: $\Omega = 0.2, \ \sigma = 0.3, \ \eta = 1.1, \ \tilde{\nu} = 0.001$. Numbers on the figures just near the curves corresponds to different values of the non-uniformity parameter δ . The normalized frequency ϖ of the m = +1 mode decreases significantly with the increase of the plasma density radial non-uniformity parameter δ , especially in the region of large axial wavenumber x values, when plasma density radial profile is similar to that of the ambipolar diffusion regime (see Fig. 9). Simultaneously the spatial attenuation of the m = +1 mode increases significantly (see Fig. 10). The studies have shown that the influence of the radial non-uniformity of plasma density on the frequency ϖ and attenuation coefficient α for the m = -1 mode is similar to that of the m = +1mode (see Figs. 11, 12).



Fig. 10. The eigen wave attenuation coefficient α via axial wavenumber x for the dipolar mode m=+1 for different non-uniformity parameter δ values



Fig. 11. The eigen wave normalized frequency ϖ via axial wavenumber x for the dipolar modes m=-1 for different non-uniformity parameter δ values



Fig. 12. The eigen wave attenuation coefficient α via axial wavenumber x for the dipolar mode with m=-1 for different non-uniformity parameter δ values

CONCLUSIONS

The article presents the results of detailed theoretical analysis of the phase and attenuation properties of the electromagnetic modes with azimuth wavenumber $m = \pm 1$ in magnetized cylindrical plasma-vacuum-metal waveguide structure. The study was carried out in the framework of hydrodynamic approach taking into account slightly axial and strongly radial non-uniformity of plasma density.

It was studied the influence of the external magnetic field value, the electron effective collision frequency and geometrical waveguide parameters on the phase and attenuation properties of the considered waves. It was found the plasma density ranges in which the studied eigen waves of the given frequency can propagate in the waveguide structure and also the dependence of the sizes of these ranges upon the score parameters for different plasma density radial profiles. It was also found the dependence of spatial attenuation coefficient of the wave upon the problem parameters to estimate the region, where the eigen wave can sustain the discharge. This research is the first step in studying of the plasma density stationary axial distribution in rather long gas discharges supported by the eigen waves of the considered waveguide structure in the framework of electrodynamic approach.

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ЕЛЕКТРОМАГНІТНІ ДИПОЛЬНІ ХВИЛІ В МАГНІТОАКТИВНОМУ НЕОДНОРІДНОМУ ПЛАЗМОВО-ВАКУУМНО-МЕТАЛЕВОМУ ХВИЛЕВОДІ

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Наведено результати теоретичних досліджень фазових властивостей та просторового загасання дипольних електромагнітних хвиль, що поширюються в циліндричній хвилеводній структурі плазма – вакуум – метал. Плазма розглядається в гідродинамічному наближенні та вважається слабко неоднорідною в аксіальному напрямку та сильно неоднорідною в радіальному напрямку. Досліджено вплив величини зовнішнього магнітного поля, ефективної частоти зіткнень електронів та інших параметрів хвилеводної структури на фазові властивості та просторове загасання дипольних хвиль.