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ENVIRONMENT GUARD MODEL AS DYADIC THREE-PERSON GAME WITH THE GENERALIZED FINE FOR THE RESERVOIR POLLUTION

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There is investigated the noncooperative dyadic three-person game with the parameter, defining the penalty for the reservoir pollution. Given NE-solutions of this game as the environment guard model not only contain simultaneously symmetrical and favorable NE-situations for the pollution subjects, but also may be used to force these subjects to apply the cleaning installations completely within the non-equilibrant situation, allowing to lose minimally though.

Досліджується безкоаліційна діадична гра трьох осіб з параметром, що визначає стягнення за забруднення водойми. Наведені NE-розв'язки цієї гри як моделі охорони довкілля не тільки містять одночасно симетричні та вигідні NE-ситуації для суб'єктів забруднення, але також можуть бути використані для того, щоб змусити ці суб'єкти застосовувати очисні споруди у рамках нерівноважної ситуації, що, втім, дозволяє досягати мінімальних втрат.

Исследуется бескоалиционная диадическая игра трёх лиц с параметром, что определяет взыскание за загрязнение водоёма. Приведённые NE-решения этой игры как модели охраны окружающей среды не только содержат одновременно выгодные NE-ситуации для субъектов загрязнения, но также могут быть использованы для того, чтобы вынудить эти субъекты применять очистительные сооружения в рамках неравновесной ситуации, что, впрочем, позволяет достичь минимальных потерь.

Problem statement

Mathematical modeling of eco-defense events and systems is an actual problem of fundamental and natural sciences. The dyadic noncooperative game is the most comfortable mathematical model of conflict events by the interaction amongst several subjects of the environment pollution. In the game of this class a subject of pollution has only two pure strategies, one of which is to apply the cleaning installations, and another is to not apply the cleaning installations. A classic dyadic game as the environment defense model is the three-person dyadic game [1, p. 193 — 197], where the j-th subject of pollution has a pure strategy $x_j \in \{0, 1\}$ for $j = \overline{1, 3}$ by the zero pure strategy, meant applying the cleaning installations. There are three industrial factories, using the water from some reservoir for their technical needs, and the natural ecological balance is restored only if not greater than the single factory pours out the wasted water into the reservoir without cleaning. The violation of this condition drives to the penalty for three units from each of factories, though the application of cleaning installations costs only one unit. Meanwhile this game doesn't have the simultaneous profitable, symmetric and equilibrant situation, its formulation is not general, as that fine in three units had been only locally fixed. And it is unknown, what solutions will be when that fine is generalized.

Last publications analysis

Besides [1, p. 193—197], there are the origins [2, 3], where the dyadic three-person game [4, 5] as the model of the environment defense has been investigated. The result of [1, p. 193—197] is that this game has the nine NE-solutions (Nash equilibrium solutions [6, 7]), four of which are in the pure strategies. Being locally fixed for the fine in the three units, there [1, p. 193—197] have been found some non-equilibrant situations [1, 7, 8], that had been appeared to be simultaneously symmetric and the most advantageous [9, 10]. However, there remains the localization in those three units, which restricts as the NE-solution, as well as the symmetric advantageous solution, needed to be generalized.

Paper aim formulation

Assign a pure strategy of the j-th factory as x_j by $x_j \in \{0, 1\}$ $\forall\ j = \overline{1, 3}$. Further, assign the probability of that the j-th factory will not use the water cleaning technology for the wasted water before pouring it out into the reservoir as α_j $\forall\ j = \overline{1, 3}$. In other words, α_j is the probability of using the pure strategy $x_j = 1$ $\forall\ j = \overline{1, 3}$. The now expanding paper aim is to find the NE-solution of the described game, if the fine for pouring the wasted unclean water out by two or three factories is $a \in (1; d]$, where d is the maximal fine, that is possible, as d cannot be infinitely large because of the law. On the other hand, the great fines may influence negatively upon the factory functioning [11, 12], and it may want to be stopped in it, and this yet is not profitable for the state (or its regional economics).

Main section of investigation

Assign the payoff of the j-th factory in the pure strategies situation $\{x_1, x_2, x_3\}$ as $K_j(x_1, x_2, x_3)$ $\forall\ j = \overline{1, 3}$. Then the payoff of the j-th factory in the mixed strategies situation $\{\alpha_1, \alpha_2, \alpha_3\}$ is

$$V_j(\alpha_1, \alpha_2, \alpha_3) = (1-\alpha_1)(1-\alpha_2)(1-\alpha_3)K_j(0, 0, 0) +$$
$$+(1-\alpha_1)(1-\alpha_2)\alpha_3 K_j(0, 0, 1) + (1-\alpha_1)\alpha_2(1-\alpha_3)K_j(0, 1, 0) +$$
$$+(1-\alpha_1)\alpha_2\alpha_3 K_j(0, 1, 1) + \alpha_1(1-\alpha_2)(1-\alpha_3)K_j(1, 0, 0) +$$
$$+\alpha_1(1-\alpha_2)\alpha_3 K_j(1, 0, 1) + \alpha_1\alpha_2(1-\alpha_3)K_j(1, 1, 0) +$$
$$+\alpha_1\alpha_2\alpha_3 K_j(1, 1, 1) \qquad (1)$$

for $j = \overline{1, 3}$ by

$$K_1(0, 0, 0) = K_2(0, 0, 0) = K_3(0, 0, 0) = -1, \qquad (2)$$

$$K_1(0, 0, 1) = K_2(0, 0, 1) = -1, \quad K_3(0, 0, 1) = 0, \qquad (3)$$

$$K_1(0, 1, 0) = K_3(0, 1, 0) = -1, \quad K_2(0, 1, 0) = 0, \qquad (4)$$

$$K_1(0, 1, 1) = -1-a, \quad K_2(0, 1, 1) = K_3(0, 1, 1) = -a, \qquad (5)$$

$$K_1(1, 0, 0) = 0, \quad K_2(1, 0, 0) = K_3(1, 0, 0) = -1, \qquad (6)$$

$$K_1(1, 0, 1) = K_3(1, 0, 1) = -a, \quad K_2(1, 0, 1) = -1-a, \qquad (7)$$

$$K_1(1, 1, 0) = K_2(1, 1, 0) = -a, \quad K_3(1, 1, 0) = -1-a, \qquad (8)$$

$$K_1(1, 1, 1) = K_2(1, 1, 1) = K_3(1, 1, 1) = -a. \qquad (9)$$

Putting (2)—(9) into (1) gives the following payoffs:

$$V_1(\alpha_1, \alpha_2, \alpha_3) = -1 - a\alpha_1\alpha_2 + 2a\alpha_1\alpha_2\alpha_3 - a\alpha_2\alpha_3 - a\alpha_1\alpha_3 + \alpha_1, \quad (10)$$

$$V_2(\alpha_1, \alpha_2, \alpha_3) = -1 - a\alpha_1\alpha_2 + 2a\alpha_1\alpha_2\alpha_3 - a\alpha_2\alpha_3 - a\alpha_1\alpha_3 + \alpha_2, \quad (11)$$

$$V_3(\alpha_1, \alpha_2, \alpha_3) = -1 - a\alpha_1\alpha_2 + 2a\alpha_1\alpha_2\alpha_3 - a\alpha_2\alpha_3 - a\alpha_1\alpha_3 + \alpha_3. \quad (12)$$

It is interesting to underline, that the payoffs (10) — (12) differ from each other only for the corresponding probability in the mixed strategy. This means that the equal payoffs in the being investigated game are only on the symmetric situations $\{\alpha, \alpha, \alpha\}$ by $\alpha_1 = \alpha_2 = \alpha_3$.

For searching the set $\mathscr{A}_j$ of all the acceptable situations of the j-th factory it is sufficient to find the solutions of the corresponding nonstrict inequality [1, p. 192]. In this way, the set $\mathscr{A}_1$ is found from the inequality

$$(1-\alpha_2)(1-\alpha_3)K_1(0, 0, 0) + (1-\alpha_2)\alpha_3 K_1(0, 0, 1) +$$
$$+\alpha_2(1-\alpha_3)K_1(0, 1, 0) + \alpha_2\alpha_3 K_1(0, 1, 1) \leqslant$$
$$\leqslant (1-\alpha_2)(1-\alpha_3)K_1(1, 0, 0) + (1-\alpha_2)\alpha_3 K_1(1, 0, 1) +$$
$$+\alpha_2(1-\alpha_3)K_1(1, 1, 0) + \alpha_2\alpha_3 K_1(1, 1, 1). \quad (13)$$

Then the situations $\{1, \alpha_2, \alpha_3\}$ which satisfy the inequality (13) with the strict inequality sign, being coalesced into the set $\mathscr{A}_{11}$, and the situations $\{\alpha_1, \alpha_2, \alpha_3\}$ which satisfy the inequality (13) with the equality sign, being coalesced into the set $\mathscr{A}_{1\alpha_1}$, and the situations $\{0, \alpha_2, \alpha_3\}$ which do not satisfy the inequality (13), being coalesced into the set $\mathscr{A}_{10}$, will compound the set

$$\mathscr{A}_1 = \mathscr{A}_{11} \cup \mathscr{A}_{1\alpha_1} \cup \mathscr{A}_{10}. \quad (14)$$

As the sets of the pure strategies of each of the three factories are identical, then their sets of the mixed strategies are identical also, and the sets

$$\mathscr{A}_2 = \mathscr{A}_{21} \cup \mathscr{A}_{2\alpha_2} \cup \mathscr{A}_{20} \quad (15)$$

and

$$\mathscr{X}_3 = \mathscr{X}_{31} \cup \mathscr{X}_{3\alpha_3} \cup \mathscr{X}_{30} \qquad (16)$$

are found analogously from the inequalities

$$(1-\alpha_1)(1-\alpha_3)K_2(0, 0, 0)+(1-\alpha_1)\alpha_3K_2(0, 0, 1)+$$
$$+\alpha_1(1-\alpha_3)K_2(1, 0, 0)+\alpha_1\alpha_3K_2(1, 0, 1)\leqslant$$
$$\leqslant(1-\alpha_1)(1-\alpha_3)K_2(0, 1, 0)+(1-\alpha_1)\alpha_3K_2(0, 1, 1)+$$
$$+\alpha_1(1-\alpha_3)K_2(1, 1, 0)+\alpha_1\alpha_3K_2(1, 1, 1) \qquad (17)$$

and

$$(1-\alpha_1)(1-\alpha_2)K_3(0, 0, 0)+(1-\alpha_1)\alpha_2K_3(0, 1, 0)+$$
$$+\alpha_1(1-\alpha_2)K_3(1, 0, 0)+\alpha_1\alpha_2K_3(1, 1, 0)\leqslant$$
$$\leqslant(1-\alpha_1)(1-\alpha_2)K_3(0, 0, 1)+(1-\alpha_1)\alpha_2K_3(0, 1, 1)+$$
$$+\alpha_1(1-\alpha_2)K_3(1, 0, 1)+\alpha_1\alpha_2K_3(1, 1, 1) \qquad (18)$$

respectively.

The set $\mathscr{X}_3$ may be depicted graphically, as it was in [1, p. 194], within the cube of situations (figure 1).

Before making it, there must be considered the inequality (18), which is

$$2a\alpha_1\alpha_2 - a\alpha_1 - a\alpha_2 + 1 = a(2\alpha_1\alpha_2 - \alpha_1 - \alpha_2) + 1 \geqslant 0. \qquad (19)$$

The inequality (19), taken as the strict inequality, depicts the subset $\mathscr{X}_{31} \subset \mathscr{X}_3$ lying in the plane $\alpha_3 = 1$. As from (19)

$$\alpha_1 a(2\alpha_2 - 1) \geqslant a\alpha_2 - 1 \qquad (20)$$

then

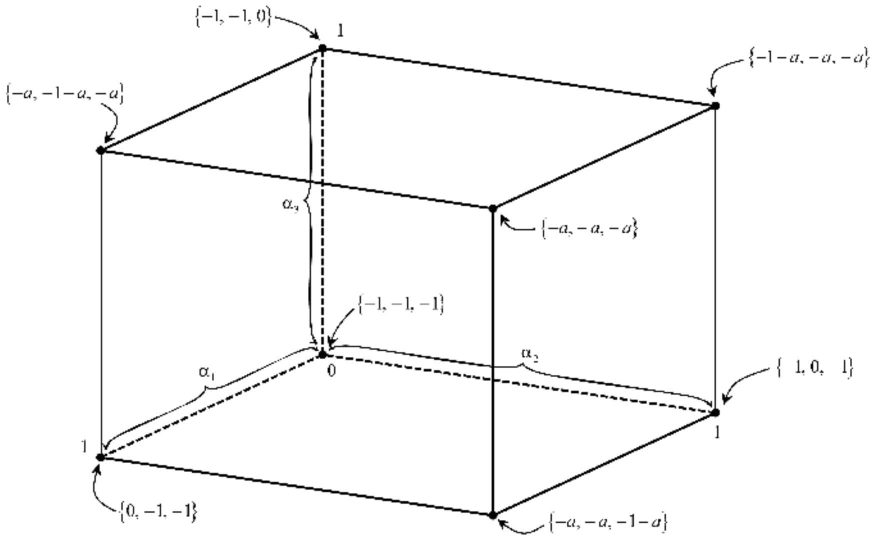

Figure 1. The cube of situations in the dyadic game with the three factories and their payoffs $\left\{K_j\left(x_1,\, x_2,\, x_3\right)\right\}_{j=1}^{3}$

$$\alpha_1 \geqslant \frac{a\alpha_2 - 1}{a\left(2\alpha_2 - 1\right)} \tag{21}$$

for $\alpha_2 \in \left(\frac{1}{2};\, 1\right]$ and

$$\alpha_1 \leqslant \frac{a\alpha_2 - 1}{a\left(2\alpha_2 - 1\right)} \tag{22}$$

for $\alpha_2 \in \left[0;\, \frac{1}{2}\right)$. The character of the transformation of the subset $\mathscr{A}_{31} \subset \mathscr{A}_3$ is shown on the frames of the figures 2—8, plotted by the conditions (21) and (22) in the strict form. It is certain that by the fine $a \in (1;\, 2)$ this subset becomes the inseparable set, though by

$a > 2$ the set $\mathscr{A}_{31}$ consists of the two separate parts, and by $a = 2$ the set $\mathscr{A}_{31}$ consists of the two half-open squares, not having the adjacent points within the unit square.

Figure 2. The set $\mathscr{A}_{31}$ of the two separated hyperbolic parts on the cube of situations in the plane $\alpha_3 = 1$ by $a \in \{5, 4, 3\}$

Figure 3. The set $\mathscr{A}_{31}$ of the two separated hyperbolic parts on the cube of situations in the plane $\alpha_3 = 1$ by $a \in \{2.5, 2.4, 2.3\}$

Figure 4. The set $\mathscr{A}_{31}$ of the two separated hyperbolic parts (continued to be evolved) on the cube of situations in the plane $\alpha_3 = 1$ by $a \in \{2.2, 2.1, 2.075\}$

Figure 5. The set $\mathscr{A}_{31}$ of the two half-open squares,
not having the adjacent points within the unit square
on the cube of situations
in the plane $\alpha = 1$ by $a = 2$,
and the inseparable set $\mathscr{A}_{31}$ on the cube of situations
in the plane $\alpha_3 = 1$ by $a \in \{1.925, 1.9\}$

Figure 6. The inseparable set $\mathscr{A}_{31}$
on the cube of situations
(completion of its evolution)
in the plane $\alpha_3 = 1$ by $a \in \{1.8, 1.7, 1.6\}$

Figure 7. The inseparable set $\mathscr{A}_{31}$ on the cube of situations
in the plane $\alpha_3 = 1$ by $a \in \{1.5, 1.4, 1.3\}$

Figure 8. The inseparable set $\mathcal{SH}_{31}$ on the cube of situations in the plane $\alpha_3 = 1$ by $a \in \{1.2, 1.1, 1.05\}$

By $a > 2$ the set $\mathcal{SH}_{30}$ consists of the curvilinear hexagon, lying in the plane $\alpha_3 = 0$ (it is seen as the white-colored area on the figures 2— 4), where the continuous margin belongs to this set. And the set $\mathcal{SH}_{3\alpha_3}$ consists of the two parts of the parabolic cylinders with generatrices along the α_3 axis, where the projections of these parabolic cylinders on the plane $\alpha_3 = 0$ may be seen as the inner parabolic borders on the figures 2—4.

After having depicted the set (16) within the cube of situations by $a > 2$, there yet may be pictured the sets (14) and (15), which are obtained by the corresponding right-angle rotation of the set (16). Thus the set of all the equilibrium situations

$$\mathcal{SH} = \mathcal{SH}_1 \cap \mathcal{SH}_2 \cap \mathcal{SH}_3 = \left\{ \mathcal{SH}_{11} \cup \mathcal{SH}_{1\alpha_1} \cup \mathcal{SH}_{10} \right\} \cap$$

$$\cap \left\{ \mathcal{SH}_{21} \cup \mathcal{SH}_{2\alpha_2} \cup \mathcal{SH}_{20} \right\} \cap \left\{ \mathcal{SH}_{31} \cup \mathcal{SH}_{3\alpha_3} \cup \mathcal{SH}_{30} \right\} \qquad (23)$$

for the case $a > 2$ turns out to be the same structure as it is in [1, p. 193—197] with $a = 3$. That is the set (23) for the case $a > 2$ consists of the following pure strategies situations:

$$\{0, 0, 1\},\ \{0, 1, 0\},\ \{1, 0, 0\},\ \{1, 1, 1\}. \qquad (24)$$

The payoffs in the situations (24) are

$$\{K_1(0, 0, 1), K_2(0, 0, 1), K_3(0, 0, 1)\} = \{-1, -1, 0\}, \qquad (25)$$

$$\{K_1(0, 1, 0), K_2(0, 1, 0), K_3(0, 1, 0)\} = \{-1, 0, -1\}, \qquad (26)$$

$$\left\{K_1(1,\,0,\,0),\;K_2(1,\,0,\,0),\;K_3(1,\,0,\,0)\right\}=\{0,\,-1,\,-1\}, \tag{27}$$

$$\left\{K_1(1,1,1),\;K_2(1,1,1),\;K_3(1,1,1)\right\}=\{-a,\,-a,\,-a\}. \tag{28}$$

Besides, there are the three mixed strategies situations, lying on the situations cube faces:

$$\left\{0,\,\frac{1}{a},\,\frac{1}{a}\right\},\;\left\{\frac{1}{a},\,\frac{1}{a},\,0\right\},\;\left\{\frac{1}{a},\,0,\,\frac{1}{a}\right\}. \tag{29}$$

The payoffs in the situations (29) due to (1)—(12) are

$$\left\{V_1\left(0,\,\frac{1}{a},\,\frac{1}{a}\right),V_2\left(0,\,\frac{1}{a},\,\frac{1}{a}\right),V_3\left(0,\,\frac{1}{a},\,\frac{1}{a}\right)\right\}=\left\{-\frac{1+a}{a},\,-1,\,-1\right\}, \tag{30}$$

$$\left\{V_1\left(\frac{1}{a},\,\frac{1}{a},\,0\right),V_2\left(\frac{1}{a},\,\frac{1}{a},\,0\right),V_3\left(\frac{1}{a},\,\frac{1}{a},\,0\right)\right\}=\left\{-1,\,-1,\,-\frac{1+a}{a}\right\}, \tag{31}$$

$$\left\{V_1\left(\frac{1}{a},\,0,\,\frac{1}{a}\right),V_2\left(\frac{1}{a},\,0,\,\frac{1}{a}\right),V_3\left(\frac{1}{a},\,0,\,\frac{1}{a}\right)\right\}=\left\{-1,\,-\frac{1+a}{a},\,-1\right\}. \tag{32}$$

Those six situations in (24) and (29), where the situation $\{1,1,1\}$ is excluded, are non-symmetrical and, as a corollary, they are not favorable for the pair of factories, having greater losses in (25) — (27), and not favorable for the single factory, having greater loss in (30) — (32). But the symmetrical situation $\{1,1,1\}$ is not favorable for the three factories simultaneously, as each of them would have been fined for a units, where $a > 2$.

Nevertheless, there are two mixed strategies situations inside the situations cube, which are constituted due to the nonempty intersection (23) inside the cube, belonged to the set $\mathscr{H}_{1\alpha_1} \bigcap \mathscr{H}_{2\alpha_2} \bigcap \mathscr{H}_{3\alpha_3}$ as the intersection of the three couples of parabolic cylinders generatrices. Surely, they should be symmetrical, so then assign them as

$$\{\alpha_1,\,\alpha_2,\,\alpha_3\}=\{\alpha,\,\alpha,\,\alpha\}=\left\{\alpha^{\langle 1\rangle},\,\alpha^{\langle 1\rangle},\,\alpha^{\langle 1\rangle}\right\}, \tag{33}$$

$$\{\alpha_1, \alpha_2, \alpha_3\} = \{\alpha, \alpha, \alpha\} = \left\{\alpha^{\langle 2\rangle}, \alpha^{\langle 2\rangle}, \alpha^{\langle 2\rangle}\right\}. \tag{34}$$

Letting $\alpha_j = \alpha \;\; \forall\, j = \overline{1, 3}$ into the inequality (18), will find the situations (33) and (34), transforming (18) into (19) and stating it as the equality:

$$a\left(2\alpha^2 - 2\alpha\right) + 1 = 2\alpha a\left(\alpha - 1\right) + 1 = 0\,. \tag{35}$$

Further, from (35) have the quadratic equation

$$2a\alpha^2 - 2a\alpha + 1 = 0, \tag{36}$$

which roots are the probabilities $\alpha^{\langle 1\rangle}$ and $\alpha^{\langle 2\rangle}$ for (33) and (34):

$$\alpha^{\langle 1\rangle} = \frac{2a - \sqrt{4a\left(a-2\right)}}{4a} = \frac{a - \sqrt{a\left(a-2\right)}}{2a}, \tag{37}$$

$$\alpha^{\langle 2\rangle} = \frac{2a + \sqrt{4a\left(a-2\right)}}{4a} = \frac{a + \sqrt{a\left(a-2\right)}}{2a}. \tag{38}$$

It is easy to check, that $\alpha^{\langle 1\rangle} \in \left(0; 1\right)$ and $\alpha^{\langle 2\rangle} \in \left(0; 1\right)$ by $a > 2$, so the symmetrical situations (33) and (34) are

$$\left\{\frac{a - \sqrt{a\left(a-2\right)}}{2a}, \frac{a - \sqrt{a\left(a-2\right)}}{2a}, \frac{a - \sqrt{a\left(a-2\right)}}{2a}\right\}, \tag{39}$$

$$\left\{\frac{a + \sqrt{a\left(a-2\right)}}{2a}, \frac{a + \sqrt{a\left(a-2\right)}}{2a}, \frac{a + \sqrt{a\left(a-2\right)}}{2a}\right\}. \tag{40}$$

The situation (39) payoffs

$$\left\{V_1\left(\alpha^{\langle 1\rangle}, \alpha^{\langle 1\rangle}, \alpha^{\langle 1\rangle}\right), V_2\left(\alpha^{\langle 1\rangle}, \alpha^{\langle 1\rangle}, \alpha^{\langle 1\rangle}\right), V_3\left(\alpha^{\langle 1\rangle}, \alpha^{\langle 1\rangle}, \alpha^{\langle 1\rangle}\right)\right\} =$$

$$=\left\{\frac{\sqrt{a(a-2)}-1-a}{2}, \frac{\sqrt{a(a-2)}-1-a}{2}, \frac{\sqrt{a(a-2)}-1-a}{2}\right\} \quad (41)$$

are obviously identical. They are less than the situation (40) payoffs

$$\left\{V_1\left(\alpha^{\langle 2\rangle}, \alpha^{\langle 2\rangle}, \alpha^{\langle 2\rangle}\right), V_2\left(\alpha^{\langle 2\rangle}, \alpha^{\langle 2\rangle}, \alpha^{\langle 2\rangle}\right), V_3\left(\alpha^{\langle 2\rangle}, \alpha^{\langle 2\rangle}, \alpha^{\langle 2\rangle}\right)\right\}=$$

$$=\left\{-\frac{\sqrt{a(a-2)}+1+a}{2}, -\frac{\sqrt{a(a-2)}+1+a}{2}, -\frac{\sqrt{a(a-2)}+1+a}{2}\right\}, \quad (42)$$

speaking that this situation is not favorable entirely.

Thus the being investigated game by $a>2$ has the nine NE-situations (24), (29), (39), (40), where only three situations are symmetrical (absolutely fair): situation $\{1, 1, 1\}$ and (39), (40). Meanwhile, the situation (40) is such that the factories bear losses, which are apparently greater than the losses in the situation (39). Will give evidence of that the situation (39) payoffs are greater than the situation (24) payoffs:

$$V_j\left(\alpha^{\langle 1\rangle}, \alpha^{\langle 1\rangle}, \alpha^{\langle 1\rangle}\right)-K_j(1, 1, 1)=\frac{\sqrt{a(a-2)}-1-a}{2}-(-a)=$$

$$=\frac{\sqrt{a(a-2)}-1+a}{2} \quad \forall\ j=\overline{1, 3},$$

whence $\sqrt{a(a-2)}-1+a \geqslant 0$ by $\sqrt{a(a-2)} \geqslant 1-a$, that for $a>2$ is true. So, the situation (39) payoffs (41) are the most favorable for the three factories simultaneously.

Noticing that

$$\lim_{a\to\infty}\alpha^{\langle 1\rangle}=\lim_{a\to\infty}\frac{a-\sqrt{a(a-2)}}{2a}=0$$

and $\forall\ j=\overline{1, 3}$

$$V_j(0,0,0)=K_j(0,0,0)=-1$$

due to (2), will conclude, that the huge fines force the factories, wanting the symmetrical fairness, to convert totally into the water cleaning technology for the wasted water before pouring it out into the reservoir. The maximal fine d may be determined from the reason of the supportable and negligible volume of the unclean wasted water sink. For instance, if $\alpha^{\langle 1\rangle}=0.1$ that means a factory will pour out only one tenth of the unclean wasted water, then completely there will be poured only 30 percent of the unclean wasted water in relation to the bearable volume of the unclean wasted water sink (which is equivalent to the case when only one factory pollutes the reservoir). By that the rate of the loss for a factory should be closely approximated to one unit, having been fixed as l_d. That is the equation

$$\alpha^{\langle 1\rangle}=\frac{a-\sqrt{a(a-2)}}{2a}=\alpha_d \tag{43}$$

by the fixed α_d will give the value $a=d$ of the maximal fine, satisfying the condition

$$V_j(\alpha_d,\alpha_d,\alpha_d)=$$

$$=V_j\left(\frac{d-\sqrt{d(d-2)}}{2d},\frac{d-\sqrt{d(d-2)}}{2d},\frac{d-\sqrt{d(d-2)}}{2d}\right)\geqslant -l_d\,. \tag{44}$$

Exemplifying the above further, take $l_d=1.05$, and from (43) by $\alpha_d=0.1$ obtain that $d\approx 5.5556$. Then

$$V_j(0.1,0.1,0.1)\approx -1.0556<-l_d=-1.05$$

and the condition (44) is not true, so the 30 percent unclean wasted water sink is not negligible here. But taking $\alpha_d=0.09$ will give

$$V_j(0.09,0.09,0.09)\approx -1.04945\geqslant -l_d=-1.05,$$

that satisfies (44) with the fine $d\approx 6.105$, what restricts the factories at their 27 percent unclean wasted water sink.

Now will look at the NE-solution of the being investigated game by the fine $a = 2$. In this case, might have been called marginal, there are still the four NE-situations (24) in the pure strategies, and there appear the three continuous sets of the NE-situations in the mixed strategies

$$\left\{\alpha_1, \frac{1}{2}, \frac{1}{2}\right\} \quad \forall\, \alpha_1 \in [0; 1], \tag{45}$$

$$\left\{\frac{1}{2}, \alpha_2, \frac{1}{2}\right\} \quad \forall\, \alpha_2 \in [0; 1], \tag{46}$$

$$\left\{\frac{1}{2}, \frac{1}{2}, \alpha_3\right\} \quad \forall\, \alpha_3 \in [0; 1]. \tag{47}$$

The payoffs in the situations (45)—(47) are

$$\left\{V_1\left(\alpha_1, \frac{1}{2}, \frac{1}{2}\right), V_2\left(\alpha_1, \frac{1}{2}, \frac{1}{2}\right), V_3\left(\alpha_1, \frac{1}{2}, \frac{1}{2}\right)\right\} = $$

$$= \left\{-\frac{3}{2}, -1-\alpha_1, -1-\alpha_1\right\}, \tag{48}$$

$$\left\{V_1\left(\frac{1}{2}, \alpha_2, \frac{1}{2}\right), V_2\left(\frac{1}{2}, \alpha_2, \frac{1}{2}\right), V_3\left(\frac{1}{2}, \alpha_2, \frac{1}{2}\right)\right\} = $$

$$= \left\{-1-\alpha_2, -\frac{3}{2}, -1-\alpha_2\right\}, \tag{49}$$

$$\left\{V_1\left(\frac{1}{2}, \frac{1}{2}, \alpha_3\right), V_2\left(\frac{1}{2}, \frac{1}{2}, \alpha_3\right), V_3\left(\frac{1}{2}, \frac{1}{2}, \alpha_3\right)\right\} = $$

$$= \left\{-1-\alpha_3, -1-\alpha_3, -\frac{3}{2}\right\}. \tag{50}$$

Here remains only one symmetrical situation $\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$ in the mixed strategies, consequential to the symmetrical situations (39) and (40) junction, and giving the equal payoffs $\left\{-\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}\right\}$.

The NE-solution of the being investigated game by the fines $a \in (1; 2)$ may be deduced right from the discourse on the NE-situations (24), (29), (39), (40), using the case $a = 2$ border conception. So, by $a \in (1; 2)$ there are still the four NE-situations (24) in the pure strategies, and there the three mixed strategies situations, lying on the situations cube faces:

$$\left\{1, \frac{a-1}{a}, \frac{a-1}{a}\right\}, \quad \left\{\frac{a-1}{a}, \frac{a-1}{a}, 1\right\}, \quad \left\{\frac{a-1}{a}, 1, \frac{a-1}{a}\right\}. \tag{51}$$

The payoffs in the situations (51) due to (1)—(12) are

$$\left\{V_1\left(1, \frac{a-1}{a}, \frac{a-1}{a}\right), V_2\left(1, \frac{a-1}{a}, \frac{a-1}{a}\right), V_3\left(1, \frac{a-1}{a}, \frac{a-1}{a}\right)\right\} =$$

$$= \left\{\frac{1-a^2}{a}, -a, -a\right\}, \tag{52}$$

$$\left\{V_1\left(\frac{a-1}{a}, \frac{a-1}{a}, 1\right), V_2\left(\frac{a-1}{a}, \frac{a-1}{a}, 1\right), V_3\left(\frac{a-1}{a}, \frac{a-1}{a}, 1\right)\right\} =$$

$$= \left\{-a, -a, \frac{1-a^2}{a}\right\}, \tag{53}$$

$$\left\{V_1\left(\frac{a-1}{a}, 1, \frac{a-1}{a}\right), V_2\left(\frac{a-1}{a}, 1, \frac{a-1}{a}\right), V_3\left(\frac{a-1}{a}, 1, \frac{a-1}{a}\right)\right\} =$$

$$= \left\{-a, \frac{1-a^2}{a}, -a\right\}. \tag{54}$$

As for the NE-situations within the situations cube, then their set is empty because of the quadratic equation (36) by $a \in (1; 2)$ does not have real roots.

Conclusions and perspective of further investigations of the noncooperative models of the environment guard

The investigated dyadic game for $a > 2$ has the single simultaneously symmetrical and favorable NE-situation (39). This situation must be favorable also for the local government, wanting the factories to be in functioning state, as well as the reservoir to be clean. It draws the fine $a > 2$ to be increased up to the maximal fine d, being fixed with the conditions (43) and (44). In the marginal case $a = 2$ there is the single symmetrical NE-situation $\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$ with the corresponding payoffs $\left\{-\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}\right\}$. Obviously, this situation is pretty unfavorable for each factory [13—15], and the reservoir would be polluted much irreversibly. Furthermore, the investigated dyadic game does not have the simultaneously symmetrical and favorable NE-situations by any $a \in (1; 2)$. But its NE-solution as the aggregate of (24) and (51) may occur useful anyway. Hence for both the environment guard and the factories the selection of some fine $a > 2$ will be the most profitable, as then the factories most likely will use the strategies from the symmetrical favorable NE-situation (39), giving the losses, which are greater for the one unit loss: $\frac{\sqrt{a(a-2)}-1-a}{2} \leqslant -1$, what is checked elementarily. But for the situation (39) equilibrium, all the stated conditions may influence on the factories to claim the treaty about the joint application of the cleaning installations, that is the non-equilibrant situation $\{0, 0, 0\}$ with the minimal payoffs $\{-1, -1, -1\}$ (or losses just in the one unit). And the greater fine a the more probable that the factories will make their pact on the joint holding the reservoir clean.

Further investigations of the noncooperative models of the environment guard should apparently be directed to increasing the number of the factories (or, generally, potential subjects of the environment pollution), taking there general fines and finding the most acceptable fine for both the factories and the environment guard or local government.

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1. Воробьёв Н. Н. Теория игр для экономистов-кибернетиков / Н.Н. Воробьёв. — М.: Наука, Главная редакция физико-математической литературы, 1985. — 272 с.

2. Романюк В. В. Практична реалізація стратегії у найвигіднішій симетричній ситуації у діадичній грі з трьома суб'єктами забруднення водойми / В.В. Романюк. — Екологічна безпека. — 2009. — № 4 (8). — С. 49—56.

3. Горелик В. А. Теоретико-игровые модели принятия решений в эколого-экономических системах / В.А. Горелик, А.Ф. Кононенко. — М.: Радио и связь, 1982. — 144 с.

4. Browning L. Evolution of coordinated alternating reciprocity in repeated dyadic games / L. Browning, Andrew M. Colman // Journal of Theoretical Biology. — 2004. — Vol. 229, Iss. (4). — P. 549—557.

5. Sparkes Kathleen K. Cooperative and competitive behavior in dyadic game-playing: a comparison of Anglo-American and Chinese children / Kathleen K. Sparkes // Early Child Development and Care. — 1991. — Vol. 68, Iss. 1. — P. 37—47.

6. Оуэн Г. Теория игр / Оуэн Г.; [пер. с англ.]. — 2-е изд. — М. : Едиториал УРСС, 2004. — 216 с.

7. Теория игр : [учеб. пособие для ун-тов] / Л.А. Петросян, Н.А. Зенкевич, Е.А. Семина— М.: Высшая школа, Книжный дом «Университет», 1998. — 304 с. : ил.

8. Colman Andrew M. Asymmetric dominance and phantom decoy effects in games / Andrew M. Colman, Briony D. Pulford, Fergus Bolger // Organizational Behavior and Human Decision Processes. — 2007. — Vol 104, Iss. (2). — P. 193—206.