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MOVEMENT OF CHARGED PARTICLES IN MAGNETIC AND NONUNIFORM STOCHASTIC ELECTRIC FIELDS

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The equation of motion of charged plasma particles in a homogeneous magnetic field and in an inhomogeneous stochastic electric field with a characteristic oscillation frequency much lower than the electron cyclotron frequency and much higher than the ion cyclotron frequency is solved. The diffusion motion, as well as the drift of ions and guiding center of electrons, due to the inhomogeneity of the stochastic electric field, is considered. The obtained values of the diffusion coefficient and drift velocity are used in the Fokker-Planck equation to determine the stationary distribution of the plasma density due to the effect of an inhomogeneous stochastic field.

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INTRODUCTION

It was shown in Ref. [1] that inhomogeneous high-frequency harmonic electric field in plasma which has the form

$$\vec{E}\left(\vec{r},t\right)=\vec{E}\left(\vec{r}\right)\sin\omega t \qquad (1)$$

leads to the appearance of a nonlinear force acting on a charged particles

$$\vec{F}\left(\vec{r}\right)=-\frac{e^2}{4m\omega^2}\nabla E^2\left(\vec{r}\right), \qquad (2)$$

which called the RF pressure force. This force pushes the charged particles out of the region of the increased level of high-frequency oscillations, which leads to a depletion of the plasma density in this region.

In Ref. [2] the effect of inhomogeneous stochastic electric fields on charged particles is also found. Such fields can occur due to the excitation of various instabilities in plasma. It was shown that, as in the case of a harmonic field, a ponderomotive force acts on the particles, which leads to the drift motion of particles from the region of an increased level of oscillations. However, in addition to drift motion, stochastic electric fields also lead to increased diffusion motion of particles. This motion is analogous to the motion of Brownian particles caused by their collisions with the molecules of the liquid in which they are placed. It was suggested that localized stochastic electric fields may be responsible for the appearance of lower hybrid cavities in the plasma of the earth's ionosphere, that is, regions with a depleted density, which were often observed by spacecraft [3 - 6]. It was shown [7 - 9] that such cavities can also form in plasma due to the propagation of electron beams in it and the excitation of plasma instabilities there.

It was assumed in [2] that there is no the magnetic field in plasma, or otherwise the characteristic values of the frequencies of stochastic oscillations significantly exceed the cyclotron frequency of charged particles. This condition is satisfied the ions of the lower hybrid cavities, where the frequency of the lower hybrid oscillations significantly exceeds the ion cyclotron frequency. However, the frequency of the lower hybrid oscillations turns out to be much lower than the cyclotron frequency of electrons. In this regard, the problem arises of studying the motion of particles in inhomogeneous stochastic fields, the cyclotron frequency of which significantly exceeds the characteristic frequency of oscillations of the electric field. This problem is considered in this work. In contrast to the case of high-frequency oscillations, when the cyclotron motion of particles during the period of oscillations can be neglected, at low-frequency oscillations the cyclotron motion is important, and therefore, in this consideration, not the motion of the particle, but the motion of its guiding center, i.e., the center of the Larmor orbit is investigated. Moreover, since the behavior of ions and electrons is different, we investigate the effect of a non-uniform stochastic electric field on the motion of not only electrons, but also ions and also compare their characteristics of motion. This work is a development of [2], however, in contrast to it where a cylindrical plasma model was considered, the slab plasma model is used here.

1. HEATING AND DIFFUSION

Consider homogeneous plasma in the magnetic field $\vec{B}$ directed along the z axis, in which there is a region with a stochastic electric field inhomogeneous along the x-axis and homogeneous in other directions. It is assumed that the characteristic frequency of the stochastic field ω is much lower than the electron cyclotron frequency ω_{ce} and much higher than the ion cyclotron frequency ω_{ci}. We assume that the turbulent state of the plasma arises due to an external source, for example, as a result of the passage of a high-energy particle beam through the plasma and the excitation of instability in it.

The equation of motion of the charged particles in the magnetic field, taking into account the stochastic electric field, is

$$\frac{d\vec{v}}{dt}=\frac{e_\alpha}{m_\alpha}F\left(x\right)\vec{E}\left(\vec{r},t\right)+\frac{e_\alpha}{m_\alpha}\left[\vec{v},\vec{B}\right], \qquad (3)$$

where e_α and m_α are the charge and mass of α species particles ($\alpha=i$ for ions and $\alpha=e$ for electrons),

$\vec{E}(\vec{r},t)$ is the electric field strength of turbulence, far from the region with a high level of turbulence, $F(x) \geq 1$ is the envelope of turbulent pulsations having a maximum at $x=0$ and $F(\infty)=1$. Equation (3) is a stochastic differential equation with random force acting on the charged particle.

1.1. HEATING AND DIFFUSION OF IONS

First we consider heating and diffusion of ions. Since the frequencies of stochastic oscillations significantly exceed the ion cyclotron frequency, we neglect the effect of the magnetic field on the motion of ions, and then the solution to Eq. (3) is

$$\vec{v}(t)=\frac{e_i}{m_i}F(r)\int_{t_0}^{t}\vec{E}(\vec{r},t')dt', \qquad (4)$$

where $t=t_0$ is the time of occurrence of turbulence in plasma. The rate of change of the mean square of the velocity ion velocity we find by multiplying (3) by $\vec{v}(t)$ (4) and averaging this product over a long period of time

$$\left\langle \vec{v}\frac{d\vec{v}}{dt}\right\rangle=\frac{1}{2}\frac{d\langle v^2\rangle}{dt}=\frac{e_i^2}{m_i^2}F^2(r)\int_{t_0}^{t}\left\langle \vec{E}(\vec{r},t')\vec{E}(\vec{r},t)\right\rangle dt'. \quad (5)$$

Assume that an electrostatic turbulence satisfies the following conditions

$$\left\langle \vec{E}(\vec{r},t)\right\rangle=0,\ \left\langle \vec{E}(\vec{r},t')\vec{E}(\vec{r},t)\right\rangle=\left\langle \vec{E}^2(\vec{r},t)\right\rangle\delta(t'-t), \quad (6)$$

that is the electric-field at a time t is considered to be completely uncorrelated with it at any other time (white noise). Here $\left\langle \vec{E}^2(\vec{r},t)\right\rangle$ is the average value of the square of the amplitude of electric-field noise far from the region with a high level of turbulence. Accounting (6) in (5) gives

$$\frac{d\langle v^2\rangle}{dt}=\frac{e_i^2}{m_i^2}F^2(r)\left\langle \vec{E}^2(\vec{r},t)\right\rangle. \qquad (7)$$

The solution of this equation,

$$\langle v^2\rangle=\langle v_0^2\rangle+\frac{e_i^2}{m_i^2}F^2(r)\left\langle \vec{E}^2(\vec{r},t)\right\rangle t, \qquad (8)$$

shows that the mean square velocity, in fact the ion temperature, in the region of an increased level of turbulent pulsations of the electric field increases linearly with time. In (8) $\langle v_0^2\rangle$ is the initial value of the mean square of the ion velocity before the appearance of turbulence, that is, the corresponding value for the environment.

Now we find the value of the random displacement $\vec{r}(t)$ of ion by integrating (4) over time,

$$\vec{r}(t)=\int_{t_0}^{t}\delta\vec{v}dt'=\frac{e_i}{m_i}F(r)\int_{t_0}^{t}\int_{t_0}^{t'}\vec{E}(\vec{r},t'')dt''dt', \quad (9)$$

and then obtain the rate of change in mean square displacement by multiplying (4) by (9) and averaging over a large time interval

$$\left\langle \vec{r}(t)\frac{d\vec{r}(t)}{dt}\right\rangle=\frac{1}{2}\frac{d\left\langle \vec{r}(t)^2\right\rangle}{dt}=\frac{e_i^2}{m_i^2}F^2(r)\times$$

$$\times\left\langle \int_{t_0}^{t}\int_{t_0}^{t'}\vec{E}(\vec{r},t'')dt''dt'\int_{t_0}^{t}\vec{E}(\vec{r},t')dt'\right\rangle. \qquad (10)$$

Write $\vec{E}(\vec{r},t)$ in the form of a Fourier integral

$$\vec{E}(\vec{r},t)=\int_{-\infty}^{\infty}\vec{E}(\vec{r},\omega)\exp(-i\omega t)d\omega. \qquad (11)$$

Substituting (11) into (10) we get

$$\frac{1}{2}\frac{d\left\langle \vec{r}^2(t)\right\rangle}{dt}=$$

$$=\frac{e_i^2}{m_i^2}F^2(r)\left\langle \int_{t_0}^{t}\int_{t_0}^{t'}\int_{-\infty}^{\infty}\vec{E}(\vec{r},\omega)\exp(-i\omega t'')d\omega dt''dt'\times\right.$$

$$\left.\times\int_{t_0}^{t}\vec{E}(\vec{r},t')dt'\right\rangle. \qquad (12)$$

Integrate (12) over t

$$\frac{1}{2}\frac{d\left\langle \vec{r}(t)^2\right\rangle}{dt}=$$

$$=\frac{e_i^2}{m_i^2}F^2(r)\left\langle \int_{t_0}^{t}\int_{-\infty}^{\infty}\frac{\vec{E}(\vec{r},\omega)}{\omega^2}\exp(-i\omega t')d\omega dt'\times\right.$$

$$\left.\times\int_{t_0}^{t}\vec{E}(\vec{r},t')dt'\right\rangle,$$

and find the inverse Fourier transform

$$\frac{1}{2}\frac{d\left\langle \vec{r}(t)^2\right\rangle}{dt}\approx\frac{e_i^2}{m_i^2\Delta\omega^2}F^2(r)\int_{t_0}^{t}\left\langle \vec{E}(\vec{r},t')\vec{E}(\vec{r},t)\right\rangle dt',$$

where $\Delta\omega$ is the width of the frequency spectrum of the turbulent pulsations of an electric field. We assume this is in order of magnitude $\Delta\omega\approx\omega$. Accounting for (6), we finally obtain

$$\frac{1}{2}\frac{d\left\langle \vec{r}(t)^2\right\rangle}{dt}\approx\frac{e_i^2}{2m_i^2\Delta\omega^2}F^2(r)\left\langle \vec{E}^2(\vec{r},t)\right\rangle. \qquad (13)$$

Equation (13) determines the diffusion coefficient of the plasma in the turbulent field of electrostatic turbulence.

1.2. HEATING AND DIFFUSION OF ELETRONS

For electrons, we take into account the magnetic field. First, we assume that the effect of the stochastic electric field is small and write down the integrals of motion from the equation (3)

$$\frac{d\vec{v}}{dt}=\frac{e_e}{m_e}\left[\vec{v},\vec{B}\right]. \qquad (14)$$

One of the integrals of motion obtained from equation (14) is the kinetic energy of the electron

$$W=\frac{m_e v^2}{2}. \qquad (15)$$

The integrals of motion are also the coordinates of the guiding center

$$X=x+\frac{v_y}{\omega_{ce}},\ \ Y=y-\frac{v_x}{\omega_{ce}}, \qquad (16)$$

which mean the invariability in time of the coordinates of the guiding center of an electron rotating in a magnetic field.

However, the existence of stochastic electric fields in plasma leads to slight distortions in these integrals of motion. Now we consider the solution of equation (3) in the first approximation and find the changes in the integrals of motion caused by stochastic electric fields. We consider this effect only on the motion of the electron across the magnetic field, neglecting the motion along the magnetic field.

Represent $\vec{v}$ in the form:

$$\vec{v} = \vec{v}_0 + \vec{v}_1 ,$$

where $\vec{v}_1$ is the velocity fluctuation caused by stochastic fields which is determined by the equation

$$\frac{d\vec{v}_1}{dt} = \frac{e_e}{m_e} F(x)\vec{E}(\vec{r},t) + \frac{e_e}{m_e}\left[\vec{v}_1,\vec{B}\right]. \quad (17)$$

In order to estimate the change in the kinetic energy of electrons we find the dot product $\vec{v}_1$ and (17) averaging it over time

$$\left\langle \vec{v}_1 \frac{d\vec{v}_1}{dt} \right\rangle = \frac{e_e}{m_e} F(x)\left\langle \vec{v}_1 \vec{E}(\vec{r},t)\right\rangle . \quad (18)$$

Find $\vec{v}_1$ from (17). The solutions of the eq. (17) for the components v_{1x} and v_{1y} are

$$v_{1x} = \frac{e_e F(x)}{m_e \omega_{ce}^2}\frac{dE_x(\vec{r},t)}{dt} + \frac{e_e F(x)}{m_e \omega_{ce}} E_y(\vec{r},t), \quad (19)$$

$$v_{1y} = \frac{e_e F(x)}{m_e \omega_{ce}^2}\frac{dE_y(\vec{r},t)}{dt} - \frac{e_e F(x)}{m_e \omega_{ce}} E_x(\vec{r},t). \quad (20)$$

Substituting (19) and (20) into (18), we obtain

$$\frac{1}{2}\frac{d\left\langle \vec{v}_1^2\right\rangle}{dt} = \frac{e_e^2}{m_e^2}\frac{\omega}{\omega_{ce}^2} F^2(x)\frac{1}{T}\left\langle \vec{E}^2(\vec{r},t)\right\rangle, \quad (21)$$

where T is the time over which averaging is performed. Assuming that T is much larger than the period of field oscillations, we find that there is no heating of the electrons by low-frequency stochastic electric fields, whereas ions, the cyclotron frequency of which is much lower than the frequency of the stochastic field, are heated.

Calculate the change in the coordinates of the guiding center due to stochastic electric fields. Represent the X coordinate in the form

$$X = X_0 + X_1 , \quad (22)$$

where X_1 is the random displacement of the coordinate of the guiding center due to stochastic electric fields. Using (16) we write

$$X_1 = x_1 + \frac{1}{\omega_{ce}} v_{1y} . \quad (23)$$

In (23), the value of x_1 is determined by integrating v_{1x} (19) over time

$$x_1 = \int_0^t v_{1x}(t')dt' =$$

$$= \frac{e_e F(x)}{m_e \omega_{ce}^2}\left(E_x(\vec{r},t) + \omega_{ce}\int_0^t E_y(\vec{r},t')\right). \quad (24)$$

The value of v_{1y} in (23) is given by (20).

Thus, the random displacement of the x-coordinate of the guiding center is equal to

$$X_1 = \frac{e_e F(x)}{m_e \omega_{ce}}\int_0^t E_y(\vec{r},t')dt' + \frac{e_e F(x)}{m_e \omega_{ce}^3}\frac{dE_y(\vec{r},t)}{dt} . \quad (25)$$

Next, we find the rate of change of X_1 by differentiating (25) with respect to time

$$\frac{dX_1}{dt} = v_{x1} + \frac{1}{\omega_{ce}}\frac{dv_{y1}}{dt} = \frac{e_e}{m_e}\frac{1}{\omega_{ce}} F(x)E_y(\vec{r},t), \quad (26)$$

and then obtain the rate of change in mean square displacement multiplying (26) by (25) and averaging over a large time interval

$$\left\langle X_1 \frac{dX_1}{dt}\right\rangle = \frac{1}{2}\frac{d\left\langle X_1^2\right\rangle}{dt} = \frac{e_e^2}{m_e^2}F^2(x)\times$$

$$\times\frac{1}{\omega_{ce}^2}\left(\int_0^t \left\langle E_y(\vec{r},t')E_y(\vec{r},t)\right\rangle dt' + \frac{1}{\omega_{ce}^2}\frac{1}{2}\frac{d\left\langle E_y(\vec{r},t)\right\rangle}{dt}\right). \quad (27)$$

Neglecting the second term in (27) which is much smaller than the first one, we obtain

$$\frac{1}{2}\frac{d\left\langle X_1^2\right\rangle}{dt} = \frac{e_e^2}{m_e^2}F^2(x)\frac{1}{\omega_{ce}^2}\int_0^t \left\langle E_y(\vec{r},t')E_y(\vec{r},t)\right\rangle dt'. \quad (28)$$

Taking into account the condition (6) we obtain

$$\frac{1}{2}\frac{d\left\langle X_1^2\right\rangle}{dt} = \frac{e_e^2}{2m_e^2\omega_{ce}^2}F^2(x)\left\langle E_y^2(\vec{r},t)\right\rangle . \quad (29)$$

Thus, the rate of mean square displacement of the guiding center coordinate along the x-axis is determined by the mean value of the square of y-component of the stochastic electric field. Equation (29) can also be written as

$$\frac{1}{2}\frac{d\left\langle X_1^2\right\rangle}{dt} = \frac{c^2}{2B^2}F^2(x)\left\langle E_y^2(\vec{r},t)\right\rangle, \quad (30)$$

or, label $v_{dx} = cE_y / B$, which is equal to the velocity of the drift motion of a particle in crossed fields along the x-axis we obtain

$$\frac{1}{2}\frac{d\left\langle X_1^2\right\rangle}{dt} = \frac{F^2(x)}{2}\left\langle v_{dx}^2(\vec{r},t)\right\rangle . \quad (31)$$

Then the rate of change in the mean square displacement is equal to the mean value of the square of the drift rate of particles in crossed fields.

Similarly, for the mean square displacement of the guiding center coordinate along the y-axis we obtain

$$\frac{1}{2}\frac{d\left\langle Y_1^2\right\rangle}{dt} = -\frac{e_e^2}{2m_e^2\omega_{ce}^2}F^2(x)\left\langle E_x^2(\vec{r},t)\right\rangle, \quad (32)$$

or, otherwise, by analogy with the displacement along the x-axis

$$\frac{1}{2}\frac{d\left\langle Y_1^2\right\rangle}{dt} = \frac{F^2(x)}{2}\left\langle v_{dy}^2(\vec{r},t)\right\rangle, \quad (33)$$

where $v_{dy} = cE_x / B$ is the drift velocity of a particle in crossed fields along the y-axis.

Summing (30) and (32) we obtain

$$\frac{1}{2}\frac{d\left\langle \vec{R}_1^2\right\rangle}{dt} = \frac{e_e^2}{2m_e^2\omega_{ce}^2}F^2(x)\left\langle \vec{E}^2(\vec{r},t)\right\rangle, \quad (34)$$

where $\vec{R}$ is the position vector of the guiding center of electron with coordinates (X,Y) and $\vec{R}_1(X_1,Y_1)$ is its perturbation caused by stochastic electric fields. Equation (34) determines the diffusion coefficient of electrons in the stochastic electric fields of low frequency turbulence.

Compare the diffusion coefficients of electrons and ions for this divide (34) by (13)

$$\frac{1}{2}\frac{d\left\langle \vec{R}_1^2\right\rangle}{dt}\Bigg/\frac{1}{2}\frac{d\left\langle \vec{r}(t)^2\right\rangle}{dt}=\frac{m_i^2\Delta\omega^2}{m_e^2\omega_{ce}^2}=\frac{\Delta\omega^2}{\omega_{ci}^2}>>1. \quad (35)$$

Thus, the diffusion coefficient of the guiding centers of electrons significantly exceeds the diffusion coefficient of ions.

2. DRIFT MOTION OF PARTICLES

In addition to diffusion, there is also quasilinear motion of a particle caused by a ponderomotive force in this region due to the radial gradient of the envelope of the amplitude of the turbulent field. Ponderomotive force affecting on the particle in the case of an inhomogeneous coherent electric field E oscillating with the frequency ω is determined by (2).

Now we obtain the equation of motion of the particles with the ponderomotive force affecting on a particle in an inhomogeneous electrostatic turbulence.

2.1. DRIFT MOTION OF IONS

The equation of motion of ion along x-axis is

$$m_i\frac{d^2x}{dt^2}=e_iF(x)E_x(\vec{r},t), \quad (36)$$

where $E_x(\vec{r},t)$ is the projection of the electric field strength onto the x-axis. We represent the x-coordinate of ion as the sum of the oscillatory and quasilinear changes

$$x=\tilde{x}+\bar{x}, \quad (37)$$

where quasilinear change means the mean value of the x-coordinate of ion over a long period of time, $\bar{x}=\langle x\rangle$. If the initial position of ion is x_0, and $\tilde{x}<<x_0$ then we can use Taylor expansion on the force equation about x_0. Substituting (37) into (36), we obtain

$$m_i\frac{d^2x}{dt^2}=e_i\left(F(x_0)+\tilde{x}\nabla F(x_0)\right)E_x(\vec{r},t). \quad (38)$$

For the oscillating part of trajectory, we obtain

$$m_i\frac{d^2\tilde{x}}{dt^2}=e_iF(x_0)E_x(\vec{r},t).$$

This equation is the x-component of equation (3) at $\vec{B}=0$ and therefore has the solution (9)

$$\tilde{x}(t)=\frac{e_i}{m_i}F(x_0)\int_{t_0}^{t}\int_{t_0}^{t'}E_x(\vec{r},t'')dt''dt'. \quad (39)$$

Writing $E_x(\vec{r},t)$ in the form of a Fourier integral (11) we obtain for $\tilde{x}(t)$

$$\tilde{x}(t)=\frac{e_i}{m_i}F(r_0)\int_{t_0}^{t}\int_{t_0}^{t'}\int_{-\infty}^{\infty}E_x(\vec{r},\omega)\exp(-i\omega t'')d\omega dt''dt'. \quad (40)$$

Integrating (40) over t'' and t' we get

$$\tilde{x}(t)=\frac{e_i}{m_i}F(r_0)\int_{-\infty}^{\infty}\frac{E_x(\vec{r},\omega)}{\omega^2}\exp(-i\omega t)d\omega\approx$$
$$\approx\frac{e_i}{m_i}F(x_0)\frac{E_x(\vec{r},t)}{\Delta\omega^2}. \quad (41)$$

Now we average (38) over a large period of time

$$m_i\frac{d^2\bar{x}}{dt^2}=e_i\nabla F(x_0)\left\langle\tilde{x}(t)E_x(\vec{r},t')\right\rangle. \quad (42)$$

Substitute (41) into (42)

$$m_i\frac{d^2\bar{x}}{dt^2}=\frac{e_i^2}{m_i\Delta\omega^2}F(x_0)\nabla F(x_0)\left\langle E_x(\vec{r},t)E_x(\vec{r},t')\right\rangle,$$

and use (6)

$$m_i\frac{d^2\bar{x}}{dt^2}=\frac{e_i^2}{2m_i\Delta\omega^2}\nabla F^2(x_0)\left\langle E_x^2(\vec{r},t)\right\rangle\delta(t'-t). \quad (43)$$

Thus, we have obtained the equation for the drift motion of ion under the action of a non-uniform electrostatic turbulent field. The expression on the right side of (43) is a ponderomotive force which is the effect of the radial inhomogeneity of the electrostatic turbulent field. This force is proportional to the delta function. This is due to the fact that the affecting of this force occurs only at certain instants of time, corresponding to the moments of "collision" of particles with random pulsations of the electrostatic field.

Integrating (43), we obtain the velocity of the drift motion of ion along the x-axis

$$\frac{d\bar{x}}{dt}=\int_{t_0}^{t}\frac{d^2\bar{x}}{dt^2}dt'=\frac{e_i^2}{4m_i^2\Delta\omega^2}\nabla F^2(x_0)\left\langle E_x^2(\vec{r},t)\right\rangle. \quad (44)$$

2.2. DRIFT MOTION OF ELECTRONS

Now we obtain the equation of motion of the guiding center of electron with the ponderomotive force in an inhomogeneous electrostatic turbulence. We represent the random displacement of the coordinate of the guiding center X_1 as the sum of oscillatory $\tilde{X}$ and quasilinear $\bar{X}$ components

$$X_1=\tilde{X}+\bar{X}, \quad (45)$$

where $\langle X_1\rangle=\bar{X}$ and $\langle\tilde{X}\rangle=0$. To determine the drift velocity of the guiding center we use the equation (16)

$$\frac{dX_1}{dt}=\frac{d\tilde{X}}{dt}+\frac{d\bar{X}}{dt}=\frac{e_e}{m_e}\frac{1}{\omega_{ce}}F(x)E_y(\vec{r},t). \quad (46)$$

Expand envelop function $F(x)$ in a Taylor series about the initial value of the position of the coordinate of the guiding center

$$F(x)=F(X_0)+\nabla F(X_0)\cdot x_1 \quad (47)$$

and substitute it into (46)

$$\frac{dX_1}{dt}=\frac{e_e}{m_e}\frac{1}{\omega_{ce}}\left(F(X_0)+x_1\nabla F(X_0)\right)E_y(\vec{r},t). \quad (48)$$

Then averaging over a large time interval we obtain the rate of quasi-linear change in the coordinate of the guiding center

$$\frac{d\langle X_1\rangle}{dt}=\frac{d\bar{X}}{dt}=\frac{e_e}{m_e}\frac{1}{\omega_{ce}}\nabla F(X_0)\left\langle x_1E_y(\vec{r},t)\right\rangle. \quad (49)$$

Calculate $\langle x_1 E_y(\vec{r},t)\rangle$ by substituting here the value x_1 (24):

$$\langle x_1 E_y(\vec{r},t)\rangle =$$

$$= \frac{e_e F(X_0)}{m_e \omega_{ce}^2}\left\langle \left(\vec{E}_x(\vec{r},t) + \omega_{ce}\int_0^t \vec{E}_y(\vec{r},t')dt' \right) E_y(\vec{r},t) \right\rangle .$$

Since $\langle \vec{E}_x(\vec{r},t) E_y(\vec{r},t)\rangle = 0$ we obtain

$$\frac{d\bar{X}}{dt} = \frac{e_e^2}{m_e^2 \omega_{ce}^2} F(X_0)\nabla F(X_0)\int_0^t \langle E_y(\vec{r},t) E_y(\vec{r},t')\rangle dt' . \quad (50)$$

Integration (50) using condition (6), we obtain

$$\frac{d\bar{X}}{dt} = \frac{e_e^2}{4m_e^2 \omega_{ce}^2}\nabla F^2(X_0)\langle E_y^2(\vec{r},t)\rangle . \quad (51)$$

Equation (51) determines the velocity of the drift motion of the guiding center of electron along the x-axis.

Now compare the drift velocities of ions and guiding centers of electrons. Dividing (51) by (44) we get

$$\frac{d\bar{X}}{dt}\Big/\frac{d\bar{x}}{dt} = \frac{m_i^2 \Delta\omega^2}{m_e^2 \omega_{ce}^2} = \frac{\Delta\omega^2}{\omega_{ci}^2} >> 1 . \quad (52)$$

Inequality (52) means that electrons leave the region of increased turbulence level much faster than ions.

Note also that the ratio of the diffusion coefficients of the guiding centers of electrons and ions is of the same order as the ratio of their drift velocities, namely $\Delta\omega^2/\omega_{ci}^2$.

3. STATIONARY DENSITY DISTRIBUTION

Inhomogeneous electrostatic turbulence leads to a change in the plasma density distribution. We find only stationary distribution of the electron density, since this process occurs faster than for ions. The evolution of the distribution function $f(x,t)$ as a result of diffusion as well as the drift motion of particles is governed by the Fokker-Planck equation

$$\frac{\partial f(x,t)}{\partial t} = -\frac{\partial}{\partial x}\big(A(x) f(x,t)\big) +$$

$$+\frac{\partial^2}{\partial x^2}\left(\frac{B(x)}{2} f(x,t)\right), \quad (53)$$

where A is the drift velocity, $B/2$ is the diffusion coefficient. Above it was obtained that diffusion coefficient is (34)

$$\frac{B}{2} = \frac{1}{2}\frac{d\langle \vec{R}_1^2\rangle}{dt} = \frac{e^2}{2m^2\omega_c^2} F^2(x)\langle \vec{E}^2(\vec{r},t)\rangle , \quad (54)$$

and drift velocity is (51)

$$A = \frac{d\bar{X}}{dt} = \frac{e_e^2}{4m_e^2 \omega_{ce}^2}\nabla F^2(X_0)\langle E_y^2(\vec{r},t)\rangle . \quad (55)$$

We now find the dependence of the plasma density on the x-axis in a stationary state, $n(x) = f(x)$, assuming that the evolution of the distribution function has ended. Equating in (53) the derivative of the distribution function with respect to time to zero, we obtain the equation

$$-\frac{\partial}{\partial x}\big(A(x) n(x)\big) + \frac{\partial^2}{\partial x^2}\left(\frac{B(x)}{2} n(x)\right) = 0 . \quad (56)$$

This equation is simplified and reduces to the following

$$-A(x) n(x) + \frac{\partial}{\partial x}\left(\frac{B(x)}{2} n(x)\right) = 0 , \quad (57)$$

and then

$$\frac{2A(x)dr}{B(x)} = \frac{d\big(B(x) n(x)\big)}{B(x) n(x)} . \quad (58)$$

Substituting (54) and (55) into (58) we get

$$\frac{dF(r)}{F(r)}\frac{\langle \vec{E}_r^2(\vec{r},t)\rangle}{\langle \vec{E}^2(\vec{r},t)\rangle} = \frac{d\big(F^2(r) n\big)}{nF^2(r)} . \quad (59)$$

Assuming that $\langle \vec{E}_x^2(\vec{r},t)\rangle / \langle \vec{E}^2(\vec{r},t)\rangle = 1/2$ and integrating (59), we obtain

$$n(x) = \frac{C}{F^{3/2}(x)} . \quad (60)$$

We choose the integration constant C using the condition $n(\infty) = n_0$, where n_0 is the plasma density far from the region with increased turbulence. To the same value, the plasma density was equal in this region before the appearance of turbulence, that is, when $t < t_0$. So we get $C = n_0$ since, as we suggested, $F(\infty) = 1$. Finally, we obtain the plasma density distribution long after the appearance of a region with an increased level of turbulence in homogeneous plasma

$$n(x) = \frac{n_0}{F^{3/2}(x)} . \quad (61)$$

In accordance with (61), the minimum plasma density is reached in the region with the maximum level of low-frequency turbulence. Thus a region with a depleted electron density is formed. Distribution of the plasma density (61) is the result of the evolution of initially homogeneous plasma due to inhomogeneous electrostatic turbulence.

CONCLUSIONS

The inhomogeneous stochastic electric fields in plasma in a magnetic field leads to the occurrence of ponderomotive force, which causes a quasilinear drift motion of particles outward from the region of an increased level of stochastic oscillations of the electric field. This effect takes place both for ions whose cyclotron frequency is lower than the frequency of stochastic oscillations, and for electrons, whose cyclotron frequency significantly exceeds this frequency. It is shown that the drift rate of the guiding centers of electrons exceeds the ions drift rate by a factor $\Delta\omega^2/\omega_{ci}^2 >> 1$, where $\Delta\omega$ is the width of the spectrum of stochastic oscillations of an electric field, and thus electrons leave the region of increased turbulence much faster than ions.

Apart to drift motion, the increased diffusion of particles also occurs caused by their collisions with random pulsations of electrostatic turbulence, and the diffusion coefficient of electrons exceeds the diffusion coefficient of ions by a factor $\Delta\omega^2/\omega_{ci}^2 >> 1$.

The drift and diffusion of particles lead to a decrease in the plasma density in the region of an increased level of stochastic oscillations of the electric field. The stationary distribution of the plasma density was determined from the Fokker-Planck equation, where the obtained values of the drift velocity and the diffusion coefficient were used. It is shown that the plasma density distribution is ultimately determined by (61).

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ДВИЖЕНИЕ ЗАРЯЖЕННЫХ ЧАСТИЦ В МАГНИТНОМ И НЕОДНОРОДНОМ СТОХАСТИЧЕСКОМ ЭЛЕКТРИЧЕСКОМ ПОЛЯХ

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Решается уравнение движения заряженных частиц плазмы в однородном магнитном поле и неоднородном стохастическом электрическом поле с характерной частотой колебаний, много меньшей электронной циклотронной частоты и много большей ионной циклотронной частоты. Рассмотрены диффузия, дрейфовое движение ионов и ведущих центров электронов, вызванные неоднородностью стохастического электрического поля. Полученные значения коэффициента диффузии и скорости дрейфа используются в уравнении Фоккера-Планка для определения стационарного распределения плотности плазмы, обусловленного влиянием неоднородного стохастического поля.

РУХ ЗАРЯДЖЕНИХ ЧАСТИНОК У МАГНІТНОМУ І НЕОДНОРІДНОМУ СТОХАСТИЧНОМУ ЕЛЕКТРИЧНОМУ ПОЛЯХ

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Вирішується рівняння руху заряджених частинок плазми в однорідному магнітному полі і неоднорідному стохастичному електричному полі з характерною частотою коливань, значно меншої електронної циклотронної частоти і значно більшої іонної циклотронної частоти. Розглянуто дифузію, дрейфовий рух іонів і ведучих центрів електронів, спричинені неоднорідністю стохастичного електричного поля. Отримані значення коефіцієнта дифузії і швидкості дрейфу використовуються в рівнянні Фоккера-Планка для визначення стаціонарного розподілу щільності плазми, обумовленого впливом неоднорідного стохастичного поля.