MODELLING OF DUAL-POLARIZATION INTERFEROMETRY IN STELLARATORS

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The dual polarization interferometers have been designed and applied in the Uragan-3M and Uragan-2M torsatrons. However, an extensive modeling of microwave propagation through the plasma in the sheared magnetic field should be provided for correct interpretation of data. For this purpose, the system of two bounded ordinary differential equations of the second order for the electric fields of the ordinary and extraordinary waves was solved numerically. The transfer matrix from launching to receiving waveguides was obtained in the wide range of plasma parameters. This gives the possibility to understand the results of already fulfilled measurements and to propose the optimized way for dual-polarization interferometry in stellarators.

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INTRODUCTION

Ordinary wave interferometry is a well known and commonly used plasma diagnostics for fusion devices and plasma technology [1, 2]. The ordinary wave number depends only on plasma density in the case of perpendicular probing with respect to the main magnetic field. Thus, the ordinary wave phase shift is proportional to the line-integrated plasma density along the chord of probing for the wave frequency greater than plasma frequency. For the extraordinary wave perpendicular probing, the phase shift depends on plasma density and the confining magnetic field distributions. Since the magnetic field is known for stellarator/torsatron devices, additional information about plasma density profile may be inferred from the extraordinary wave phase shift measurements. For example, this may be the peakedness of the plasma density profile $\overline{n}/n_0$, where n_0 is the plasma density on magnetic axis and $\overline{n}_e$ - averaged plasma density.

1. THE DEDICATED EXPERIMENTS

The dual polarization interferometer was first installed in the Uragan-3M torsatron very close to E-E minor cross-section [3]. The horn antennas which were connected to the oversized waveguides were used for probing plasma. These waveguides allow propagation of E- and H-polarizations modes without their interference. For independent determination of the phase shifts of O- and X - waves an orthomode transducer (OMT) was fabricated and tested. OMT has one input for oversized waveguide and two outputs which operate in the single mode regime.

The classic homodyne interferometers which have double balanced detection were connected to the output waveguides of OMT. During the determination of the phase shifts some difficulties were encountered which were caused by fast changes in the amplitude of signals passing through the plasma. Also, the small differential sensitivity of $sin(\varphi)$ when $\varphi \approx 0.5\pi + m\pi,\ m = 0,1\ldots$ made difficult determination the dynamics of change of the phase shift in these areas. In part the latter problem was solved by not-in-phase signal of phase shift for the ordinary and extraordinary waves.

The dual polarization heterodyne interferometer has been designed and installed in Uragan-2M torsatron at the cross-section where magnetic surfaces are vertically elongated [4]. For data analysis, the set of phase shifts was calculated using real spatial dependence of the magnetic field and the following parameterization of plasma density, $n(\Psi) = n_0\left(e^{\xi} - e^{\xi\Psi}\right)/\left(e^{\xi} - 1\right)$. Here Ψ is the flux surface label equals to 0 at the magnetic axis and equals to 1 at the last closed magnetic surface, ξ is profile peaking parameter (Fig. 1).

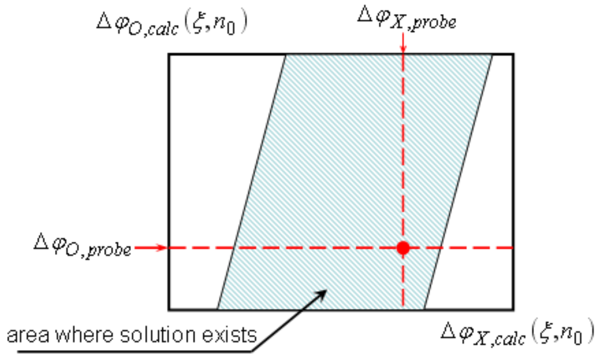

Fig. 1. Calculated phase shifts for X-wave (X axis) and O-wave (Y axis) for different values of n_0 and ξ. In the shaded area the solution for density profile exist

Herewith the WKB approximation was used for the calculations of the phase shifts of an ordinary (*O*-) and an extraordinary (*X*-) waves propagating through the plasma perpendicularly to the main magnetic field $B_0(r)$. In this approximation

$$\Delta\varphi_o = \frac{\omega}{c}\int_b^a\left[\sqrt{1-\frac{\omega_p^2(r)}{\omega^2}}-1\right]dr \approx -\frac{\overline{\omega}_p^2}{\omega c}(a-b) \sim \overline{n}_e(a-b),$$

(1)

$$\Delta\varphi_x = \frac{\omega}{c}\int_b^a\left[\sqrt{\frac{\left[1-\frac{\omega_p^2(r)}{\omega^2-\omega_c^2(r)}\right]^2-\left[\frac{\omega_c}{\omega}\,\frac{\omega_p^2(r)}{\omega^2-\omega_c^2(r)}\right]^2}{1-\frac{\omega_p^2(r)}{\omega^2-\omega_c^2(r)}}}-1\right]dr \approx$$

$$\approx -\frac{\omega}{c}\int_b^a\frac{1}{2}\,\frac{\omega_p^2(r)}{\omega^2-\omega_c^2(r)}\,dr < \Delta\varphi_o.$$

Here $\omega = 2\pi f$ is the probing wave frequency, c is the speed of light, $\omega_p(r) = \left(\frac{4\pi n_e(r) e^2}{m}\right)^{1/2}$ is the electron plasma frequency, $\omega_c(r) = \frac{eB_0(r)}{m\,c}$ is the electron cyclotron frequency, b is the position of the launching horn and a is the position of the receiving horn. It is worth noting that $abs(\Delta\varphi_X) > abs(\Delta\varphi_O)$.

Calculated phase shifts were matched with experimental results (Fig. 2) in order to determine the central density and peaking parameter of the Uragan-2M plasma.

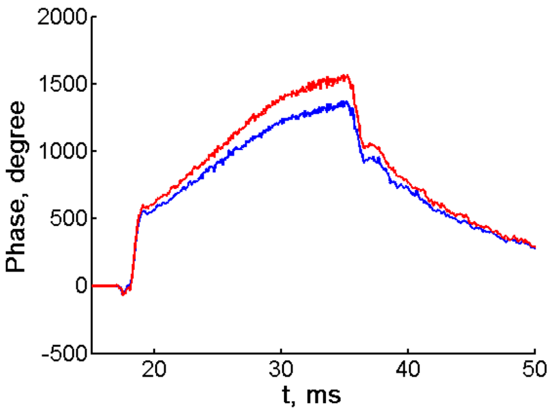

Fig. 2. Interferometry raw phase shift: blue – O-wave, red – X-wave for shot #03-24-2014-0039

This gave the possibility to reconstruct the temporary change of the plasma density profile (Fig. 3).

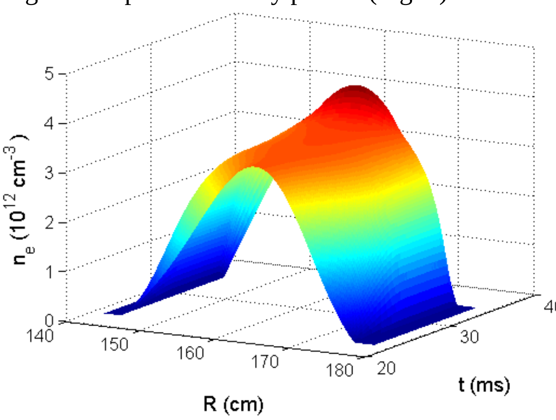

Fig. 3. Time evolution of plasma density profile from data of dual-polarization interferometry for shot #03-24-2014-0039

However, such straightforward data processing is feasible not for all shorts. There are cases when data goes beyond the shaded region of Fig. 1. That's why an extensive modeling of the microwave propagation through the plasma in the sheared magnetic field should be provided for the correct interpretation of the measurement results.

2. MODELLING OF THE MICROWAVE PROPAGATION

The model of plane inhomogeneous plasma in sheared main magnetic field (Fig. 4) was used in Section 3. Plasma is inhomogeneous in x-axis direction. Main magnetic field is located in the y-z plane. The angle between the magnetic field and z-axis ψ alters in x-direction. Probing waves propagate in x-axis direction. Such model is applicable for the interferometry in the torsatron/stellarator cross-sections with the up-down symmetry.

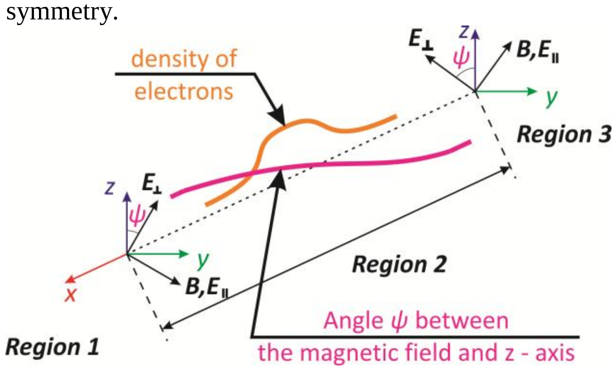

Fig. 4. The model geometry for the investigation of the probing wave propagation

Hereinafter tensor of the 'cold' plasma with components $\varepsilon_1 = 1 + \frac{\omega_p^2(r)}{\omega_c^2(r) - \omega^2}$, $\varepsilon_2 = \frac{\omega_c(r)}{\omega}\,\frac{\omega_p^2(r)}{\omega_c^2(r) - \omega^2}$ and $\varepsilon_3 = 1 - \frac{\omega_p^2(r)}{\omega^2}$ was used. Then Maxwell equations were transformed into the system of two bounded ordinary differential equations of the second order for the components of the electric field of an ordinary wave $E_{\|} = E_y \sin\psi + E_z \cos\psi$ and an extraordinary wave $E_{\perp} = E_y \cos\psi - E_z \sin\psi$:

$$E_{\|}'' + \left(\frac{\omega^2}{c^2}\varepsilon_3 - \psi'^2\right)E_{\|} - 2\psi' E_{\perp}' + \psi'' E_{\perp} = 0\,, \tag{2}$$

$$E_{\perp}'' + \left(\frac{\omega^2}{c^2}\,\frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1} - \psi'^2\right)E_{\perp} + 2\psi' E_{\|}' + \psi'' E_{\|} = 0\,. \tag{3}$$

Here prime indicates derivative by x. In the vacuum regions near the horns (see region 1 and region 3 of Fig. 4) these equations are simplified to

$$E_{\|}'' + \frac{\omega^2}{c^2}E_{\|} = 0 \text{ and } E_{\perp}'' + \frac{\omega^2}{c^2}E_{\perp} = 0\,. \tag{4}$$

Equations (4) has analytical solutions

$$E_{\|} = C_1 \exp\left(i\frac{\omega}{c}x\right) + C_2 \exp\left(-i\frac{\omega}{c}x\right) \text{ and}$$

$$E_{\perp} = C_3 \exp\left(i\frac{\omega}{c}x\right) + C_4 \exp\left(-i\frac{\omega}{c}x\right). \tag{5}$$

It gives the possibility to obtain solutions (2) and (3) numerically by means of setting running away to $x \to +\infty$ O-wave ($C_1 = 1, C_2 = C_3 = C_4 = 0$) or X-wave ($C_3 = 1$, $C_1 = C_2 = C_4 = 0$) and integrating Eqs. (2), (3) in $-x$ direction. As well, the cases of propagating to $x \to -\infty$ O- and X-waves were considered. In Fig. 5 the case of outgoing to $x \to +\infty$ X-wave is shown. By dint of combining in certain proportions of outgoing to $x \to +\infty$ X-wave with outgoing to $x \to +\infty$ O-wave, the plasma probing from $x < 0$ side by O-wave or X-

wave were recalculated. In Fig. 6 the case of *X*-wave probing is shown. As can be seen from this figure, not only the extraordinary wave, but the ordinary wave too, reaches the receiving horn at the *X*-wave probing. A similar situation takes place at the *O*-wave probing.

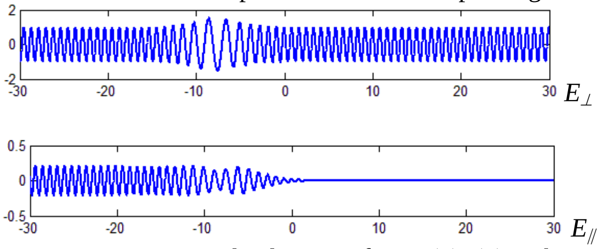

Fig. 5. Numerical solutions of Eqs. (2), (3) with outgoing to $x \to +\infty$ X-wave as boundary condition. Plasma is located from x=-20 cm to x=10 cm

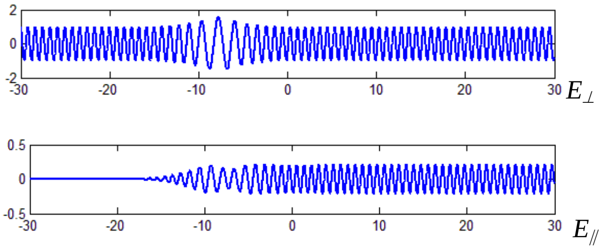

Fig. 6. Recalculated solutions for X-wave probing from the x < 0 side

As seen in the Fig. 6, there is a transformation of the probing *X*-wave into *O*-wave, which also falls into the receiving horn. The effect of the transformation is due to the shear of the magnetic field along the probing trace (last two terms in the Eqs. (2), (3)). This result is beyond the boundaries of the WKB approximation.

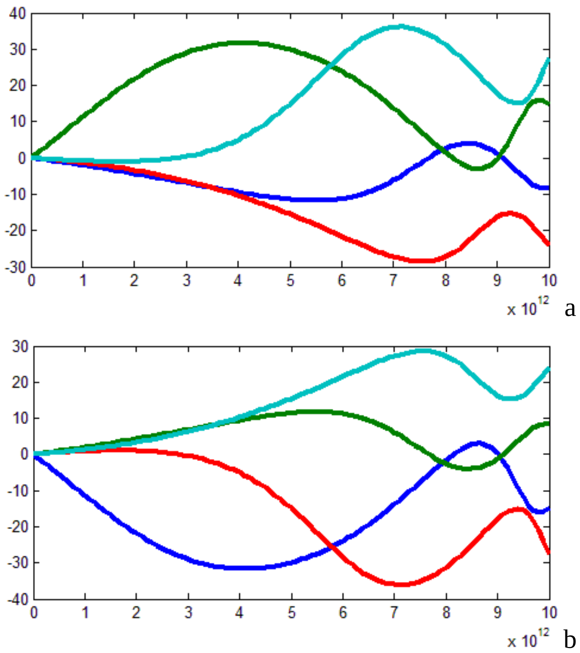

Fig. 7. The phase differences at the receiving antenna between the WKB approach and "sheared" solutions: a – for O-wave; b – for X-wave. Blue line – φ_O - φ_X = 0; red – φ_O - φ_X = 90°; green – φ_O - φ_X = 180°; light blue – φ_O - φ_X = 270° at the launching antenna

The effect of the magnetic field shear on the phase shifts of the probing signals in U-2M torsatron was investigated in detail (Figs. 7,a,b). As it was supposed, the mixture of the *O*-wave and the *X*-wave of the equal amplitudes was launched. As seen from Fig. 7, the difference between the WKB phase shifts and phase shifts accounted for magnetic shear reaches 40°. This value is comparable to the difference of the phase shifts of *O*-wave and *X*-wave measured by dual polarization interferometry. That is why they encountered difficulties during processing interferometry data based on assumption of Eq. (1) and might reconstruct density profile temporal evolution not for each shot.

From the radio point of view, the plasma can be represented as a quadrupole. In it, each pole corresponds to a receiving or launching antenna for ordinary or extraordinary wave (Fig. 8).

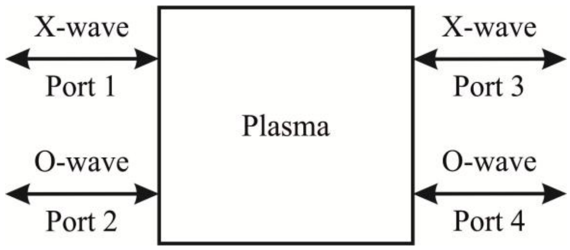

Fig. 8. Representation of plasma as radio quadrupole

In such representation, the transfer matrix $\hat{S}$ 4*4 from launching to receiving antennas was calculated in the wide range of plasma parameters. For example, if incident *X*-wave has amplitude *A=1*, then $S_{1,1}$ will be the amplitude of the transmitted *X*-wave, $S_{1,2}$ will be the amplitude of the reflected *X*-wave, $S_{1,3}$ will be the amplitude of the transmitted *O*-wave and $S_{1,4}$ will be the amplitude of the reflected *O*-wave. The left – right incidence asymmetry is a remarkable property of the transfer matrix. This asymmetry is caused by the combination of two factors. First, the spatial asymmetry of the plasma density profile according to the magnetic axis of torsatron and shift of the magnetic axis from the centre of the vessel. Second reason is the magnetic field asymmetry due to toroidal variation of the magnetic field.

3. OPTIMIZED DUAL POLARISATION INTERFEROMETER

The research performed allows one to propose the optimized design of dual polarization interferometer for Uragan-2M torsatron (Figs. 9, 10).

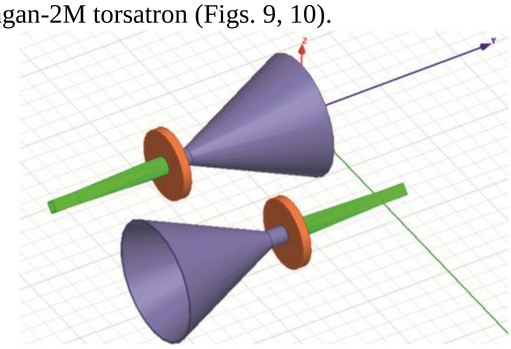

Fig. 9. Design of the launching / receiving elements of the interferometer

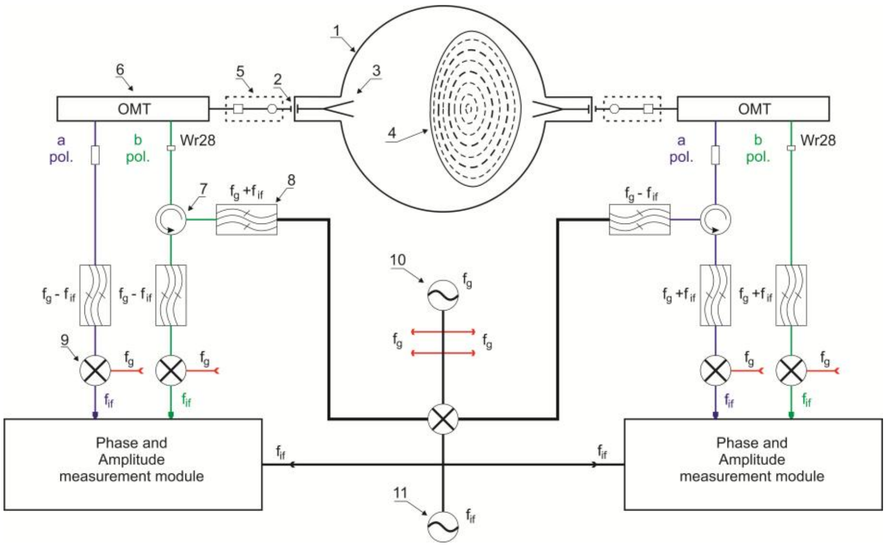

Fig. 10. Proposed circuit of the dual polarization interferometer: 1 – vacuum chamber U-2M; 2 – vacuum seals and rotating joints of waveguides; 3 – horn antennas; 4 – magnetic surfaces; 5 – transitions from round to square; 6 – polarizer selector; 7 – ferrite waveguide circulators; 8 – bandpass filters; 9 – frequency mixer; 10 – microwave oscillator (~ frequency 40 GHz); 11 – intermediate frequency generator (~ frequency 400 MHz)

CONCLUSIONS

Interaction of *O*- and *X*-waves, caused by shear of torsatron magnetic field must be taken into account while processing dual polarization interferometry data. The WKB approximation is not sufficient for correct description of the phase shifts despite the fact that requirement of unhomogeneity scale being much greater than the wavelength is met. The optimized dual polarization interferometer design was developed for the U-2M torsatron.

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МОДЕЛИРОВАНИЕ ДВУХПОЛЯРИЗАЦИОННОЙ ИНТЕРФЕРОМЕТРИИ В СТЕЛЛАРАТОРАХ

В. Филиппов, Д. Греков, Н. Азаренков

Двухполяризационные интерферометры применялись на торсатронах Ураган-3М и Ураган-2М. Для корректной интерпретации результатов экспериментов необходимо промоделировать распространение зондирующих волн через плазму в магнитном поле переменного направления. Для этого система двух связанных дифференциальных уравнений второго порядка для электрических полей обыкновенной и необыкновенной волн решалась численно. В широком диапазоне параметров плазмы была получена передаточная матрица от излучающего рупора к приемному рупору. Это дало возможность лучше понять результаты проведенных экспериментов и предложить оптимизированную схему двухполяризационного интерферометра.

МОДЕЛЮВАННЯ ДВОПОЛЯРИЗАЦІЙНОЇ ІНТЕРФЕРОМЕТРІЇ В СТЕЛАРАТОРАХ

В. Філіппов, Д. Греков, М. Азаренков

Двополяризаційні інтерферометри було застосовано в торсатронах Ураган-3М та Ураган-2М. Для коректної інтерпретації результатів експериментів, необхідно виконати моделювання поширення мікрохвиль, що зондують плазму, через плазму в магнітному полі змінного напрямку. З цією метою система двох пов’язаних диференційних рівнянь другого порядку для електричних полів звичайної та незвичайної хвиль розв’язувалась числовим методом. У широкому діапазоні параметрів плазми було отримано передаточну матрицю від рупора, що випромінює хвилі, до рупора, що їх приймає. Це дало можливість краще зрозуміти результати проведених експериментів і запропонувати оптимізовану схему двополяризаційного інтерферометра.