A MECHANICAL MODEL OF FATIGUE CRACK PROPAGATION.

REPORT 2

G. P. Karzov, V. P. Leonov,
and B. Z. Margolin

UDC 539.4:620.191.33

The propagation of a fatigue crack in the proposed model is schematized as a discrete process in which each elementary increase of crack length by a constant value takes place during the number of cycles inversely proportional to the degree of damage of the material at the given point. The analysis of deformation of the material at the crack tip is carried out using the dependences derived in report 1 [1]. The elementary crack-length increment is assumed to be equal to the dimension of the structural member, i.e., the parameter which determines the smallest volume of the material for which the mechanics of deformed continuum is applicable.

For the process of fatigue of the material described from the phenomenological positions of low-cycle strength, the size of the structural element is determined by the zone of inverse elastoplastic deformation at the crack tip in loading corresponding to the threshold value of the stress intensity factor ($\Delta K = \Delta K_{th}|_{R=\frac{K_{min}}{K_{max}}=0}$) at which the fatigue crack does not propagate. The size of the structural element in this approach is calculated from the condition $\rho_{st} > r^i_p|_{\Delta K = K_{th},\ R=0}$. Consequently, assuming that the size of the structural element is equal to $2 \cdot r^i_p$, and determining the degree of damage of the element from the range of the plastic strain intensity in its center, for $\Delta K = \Delta K_{th}$ we obtain $\Delta e^p_i = 0$.

Consequently, the endurance of the examined element will be unlimited in accordance with the accepted criterion, and the crack propagation rate is equal to zero; this indicates that the acceptable model reflects with sufficient adequacy the crack propagation rate in the area of ΔK values in the vicinity of the threshold.

From the physical point of view, the size of the structural element may be determined as follows. If it is assumed that microcracks initiate mainly as a result of pile-up of dislocations at obstacles (barriers), the size of the structural element will be determined by the minimum distance between the dislocation obstacles which are represented in the majority of cases by grain or block boundaries [2]. In fact, if the size of the zone of inverse elastoplastic deformation is larger than the grain size, the dislocations will move to its boundary and pile up there. Consequently, from the macroscopic position, plastic deformation leads to damage in this case. However, if the size of the zone is smaller than the grain size, dislocation slip during deformation does not cause pile-ups to form and, consequently, no microcracks appear.

In this case, no damage occurs; from the macroscopic point of view, this may be interpreted as the absence of cyclic plastic strain in the structural element whose size is equal to twice the diameter of the grain, at loads corresponding to the threshold values of the stress intensity factor.

The size of the zone of inverse elastoplastic deformation is determined from Eqs. (2) and (23) derived in [1]

$$r_p^i = \frac{(1-2\mu)^2 \Delta K}{18\pi S_3(r)}, \tag{1}$$

where μ is Poisson's ratio in the elastic range; ΔK is the range of the stress intensity factor; S_3 is the component of the stress deviator corresponding to the start of unloading; r is the polar coordinate.

It should be mentioned that at a constant size of the structural element which depends only on the material, the value of ΔK corresponding to r_p^i of the structural element decreases

Translated from Problemy Prochnosti, No. 8, pp. 14-18, August, 1985. Original article submitted September 17, 1984.

 0039-2316/85/1708-1044$09.50

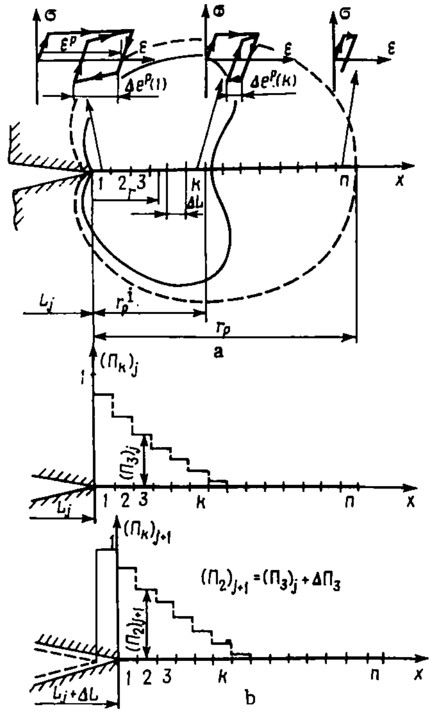

Fig. 1. Diagrams of discretization of the elastoplastic zone at the crack tip (a) and damage summation in various elements (b).

with increasing K_{max}. This effect reflects with sufficient accuracy the experimental data on the effect of the asymmetry of loading on the value of ΔK_{th}. Thus, the elementary crack length extension ΔL is determined from Eq. (1) at $\Delta K = \Delta K_{th}$ (or from the grain size of the material) and depends on the properties of the material, i.e., on ΔK_{th}.

For the known trajectory of fatigue crack propagation, it is possible to calculate, for any point of this trajectory, the range of elastoplastic deformation of the material corresponding to a specific crack length and stress state level. To determine the endurance, this region is divided into n sections with size ΔL (Fig. 1a). The stress–strain state for each of n sections is calculated on the basis of solving the cyclic elastoplastic problem [1].

It should be mentioned that under the effect of external stationary loading, the stress–strain state at the crack tip with length L can be determined without taking into account the prior stress state corresponding to crack length L – ΔL. This is associated with the fact that along the crack trajectory any elementary section ΔL travels a specific path of elastoplastic deformation which starts when the structural element enters the elastoplastic zone at the crack tip and is completed when it fails after fulfilling the fracture criterion. It may be shown, using Eq. (4) derived in [1], that if the drop gradient $\partial K_{max}/\partial L$ is lower than $K_{max}/r_p{}^i$, then any section belonging to the elastoplastic zone will be loaded during crack propagation. Consequently, in loading we can apply the strain theory of plasticity which links unambiguously the stress–strain state with load $K_{max}(L)$, and also determine the strains and stresses without taking into account the prior stress state.

The fracture criterion of the k-th section in the elastoplastic zone at fixed crack length may be expressed on the basis of the strain criterion of fracture in low-cycle loading [3] by means of fatigue damage and cumulation of one-sided plastic strain

$$\left(\frac{\Delta e_i^p(k)}{C}\right)^{1/m} \cdot N_k + \frac{\varepsilon_i^p}{\varepsilon_f^p} = 1, \qquad (2)$$

where $\Delta e_i{}^p(k)$ is the range of the intensity of the plastic strains in the k-th section of the elastoplastic zone; N_k is the number of load cycles to crack initiation; C and m are the experimental constants of the material in Coffin's equation $\Delta e_i{}^p \cdot N^m = C$ for a hard symmetric cycle of low-cycle loading; $\varepsilon_i{}^p$ is the intensity of cumulated plastic strain; $\varepsilon_f{}^p$ is the plastic strain in static loading at the instant of appearance of a crack, $\varepsilon_f{}^p \approx \ln\frac{1}{1-\psi}$ (ψ is the coefficient of the transverse reduction in area).

Taking into account that during crack propagation each k-th element is damaged to a certain extent as a result of cyclic deformation in stages preceding failure, on the basis of the law of linear damage summation the damage of the k-th element may be written in the form

$$\Pi_k = \sum_{j=0}^{n} \Pi_j^k \cdot N_{kj} + \frac{\varepsilon_i^p}{\varepsilon_f^p}, \tag{3}$$

where Π_k is the damage of the k-th element in the crack tip; N_{kj} is the number of load cycles of the k-th element in each movement of the crack from length L to L + jΔL; Π_j^k is the state of damage in the k-th element (the value reciprocal to the number of cycles to crack initiation) in each movement of the crack from length L to L + jΔL; n is the number of elementary movements of the crack.

The state of damage in any element at the crack tip for the given position of the crack is determined from the equation

$$\Pi_j^k = \frac{1}{N_k} = A\,(\Delta e_i^p\,(k))^M, \tag{4}$$

which uses the notations $A = (1/c)^{1/m}$ and $M = 1/m$. Consequently, the number of cycles required for crack propagation over length ΔL in the last step, taking into account Eq. (3), is calculated from the equation

$$N_{kk} = \frac{1 - \frac{\varepsilon_i^p}{\varepsilon_f^p} - \sum_{j=0}^{k-1} \Pi_j^k \cdot N_{kj}}{A\,(\Delta e_i^p\,(k))^M}. \tag{5}$$

Thus, using the given calculation method, it is possible to simulate the process of fatigue crack propagation on the basis of the low-cycle fatigue criteria to determine the dependence of the length of the fatigue crack on the number of load cycles.

The algorithm of calculations of the fatigue crack-propagation rate or endurance using the proposed model may be described as follows (the initial characteristics and loading parameters are the dependences $K_{max} = f(L)$ and $K_{min} = \varphi(L)$ for the crack propagation trajectory; the parameters of the deformation diagram of the material σ_T, E, E_u; the experimental coefficients of Coffin's equation for a hard symmetric load cycle; the value ΔK_{th} or the doubled diameter 2d (or block) of the material):

1. Calculation of the elementary crack-length extension ΔL from Eq. (1) for $\Delta K = \Delta K_{th}$ or from the condition of equality of ρ_{st} to the double diameter of the grain in the structure of the material.

2. Calculation of the size of the elastoplastic zone for the running value $K_{max}(L)$ using Eq. (8') from [1]; discretization of the above-mentioned zone into n elements with size ΔL.

3. Determination, from Eqs. (4), (5), (9)-(16) from [1], of the stress—strain state at point r = ΔL (k − 1/2) for each of n elements; determination of the $S_3(r)$ dependence.

4. Determination of the size of the reversible elastoplastic zone from Eq. (1).

5. Determination, from Eqs. (23), (26)-(30) from [1], of the range of the plastic strain intensity $\Delta e_i^p(k)$ in each element of the reversible elastoplastic zone.

6. Calculation of the state of damage of each element of the reversible elastoplastic zone from Eq. (4).

7. Determination, from Eq. (5), of the number of load cycles N_{kk} required for crack extension over length ΔL in the running step.

8. Determination of the state of damage in each element of the reversible elastoplastic zone for the crack length L_j + ΔL (Fig. 1b) from the equations

$$\left.\begin{aligned}
(\Pi_1)_{j+1} &= (\Pi_2)_j + A\,(\Delta e_i^p\,(2))^M \cdot N_{kk};\\
(\Pi_2)_{j+1} &= (\Pi_3)_j + A\,(\Delta e_i^p\,(3))^M \cdot N_{kk};\\
&\cdots\cdots\cdots\cdots\cdots\\
(\Pi_k)_{j+1} &= (\Pi_{k+1})_j + A\,(\Delta e_i^p\,(k+1))^M \cdot N_{kk},
\end{aligned}\right| \tag{6}$$

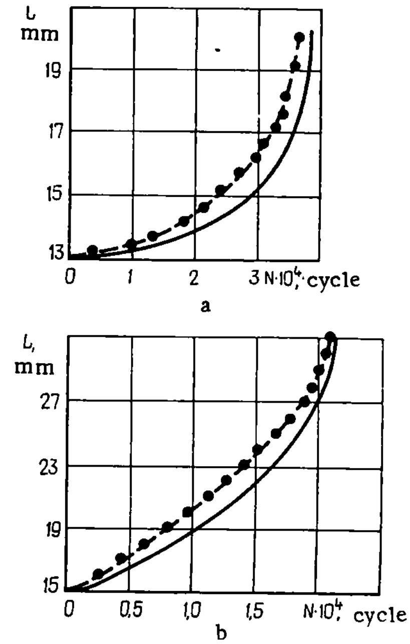

Fig. 2. Comparison of calculated (solid line) and experimental (broken line) endurances in crack propagation in type 12KhN3MD steel at loading asymmetry coefficients of R = 0.5 (a) and R = 0 (b).

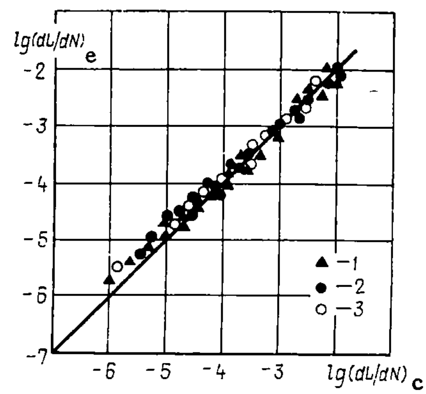

Fig. 3. Comparison of experimental and calculated data on the fatigue crack-propagation rate in the examined steels: 1) 15Kh2MFA; 2) 22K; 3) type 12KhN3MD.

where j is the number of crack movements; $(\Pi_{k+1})_j$ is the cumulated damage at crack length L_j equal to the sum $\sum_{0}^{j} \Pi_j^{k+1} N_{k+1,j}$.

9. Presentation of information on the running value of crack length and the number of load cycles

$$(L_j + \Delta L) = f\left(\sum_{1}^{j} N_{jj}\right).$$

10. Transition to paragraph 2.

The above-described algorithm of calculation of the fatigue crack-propagation rate was used to prepare a program in Fortran-IV language for an EC-type computer in which the proposed approaches were formulated.

TABLE 1. Characteristics of Examined Steels

Steel grade	σ_T, MPa	ψ, %	$E\cdot10^4$, MPa	E_u, MPa	A	M	ΔK_{th}, MPa·m$^{1/2}$	ρ_{st}, mm	Mean doubled grain diameter 2d, mm
22K	310	69	20,6	603	4,33	1,9	9,7	0,020	0,060
15Kh2MFA	640	73	20,3	1430	5,0	1,8	12,4	0,015	0,08
Type 12KhN3MD	800	60	201	2000	5,5	1,66	14,0	0,007	0,03

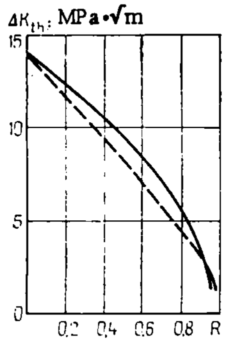

Fig. 4. Comparison of the calculated data on the effect of loading asymmetry on the threshold value of the stress intensity factor obtained from the proposed model (solid line) and the data published in [4] (broken line).

Experimental Verification of the Model of Fatigue Crack Propagation

For this purpose, experiments were carried out to determine the fatigue crack-propagation rates in three steels with various strength levels and loaded with various asymmetries of the external cyclic load. The tests were conducted in air at room temperature and at a loading frequency of 300 cycles/min. Standard bend specimens were used for determining the crack resistance characteristics with a gage cross section of 35 × 70 mm and 50 × 100 mm; the stress concentration factor of these specimens can be calculated with sufficient accuracy. The main characteristics of the examined steel grades required for calculations by the proposed model are presented in Table 1.

The experimental and calculated dependences of the length of the fatigue crack in the specimen on the number of load cycles for 12KhN3MD steel are compared in Fig. 2. At two different values of the loading asymmetry coefficients of R = 0 and R = 0.5 the results were in satisfactory agreement. The maximum relative error of the crack size equals ~10%, that of endurance ~15%. These data indicate that there is satisfactory agreement between the results of the calculations carried out using the proposed method and the experimental data for varying load asymmetry coefficients.

The results of comparison of the calculated and experimental crack-propagation rates obtained in the same conditions for the three steel grades are shown in Fig. 3. In the crack growth-rate range from $5\cdot10^{-5}$ mm/cycle and higher (in the Paris region of crack growth) the experimental and calculated data are in excellent agreement. Below the rate of $5\cdot10^{-5}$mm/cycle the data show certain systematic deviations from the theoretical curve to the region of higher values of dL/dN. However, this may be associated not only with the assumptions accepted in the calculation model but also with the procedure errors in carrying out the experiments in the given fatigue crack-propagation-rate range. For the type 12KhN3MD steel, the known value of $\Delta K_{th}/_{R\,=\,0}$ was used to determine the $\Delta K_{th}(R)$ dependence from the condition of constancy of the size of the structural element for the examined material. It was assumed that for any value of $\Delta K_{th}(R)$ at the point whose distance from the crack tip is L/2, the plastic strain range should be equal to zero. Comparison of the calculated data with the dependence derived on the basis of a large number of experimental values [4] demonstrated the satisfactory correspondence of the dependences of ΔK_{th} on the loading asymmetry (Fig. 4).

It is also necessary to mention the very satisfactory correlation of the size of the structural element calculated from Eq. (1) for $\Delta K = \Delta K_{th}$ with the mean grain size of the examined material (Table 1). The slightly smaller size of the structural element in comparison with the grain size may be caused by the fact that, in this case, the formation of dislocation pile-ups and formation of microcracks took place not only at the grain boundaries but also the block boundaries.

CONCLUSIONS

The proposed model of fatigue crack propagation based on the solution of the cyclic elastoplastic problem of the stress–strain state [1] makes it possible to take into account the effect of the triaxial stress state on the deformation of the material at the crack tip. The proposed algorithm of calculations of the state of damage on the basis of the principle of linear damage summation and also the agreement between the calculated and experimental data confirm the assumption on the controlling role of low-cycle damage in the mechanics of crack propagation in cyclic loading described from phenomenological positions. The main advantages of the proposed model are:

the possibilities of calculating endurance in crack propagation or calculating the crack propagation rate for cases in which the variation of the range of the stress intensity factor along the crack length in structural members takes place at a variable loading asymmetry;

the possibilities of describing the effect of loading asymmetry on the fatigue crack-propagation rate using only the strain criterion (Coffin's equation) since the range of the plastic strain intensity at the crack tip is, as shown in [1], a function of not only the range of the stress intensity factor ΔK but also of its maximum value K_{max};

the possibilities of describing the dependence of ΔK_{th} on loading asymmetry based on the assumption on the constancy of the size of the structural element for the given material;

the possibilities of describing the crack propagation rate in all the three sections of the $dL/dN = f(\Delta K)$ diagram, starting with the values ΔK similar to ΔK_{th} and ending with the value of K at which monotonic quasistatic fracture becomes the controlling process.

LITERATURE CITED

1. G. P. Karzov, V. P. Leonov, and B. Z. Margolin, "A mechanical model of fatigue crack propagation, Report 1," Probl. Prochn., No. 8, pp. 9-14 (1985).
2. S. Kotsan'da, Fatigue Fracture of Metals [in Russian], Metallurgiya, Moscow (1976).
3. N. A. Makhutov, Strain Criteria of Fracture and Calculation of the Strength of Structural Members [in Russian], Mashinostroenie, Moscow (1981).
4. L. R. Kaisand and D. F. Mowbray, "Relationships between low-cycle fatigue and failure-crack growth properties," J. Test. Eval., 7, No. 5, 270-280 (1979).