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A new projective exact penalty function for a general constrained optimization

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A new projective exact penalty function method is proposed for the equivalent reduction of constrained optimization problems to unconstrained ones. In the method, the original objective function is extended to infeasible points by summing its value at the projection of an infeasible point on the feasible set with the distance to the set. The equivalence means that local and global minimums of the problems coincide. Nonconvex sets with multivalued projections are admitted, and the objective function may be lower semicontinuous. The particular case of convex problems is included. So the method does not assume the existence of the objective function outside the allowable area and does not require the selection of the penalty coefficient.

Keywords: *nonconvex constrained optimization, lower semicontinuous functions, closed constraint set, exact penalty function method, projection operation.*

Introduction. The classical approach to the exact reduction of a constrained optimization problem to an unconstrained one consists in adding to the objective function some nonsmooth penalty term for the violation of constraints [1—3]. The problem in this method consists in selecting the correct penalty scale. In the present paper, we propose a new projective exact penalty function method of equivalent reduction of constrained optimization problems to unconstrained ones. The equivalence means that local and global minimums of the problems and the corresponding objective function values at the minimums coincide. In the proposed method, the original objective function is extended to infeasible points by summing its value at the projection of an infeasible point on the feasible set with the distance to the projection. Nonconvex feasible sets with multivalued projections are admitted, and the objective function may be lower semicontinuous. The special case of convex problems is included. So the method does not assume the existence of the objective function outside the allowable area and does not require the selection of the penalty coefficient. The method was introduced in [4] and was motivated by the application of the smooth-

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ing method to constrained global optimization. Later, a similar method for convex problems was proposed and studied in [14]. Here we validate it for general convex and nonconvex constrained optimization problems.

Let it be necessary to solve the problem of conditional global optimization:

$$f(x) \to \min_{x \in C \subseteq \mathbb{R}^n}, \tag{1}$$

where $f(x)$ is a lower semicontinuous (lsc) function defined on a closed set $C \subseteq \mathbb{R}^n$; $\mathbb{R}^n$ is n-dimensional Euclidian space with norm $\|\cdot\|$; for $x, y \in \mathbb{R}^n$ define $d(x,y) = \|x - y\|$ and the distance $d_C(x)$ from x to C as $d_C(x) = \min_{y \in C} d(x,y)$. For example, the set C may be given by some other lower semicontinuous function $g(x)$, $C = \{x \in \mathbb{R}^n : g(x) \leqslant 0\}$.

Proposition 1 [5, Example 1.20] (A distance function and projections). For any nonempty, closed set $C \subset \mathbb{R}^n$, the distance $d_C(x)$ of a point x from C depends continuously on x, while the projection $\pi_C(x)$, consisting of the points of C nearest to x is nonempty and compact. Whenever $y^k \in \pi_C(x^k)$ and $x^k \to x$, the sequence $\{y^k\}$ is bounded and all its cluster points lie in $\pi_C(x)$, i.e., the mapping $x \to \pi_C(x)$ is compact valued and upper semicontinuous.

Lemma 1 (A simple geometric lemma). Let y_x be a projection of point $x \in \mathbb{R}^n$ on a closed set $C \subset \mathbb{R}^n$. Then the point y_x is the unique common projection on C of all points $x_\lambda = (1-\lambda)x + \lambda y_x$, $\lambda \in (0,1]$.

There are several ways to reduce constrained problem to an equivalent nonsmooth unconstrained one.

For example, if $C = \{x \mid g_j(x) \leqslant 0, j = 1, \ldots, J; h_k(x) = 0, k = 1, \ldots, K\}$, then in the exact penalty function method the Lipschitz function $f(x)$ is replaced by

$$F(x) := f(x) + M\left(\textstyle\sum_j \max\{0, g_j(x)\} + \sum_k |h_k(x)|\right)$$

or by

$$F(x) := f(x) + M \inf_{y \in C} \|y - x\|$$

with a sufficiently large penalty parameter M and then one considers the problem of unconditional optimization of $F(x)$ [5, Proposition 9.68], [6, Theorem 18.2]. Note that here it is assumed that functions f, g_j, h_k are defined over the whole space $\mathbb{R}^n$.

Convex nonsmooth exact penalty functions were introduced in [1—3], and have been reviewed and studied, for example, in works [7—9] and many others. Recent advances in the exact penalty function method and references can be found in [10—12]. In this approach, the problem lies in the correct choice of the penalty parameter M.

The projective exact penalty function method. Consider the problem:

$$F(x) := \min_{y \in \pi_C(x)} f(y) + M\, d_C(x) \to \min_{x \in \mathbb{R}^n}, \quad M > 0. \tag{2}$$

The problem is well defined, since $d_C(x) = d(x,y)$ for all $y \in \pi_C(x) \neq \varnothing$; $f(y)$ is lsc on C, $\pi_C(x) \subset C$ is a compact, so there exists $y_x \in \pi_C(x)$ such that $\min_{y \in \pi_C(x)} f(y) = f(y_x)$. Remark that function $\varphi(x) = \min_{y \in \pi_C(x)} f(y)$ is lower semicontinuous [13, Proposition 21] and function $d_C(x)$ is continuous, so function $F(x)$ is lower semicontinuous.

Theorem 1 *(A general projective penalty function method). Let function f be lower semicontinuous on a non-empty closed set C. Any $M > 0$ is admitted. Then problems* (1) *and* (2) *are equivalent, i.e., each local (global) minimum of one problem is a local (global) minimum of the another, and the optimal values of the problems in the corresponding minima coincide.*

Proof. Let $x^* \in C$ be a global minimum of (1). Take an arbitrary $x \in \mathrm{R}^n$ and find $y_x \in \pi_C(x)$ such that $f(y_x) = \min_{y\in\pi_C(x)} f(y)$. Then $F(x) \geqslant f(y_x) \geqslant f(x^*) = F(x^*)$, thus x^* is a global minimum of (2) with the same minimal value $f(x^*)$.

Let x^{**} be a global minimum of (2). First let us show that $x^{**} \in C$, suppose the opposite, $x^{**} \notin C$. By Proposition 1, there exists a compact projection set $\pi_C(x^{**}) \subseteq C$ and $y^{**} \in \arg\min_{y\in\pi_C(x^{**})} f(y)$. Consider points $x_\lambda = (1-\lambda)x^{**} + \lambda y^{**}$, $\lambda \in [0,1]$. By Lemma 1, in the Eucleadian space $\pi_C(x_\lambda) = y^{**}$, i.e. projections of points x_λ, $\lambda \in (0,1]$, coincide with y^{**}, the projection of x^{**} on C. Define function $g(\lambda) = d(x_\lambda, y^{**})$, $\lambda \in [0,1]$. It holds

$$F(x_\lambda) = \min_{y\in\pi_C(x^{**})} f(y) + M\, d_C(x_\lambda) = f(y^{**}) + M\, d(x_\lambda, y^{**}).$$

It holds $x^{**} \in C$, otherwise $F(x_\lambda) < F(x^{**})$, $d(x_\lambda, x^{**}) = \lambda d(y^{**}, x^{**})$ for any $\lambda \in (0,1]$, a contradiction. But for $x \in C$ it holds $d_C(x) = 0$ and $f(x) = F(x) \geqslant F(x^{**}) = f(x^{**})$, hence x^{**} is a global minimum of (1).

Let x^* be a local minimum of (1). Then there exist a neighborhood $V(x^*)$ of x^* such that $f(x) \geqslant f(x^*)$ for all $x \in V(x^*) \cap C$. Let us show that x^* is a local minimum of $F(x)$. Since $\pi_C(x^*) = x^*$ and $\pi_C(\cdot)$ is upper semicontinuous, then for $V(x^*)$ there is an open vicinity $v(x^*)$ of x^* such that $\pi_C(x) \subseteq V(x^*)$ for all $x \in v(x^*)$. Consider $x \in v(x^*)$ and find $y_x \in \pi_C(x)$ such that $d_C(x) = d(x, y_x)$ and $f(y_x) = \inf_{y\in\pi_C(x)} f(y)$. Then for $x \in v(x^*)$ it holds

$$F(x) = \inf_{y\in\pi_C(x)} f(y) + M\, d_C(x) = f(y_x) + M\, d(x, y_x) \geqslant f(y_x) \geqslant f(x^*).$$

If x^{**} is a local minimum of (2), then, as was proven before, it is impossible that x^{**} does not belong to C, i. e. $x^{**} \in C$. But since on C functions $F(x)$ and $f(x)$ coincide, then x^{**} is a local minimum of (1).

Remark 1. Theorem 1 implies also that both problems (1) and (2) either have local (global) minima or do not have them.

Remark 2. The projective penalty function method is extendable to those metric spaces where statements of Proposition 1 and Lemma 1 hold.

3. The convex case. Let us consider problem (1) in the case of a convex constraint set C.

If $f(x)$ is convex on a convex set C, then $F(x)$ in (2) is not necessarily convex on $\mathbb{R}^n$. Example: if $f(x) = x$, $C = \{x \in \mathrm{R}^1 : x \leqslant 0\}$, $M < 1$, then $F(x) = \min\{x, Mx\}$.

If C is a convex closed set, then function $d_C(x)$ is continuous [5, Example 1.20], and the mapping $\pi_C(x)$ is single valued and continuous on $\mathbb{R}^n$ [5, Example 2.25]. If function f is continuous (lower semicontinuous) on a convex closed set C, then function (2) is continuous (lower semicontinuous) on $\mathbb{R}^n$.

If the non-convex feasible set $C = C_1 \cup \ldots \cup C_m$ is the union of a finite number of convex sets $C_1, \ldots, C_m$, then the original problem (1) splits into m problems of form (1) with convex feasible sets C_i, $i = 1, \ldots, m$, which can be reduced to equivalent unconstrained optimization problems of form (2).

Theorem 2. (Lipschitz property of the penalty function). *If function f is Lipschitzian with constant L on a convex closed set C, then the function $F(x)$ defined by equality* (2) *is also Lipschitzian with constant $(L+2M)$ on the whole space $\mathbb{R}^n$.*

Proof. Let us take two points $x, y \in \mathbb{R}^n$. Denote $\bar{x}, \bar{y}$ the projections of the points x, y on C. Due to the non-expanding property of the projecting operation onto a convex set C, it holds $\|\bar{x}-\bar{y}\| \leqslant \|x-y\|$ [5, Corollary 12.20], and due to the quadrilateral inequality, $\big|\|x-\bar{x}\|-\|y-\bar{y}\|\big| \leqslant \|\bar{x}-\bar{y}\|+\|x-y\|$, therefore

$$\begin{aligned} |F(x)-F(y)| &= \big|f(\bar{x})+M\|x-\bar{x}\|-f(\bar{y})-M\|y-\bar{y}\|\,\big| \leqslant \\ &\leqslant |f(\bar{x})-f(\bar{y})|+M\,\big|\|x-\bar{x}\|-\|y-\bar{y}\|\,\big| \leqslant \\ &\leqslant L\|\bar{x}-\bar{y}\|+M(\|\bar{x}-\bar{y}\|+\|x-y\|) \leqslant (L+2M)\|x-y\|. \ \square \end{aligned}$$

If some admissible point $x_0 \in C$ is known, the exact penalty function can be constructed as follows. Let $x \notin C$ and $y(x)$ be the nearest to x point from the set C lying on the segment connecting x_0 and x. Let us define the mapping

$$\pi_{x_0, C}(x) = \begin{cases} x, & x \in C, \\ y(x), & x \notin C, \end{cases}$$

and the penalty functions $r_{x_0,C}(x) = \|x - \pi_{x_0,C}(x)\|$ and $F(x) := f(\pi_{x_0,C}(x)) + Mr_{x_0,C}(x)$. Consider the unconstrained optimization problem:

$$F(x) := f(\pi_{x_0,C}(x)) + Mr_{x_0,C}(x) \to \min_{x\in\mathbb{R}^n}, \ \ M > 0. \tag{3}$$

Theorem 3 (Non-Euclidian projection). *Let C be a non-empty closed convex set. Then problems* (1) *and* (3) *are globally equivalent, i.e., the global minimum of one task is the global minimum of the other. Moreover, any local minimum of problem* (3) *is a local minimum of problem* (1). *In the case when C is a convex closed set and x_0 is an interior point of C, any local minimum of problem* (1) *is a local minimum of problem* (3).

Proof. If x^* is a global minimum of problem (1), $x^* \in C$, then for any $x \in \mathbb{R}^n$ it holds

$$F(x) = f(\pi_{x_0,C}(x)) + Mr_{x_0,C}(x) = f(\pi_{x_0,C}(x)) \geqslant f(x^*) = F(x^*). \tag{4}$$

Let x^{**} be a point of local (global) minimum of the function F, i.e., for some neighborhood $V(x^{**}) \subset \mathbb{R}^n$ the point x^{**} is the global minimum of the function F on the set $V(x^{**})$. Let us show that $x^{**} \in C$. Assume the contrary, $x^{**} \notin C$, then

$$F(x^{**}) = f(\pi_{x_0,C}(x^{**})) + M\|x^{**} - \pi_{x_0,C}(x^{**})\| > f(\pi_{x_0,C}(x^{**})).$$

Denote $x_\lambda = (1-\lambda)x^{**} + \lambda\pi_{x_0,C}(x^{**})$. Let us consider a convex function $\psi(\lambda) = F(x_\lambda)$, $\lambda \in [0,1]$. Obviously,

$$\psi(\lambda) = F(x_\lambda) \leqslant (1-\lambda)F(x^{**}) + \lambda F(\pi_{x_0,C}(x^{**})) \leqslant$$
$$= F(x^{**}) - \lambda(F(x^{**}) - F(\pi_{x_0,C}(x^{**})) =$$
$$= F(x^{**}) - \lambda(F(x^{**}) - f(\pi_{x_0,C}(x^{**})) < F(x^{**}), \qquad \lambda \in (0,1].$$

For all sufficiently small λ, we have $x_\lambda \in V(x^{**})$ and $F(x_\lambda) < F(x^{**})$, i. e., we obtain a contradiction that x^{**} is not a local minimum of the function F. In this way, $x^{**} \in C$. For all, $x \in V(x^{**}) \cap C$ it holds $f(x) = f(\pi_{x_0,C}(x)) = F(x) \geqslant F(x^{**}) = f(x^{**})$, i.e., the point x^{**} is also a local (global) minimum point for f on C and $F(x^{**}) = f(x^{**})$.

If x^* is a local minimum of problem (1), i.e., in some neighborhood $V(x^*)$ this point x^* is a global minimum on the set $V(x^*) \cap C$. Since x_0 is an interior point of a convex closed set C, the mapping $\pi_{x_0,C}(x)$ is continuous. Therefore, there is a smaller neighborhood $W(x^*) \subseteq V(x^*)$ such that for any $x \in W(x^*)$ it holds $\pi_{x_0,C}(x) \in V(x^*)$. Therefore, for any $x \in W(x^*)$, inequality (4) is true, which means that x^* is a local minimum of problem (3). The proof is complete.□

Let $f : C \to \mathbb{R}^1$ and the convex set $C \subseteq \mathbb{R}^n$ in (1) has a representation

$$C = \{x \mid g_j(x) \leqslant 0, j = 1, ..., J; h_k(x) = 0, k = 1, ..., K\},$$

where functions g_j are continuous and convex, and h_k are linear. Denote $\pi_C(x)$ the projection of point x on the set C. For a simple set C given by linear constraints, the problem of searching projection $\pi_C(x)$ is either solved analytically or reduced to a quadratic programming problem. We introduce a new penalty function

$$\Phi(x) := f(\pi_C(x)) + M\left(\textstyle\sum_{j=1}^{J} \max\{0, g_j(x)\} + \sum_{k=1}^{K} |h_k(x)|\right), \quad M > 0, \tag{5}$$

and consider the problem of unconstrained optimization:

$$\Phi(x) \to \min_{x \in \mathbb{R}^n}. \tag{6}$$

Note that in (5) function f may not be defined outside the feasible domain C.

Theorem 4. (A projective penalty function for a convex constraint set given by equalities and inequalities). *Let function f be lower semicontinuous on a non-empty closed convex set C. Then problems* (1) *and* (5)—(6) *are equivalent, i.e., each local (global) minimum of one problem is a local (global) minimum of the another, and the optimal values of the problems in the corresponding minima coincide.*

Proof. Let $x^* \in C$ be a local minimum of problem (1), i.e., for some neighborhood $V_1(x^*)$ of the point x^*, it is also a global minimum point of $f(x)$ on the set $V_1(x^*) \cap C$. Obviously, for any $x \in V_1(x^*)$, due to the non-stretching property of the projection operator $\pi_C(\cdot)$ onto a convex set [5, Corollary 12.20], it is satisfied $\pi_C(x) \in V_1(x^*)$ and, thus,

$$\Phi(x) = f(\pi_C(x)) + M(\textstyle\sum \max\{0, g_j(x)\} + \sum |h_k(x)|) \geqslant f(\pi_C(x)) \geqslant f(x^*) = \Phi(x^*),$$

i. e. x^* is a local minimum of the function Φ.

Let x^{**} be a local minimum point of function Φ, i.e., for some neighborhood $V(x^{**})\subset\mathbb{R}^n$ point x^{**} is the global minimum of the function Φ on the set $V(x^{**})$. Let's show that $x^{**}\in C$. Assume the contrary, $x^{**}\notin C$, then

$$\Phi(x^{**})=f(\pi_C(x^{**}))+M(\sum_{j=1}^{J}\max\{0,g_j(x^{**})\}+\sum_{k=1}^{K}\left|h_k(x^{**})\right|)>f(\pi_C(x^{**})).$$

Denote, $x_\lambda=(1-\lambda)x^{**}+\lambda\pi_C(x^{**})$. By Lemma 1, it holds $\pi_C(x_\lambda)=x^{**}$. Let us consider a convex function

$$\psi(\lambda)=\Phi(x_\lambda)=f(\pi_C(x_\lambda))+M(\sum_{j=1}^{J}\max\{0,g_j(x_\lambda)\}+\sum_{k=1}^{K}\left|h_k(x_\lambda)\right|)=$$
$$=f(\pi_C(x^{**}))+M(\sum_{j=1}^{J}\max\{0,g_j(x_\lambda)\}+\sum_{k=1}^{K}\left|h_k(x_\lambda)\right|),\quad\lambda\in[0,1].$$

Obviously,

$$\psi(\lambda)=\Phi(x_\lambda)\leqslant(1-\lambda)\Phi(x^{**})+\lambda\Phi(\pi_C(x^{**}))\leqslant$$
$$=\Phi(x^{**})-\lambda(\Phi(x^{**})-\Phi(\pi_C(x^{**}))=$$
$$=\Phi(x^{**})-\lambda(\Phi(x^{**})-f(\pi_C(x^{**}))<\Phi(x^{**}),\qquad\lambda\in(0,1].$$

For all sufficiently small λ, we have $x_\lambda\in V(x^{**})$ and $\Phi(x_\lambda)<\Phi(x^{**})$, i.e., we obtain a contradiction with that x^{**} is a local minimum of the function F. In this way, $x^{**}\in C$. For all $x\in V(x^{**})\cap C$, it holds $f(x)=f(\pi_C(x))=\Phi(x)\geqslant\Phi(x^{**})=f(x^{**})$, i.e., the point x^{**} is also a local minimum point for f on C and $\Phi(x^{**})=f(x^{**})$. The proof of the coincidence of global minima is carried out in a similar way. □

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REFERENCES

1. Eremin, I. I. (1966). The penalty method in convex programming, Soviet Math. Dokl., 8, pp. 459-462.
2. Eremin, I. I. (1967). The penalty method in convex programming. Cybernetics. Vol. 3 (4), pp.53-56. https://doi.org/10.1007/BF01071708
3. Zangwill, W. (1967). Non-linear programming via penalty junctions, Management Science, 13, No. 5, pp. 344-358. https://doi.org/10.1287/mnsc.13.5.344
4. Norkin, V. I. (2020). A stochastic smoothing method for nonsmooth global optimization. Cybernetics and Computer technologies, Iss. 1, pp. 5-14. https://doi.org/10.34229/2707-451X.20.1.1
5. Rockafellar, R. T., Wets, R. J.-B. (1998). Variational Analysis. Berlin, Heidelberg: Springer. (3rd printing 2009). https://doi.org/10.1007/978-3-642-02431-3
6. Mikhalevich, V. S., Gupal, A. M. & Norkin, V. I. (1987). Methods of Nonconvex Optimization. Moscow: Nauka (in Russian).
7. Di Pillo, G. (1994). Exact penalty methods. In: Algorithms for Continuous Optimization: The State of the Art, edited by E. Spedicato, pp. 1-45. Dordrecht: Kluwer Academic Publishers.

8. Boukari, D. & Fiacco, A. V. (1995) Survey of penalty, exact-penalty and multiplier methods from 1968 to 1993. Optimization: A Journal of Mathematical Programming and Operations Research, 32, Iss. 4, pp. 301-334. https://doi.org/10.1080/02331939508844053
9. Demyanov, V. F. (2005). Extremum Conditions and Calculus of Variations. Moscow: Vyschaya shkola (in Russian).
10. Dolgopolik, M. V., & Fominyh, A. V. (2019). Exact penalty functions for optimal control problems I: Main theorem and free-endpoint problems. Optimal Control Applications and Methods, 40, Iss. 6, pp. 1018-10244. https://doi.org/10.1002/oca.2530
11. Dolgopolik, M. V. (2020). Exact penalty functions for optimal control problems II: Exact penalization of terminal and pointwise state constraints. Optim. Control Appl. Methods, 41, pp. 898-947. https://doi.org/10.1002/oca.2577
12. Dolgopolik, M. V. (2022). Exact penalty functions with multidimensional penalty parameter and adaptive penalty updates. Optimization Letters, 16, pp. 1281-1300. https://doi.org/10.1007/s11590-021-01777-2
13. Aubin, J-P. & Ekeland, I. (1984). Applied Nonlinear Analysis. New York: John Wiley & Sons.
14. Galvan, G., Sciandrone, M. & Lucidi, S. (2021). A parameter-free unconstrained reformulation for nonsmooth problems with convex constraints. Comput. Optim. Appl., 80, pp. 33-53. https://doi.org/10.1007/s10589-021-00296-1

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НОВА ПРОЕКТИВНА ТОЧНА ШТРАФНА ФУНКЦІЯ
ДЛЯ ЗАГАЛЬНОЇ УМОВНОЇ ОПТИМІЗАЦІЇ

Класичний підхід до точного зведення задачі умовної оптимізації до задачі без обмежень полягає в додаванні до цільової функції деякого негладкого штрафного члена за порушення обмежень [Eremin (1966, 1967), Zangwill (1967)]. Проблема цього методу полягає у виборі правильного штрафного множника. У цій роботі ми пропонуємо нову проективну точну штрафну функцію для еквівалентного зведення задач оптимізації з обмеженнями до задач без обмежень. Еквівалентність означає, що локальні і глобальні мінімуми задач і значення цільової функції на відповідних мінімумах збігаються. У запропонованому методі вихідна цільова функція поширюється на недопустимі точки шляхом підсумовування її значення в проекції недопустимої точки на допустиму множину та відстані до множини. Допускаються багатозначні проекції, а цільова функція може бути напівнеперервною знизу. Розглядається окремий випадок опуклих задач. Таким чином, метод не передбачає існування цільової функції за межами допустимої області та не вимагає підбору штрафного коефіцієнта. Метод був запропонований у роботі [Норкін (2020)] (і пізніше вивчений у [Galavan et al. (2021)]) був мотивований застосуванням методу згладжування для умовної глобальної оптимізації. В даній статті ми обґрунтовуємо його для загальних опуклих і неопуклих задач оптимізації з обмеженнями.

Ключові слова: *неопукла умовна оптимізація, напівнеперервні знизу функції, замкнена допустима множина, метод точних штрафних функцій, операція проекції.*