

Magnetic flux locking in two weakly coupled superconducting rings

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We have analyzed the quantum interference effects in the macroscopic «superconducting molecule». The composite system consists of two massive superconducting rings, each interrupted by a Josephson junction, which are at the same time weakly coupled with one another. The special case of coupling via the Josephson four-terminal junction is considered. The structure of the macroscopic quantum states in an applied magnetic field is calculated. It is shown that, depending on the values of the magnetic fluxes through each ring, the system displays two groups of states, the «orthostates» with both induced currents going in the same direction, and the «parastates» with the opposite currents and with the total induced flux locked to zero value. The transition to the flux locked state with changing of the total applied flux is sudden and is preserved in a certain interval which is determined by the difference of the fluxes applied through each ring. It makes the system sensitive to small gradients of the external magnetic field.

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The system that we studied is shown in Fig. 1 and consists of two bulk superconducting rings, coupled via the 4-terminal Josephson junction [1,2]. The 4-terminal Josephson junction is a system of two microbridges, 1–2 and 3–4, having the common centre «0». The interference in the cross section «0» of macroscopic wave functions Ψ_j of the j th terminal ($j = 1, \dots, 4$) leads to nonlinear coupling and

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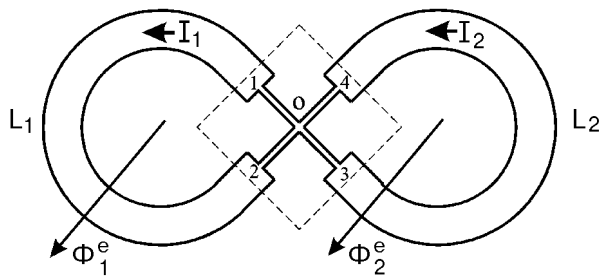


Fig. 1. The two bulk superconducting rings, coupled via the 4-terminal Josephson junction (the region closed by the dashed lines, of which the area is of the order of the coherence length squared).

o interference between the current states in each ring. The resulting current state of the whole system can be regulated by the difference of the magnetic fluxes applied through the rings, in analogy with the phase difference between two weakly coupled bulk superconductors. The studying of the macroscopic quantum states of such «superconducting molecule» is the subject of the present paper.

The free energy U of our system in an applied magnetic field contains the magnetic energy U_m and the Josephson coupling energy U_J . The energy U_m has the form [3]:

$$U_m = \frac{(\Phi_1^e - \Phi_1)^2 L_2}{2(L_1 L_2 - L_{12}^2)} + \frac{(\Phi_2^e - \Phi_2)^2 L_1}{2(L_1 L_2 - L_{12}^2)} - \frac{L_{12}}{(L_1 L_2 - L_{12}^2)} (\Phi_1^e - \Phi_1) (\Phi_2^e - \Phi_2), \quad (1)$$

where $\Phi_{1,2}^e$ are the external magnetic fluxes applied to the rings 1, 2 and $\Phi_{1,2}$ are the resulting fluxes

embraced by the rings; $L_{1,2}$ and L_{12} are the ring self-inductances and the mutual inductance ($L_{12}^2 < L_1 L_2$). The coupling energy U_J (in dimensionless units) is expressed in terms of phases ϕ_j ($j = 1, \dots, 4$) of the superconducting order parameter in the j th terminal [2]:

$$U_J = -\kappa^2 \cos^2 \frac{\phi_1}{2} - \cos^2 \frac{\phi_2}{2} - 2\kappa \cos \frac{\phi_1}{2} \cos \frac{\phi_2}{2} \cos \chi, \quad (2)$$

if we introduce the phase differences across the weak links in the rings

$$\phi_1 = \varphi_1 - \varphi_2, \quad \phi_2 = \varphi_3 - \varphi_4$$

and the «total» phase difference between the rings

$$\chi = \frac{\varphi_1 + \varphi_2}{2} - \frac{\varphi_3 + \varphi_4}{2}.$$

The coupling constant κ in (2) is the ratio of critical currents of the weak links 1–2 and 3–4. In the following for simplicity we will consider the case of identical rings with $L_1 = L_2 = L$ and the symmetrical coupling $\kappa = 1$ ($I_{c,12} = I_{c,34} = I_c$).

The phase differences $\phi_{1,2}$ are related to the magnetic fluxes $\Phi_{1,2}$ by: $\phi_{1,2} = -2e\Phi_{1,2}/\hbar$. Thus, the total energy in reduced units of the two coupled rings as function of the embraced magnetic fluxes at given values of the applied fluxes is defined as

$$U(\Phi_1, \Phi_2, \chi | \Phi_1^e, \Phi_2^e) = \frac{(\Phi_1^e - \Phi_1)^2}{2\mathcal{L}} + \frac{(\Phi_2^e - \Phi_2)^2}{2\mathcal{L}} - \frac{l}{\mathcal{L}} (\Phi_1^e - \Phi_1) (\Phi_2^e - \Phi_2) - \cos^2 \frac{\Phi_1}{2} - \cos^2 \frac{\Phi_2}{2} - 2 \cos \frac{\Phi_1}{2} \cos \frac{\Phi_2}{2} \cos \chi, \quad (3)$$

where $l = L_{12}/L$ is the normalized mutual inductance ($l < 1$), $\mathcal{L} = (2eI_c/\hbar)L(1 - l^2)$ – the dimensionless effective self-inductance; the magnetic fluxes are measured in units $\hbar/2e$. Note the dependence of the potential U on the phase χ . As we shall see, in the stable steady state the phase χ can take only the value 0 or π , which corresponds the existence of two groups of states with different symmetry.

The minima of the potential U (3) with respect to variables Φ_1, Φ_2, χ at given external fluxes Φ_1^e and Φ_2^e determine the stable steady states of our system. The minimization of U with respect to χ gives that the phase χ takes the value 0 or π , depending on the equilibrium values of Φ_1 and Φ_2

$$\cos \chi = \text{sign} \left(\cos \frac{\Phi_1}{2} \cos \frac{\Phi_2}{2} \right). \quad (4)$$

In the steady state $\partial U/\partial \Phi_1 = \partial U/\partial \Phi_2 = 0$, or:

$$\Phi_1^e - l\Phi_2^e = \Phi_1 - l\Phi_2 + \mathcal{L} \sin \frac{\Phi_1}{2} \left[\cos \frac{\Phi_1}{2} + \cos \chi \cos \frac{\Phi_2}{2} \right], \quad (5)$$

$$\Phi_2^e - l\Phi_1^e = \Phi_2 - l\Phi_1 + \mathcal{L} \sin \frac{\Phi_2}{2} \left[\cos \frac{\Phi_2}{2} + \cos \chi \cos \frac{\Phi_1}{2} \right], \quad (6)$$

with $\cos \chi$ defined by the condition (4).

The solutions of Eqs. (5) and (6) $\{\Phi_1, \Phi_2\}$ which correspond to the minima of the potential U must satisfy the requirements:

$$\frac{\partial^2 U}{\partial \Phi_1^2} > 0, \quad \frac{\partial^2 U}{\partial \Phi_2^2} > 0, \quad \frac{\partial^2 U}{\partial \Phi_1^2} \frac{\partial^2 U}{\partial \Phi_2^2} - \left(\frac{\partial^2 U}{\partial \Phi_1 \partial \Phi_2} \right)^2 > 0. \quad (7)$$

It can be shown that the conditions (7) are fulfilled for all values of Φ_1 and Φ_2 if $\mathcal{L} + l < 1$. In the following we shall consider the case when the inductances \mathcal{L} and l are small enough to satisfy this inequality. Thus, all solutions $\{\Phi_1, \Phi_2\}$ of the Eqs. (4), (5) and (6) determine the possible stable or metastable states of the system. The circulating ringcurrents $I_{1,2}$ in state $\{\Phi_1, \Phi_2\}$ are:

$$I_1 = -\frac{1}{2} \sin \Phi_1 - \sin \frac{\Phi_1}{2} \text{sign} \left(\cos \frac{\Phi_1}{2} \right) \left| \cos \frac{\Phi_2}{2} \right|, \quad (8)$$

$$I_2 = -\frac{1}{2} \sin \Phi_2 - \sin \frac{\Phi_2}{2} \text{sign} \left(\cos \frac{\Phi_2}{2} \right) \left| \cos \frac{\Phi_1}{2} \right| \quad (9)$$

in units of I_c .

The value of $\cos \chi$ in Eqs. (5), (6), which equals ± 1 , determines two possible «bindings» of the current states in individual rings. The group of states $\{\Phi_1, \Phi_2, \chi = 0\}$ we shall call symmetric, or «ortho», states and the group of states $\{\Phi_1, \Phi_2, \chi = \pi\}$ – antisymmetric, or «para», states. As we shall see, the first one corresponds to the induced ringcurrents going in the same direction, and the second one – to the currents going opposite.

We shall study the behavior of our system in an applied magnetic field as the response on the total applied magnetic flux $\Phi^e = \Phi_1^e + \Phi_2^e$ at given difference $\delta^e = \Phi_1^e - \Phi_2^e$ of the fluxes through each ring. The state of the system as whole is determined by the total embraced magnetic flux $\Phi = \Phi_1 + \Phi_2$, or

by the total orbital magnetic moment M , which is proportional to the sum of the induced ringcurrents, $I = I_1 + I_2$. Note, that the positive (negative) sign of I corresponds to the parallel (antiparallel) direction of M with respect to the external magnetic field H . From the (4)–(6) we obtain:

$$\Phi_e = \Phi + \frac{\mathcal{L}}{1-l} \sin \frac{\Phi}{2} [\cos \frac{\delta}{2} + \cos \chi], \quad (10)$$

$$\delta^e = \delta + \frac{\mathcal{L}}{1+l} \sin \frac{\delta}{2} [\cos \frac{\Phi}{2} + \cos \chi], \quad (11)$$

$$\cos \chi = \text{sign}(\cos \frac{\Phi}{2} + \cos \frac{\delta}{2}), \quad (12)$$

where $\delta = \Phi_1 - \Phi_2$.

Let us start from the case of small inductances l , $\mathcal{L} \ll 1$. If $\delta^e = 0$, from the Eqs. (11), (12) follows that $\delta = 0$ and $\chi = 0$. For $\Phi(\Phi^e)$ we have the usual equation $\Phi_e = \Phi + 2\mathcal{L} \sin(\Phi/2)$ for the case of decoupled rings [4], each interrupted by a Josephson junction. At $\delta^e \neq 0$ and consequently $\delta \neq 0$, the solutions with $\chi = \pi$ are possible in the vicinity of $\Phi \approx 2\pi$. In the limit $\mathcal{L} \rightarrow 0$ for the total induced magnetic flux $\Phi(\Phi^e, \delta^e)$ we have the expression

$$\Phi = \Phi^e - \mathcal{L} \sin \frac{\Phi^e}{2} [\cos \frac{\delta^e}{2} + \text{sign}(\cos \frac{\Phi^e}{2} + \cos \frac{\delta^e}{2})]. \quad (13)$$

In the case of small $\delta^e \ll 1$ it becomes :

$$\Phi = \begin{cases} \Phi^e - 2\mathcal{L} \sin \frac{\Phi^e}{2} & \text{if } |\Phi^e - 2\pi| > |\delta^e| \\ \Phi^e & \text{if } |\Phi^e - 2\pi| < |\delta^e|. \end{cases} \quad (14)$$

Thus, for given value of δ^e with changing of the total applied flux Φ^e the system switches from the state with $\chi = 0$ to the state with $\chi = \pi$. In the

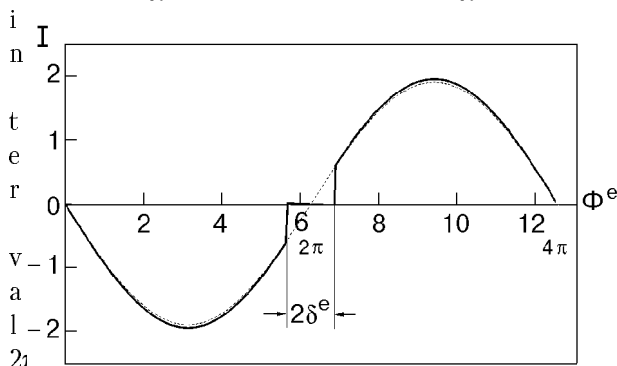


Fig. 2. The total induced current as a function of the total applied flux at given difference of applied fluxes through each ring $\delta^e = 2\pi/10$ (or $1/10$ of a flux quantum $h/2e$). $\mathcal{L} = 0$. The dashed line is the corresponding dependence in the case of decoupled rings.

total induced flux $\Phi - \Phi^e$ equals zero for $\delta^e \ll 1$. We call such behavior magnetic flux locking. It is emphasized that the transition to the flux locked state is sudden and is preserved in a certain interval of the applied magnetic flux. For the sum of the induced ringcurrents $I = I_1 + I_2$, in the limit $\mathcal{L} \rightarrow 0$ we have

$$I(\Phi^e, \delta^e) = -\sin \frac{\Phi^e}{2} [\cos \frac{\delta^e}{2} + \text{sign}(\cos \frac{\Phi^e}{2} + \cos \frac{\delta^e}{2})]. \quad (15)$$

In the flux locked state the total current I equals to zero in correspondence with (14). Thus the ringcurrents $I_{1,2}$ are going in opposite directions and compensate each other, or the system is in the «para» state. The complete compensation takes place for $\delta^e \ll 1$, with the corrections to zero value being of the order of $(\delta^e)^2$. In Fig. 2 we plot the dependence of $I(\Phi^e)$ (15) for the flux difference $\delta^e = 2\pi/10$, or in dimension units $1/10$ of a flux quantum $h/2e$. The dashed line is the sum of the currents in the same, but decoupled, rings with the same applied fluxes $\Phi_1^e = 1/2(\Phi^e + \delta^e)$ and $\Phi_2^e = 1/2(\Phi^e - \delta^e)$. The magnetic susceptibility of the system as whole is proportional to $-\partial I / \partial \Phi^e$ and will reflect the behavior of the induced currents.

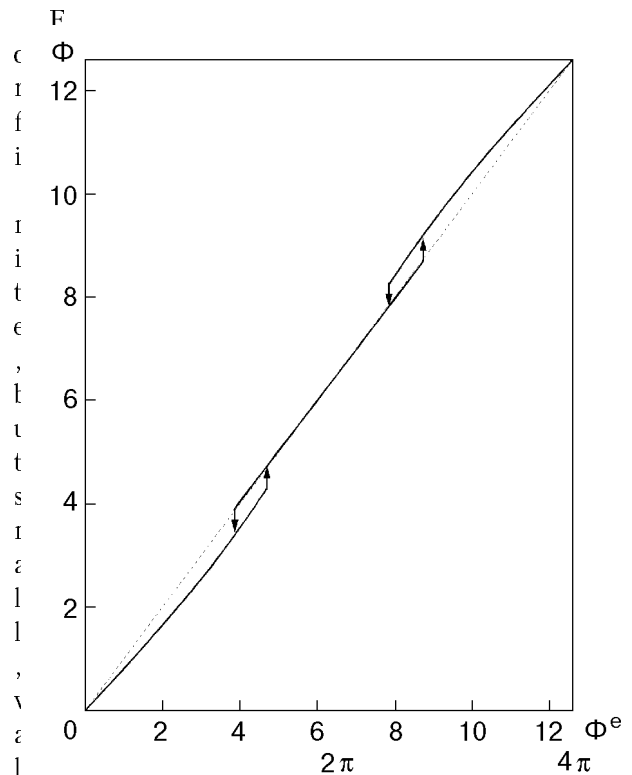


Fig. 3. The dependence of the total magnetic flux Φ on the total external flux Φ^e for $\delta^e = 1$, $\mathcal{L} = 0.25$, $l = 0$. The arrows indicate the jumps of the flux from metastable to stable states. The dashed line is $\Phi = \Phi^e$.

the inductances, the behavior described above will be qualitatively the same. Only instead of the sharp switches hysteretic regions appear, of which the width is proportional to \mathcal{L} . In Fig. 3 the dependence $\Phi(\Phi^e)$ for $\mathcal{L} = 0.25$, $l = 0$ and $\delta^e = 1$ is shown, as follows from the numerical solution of the Eqs. (10)–(12). Naturally, these hysteretic regions will be smeared by thermal fluctuations (see the analysis of the influence of noise on the similar system, the so called 4-terminal SQUID, in Ref. 5).

In conclusion, we have studied the macroscopic quantum states in the system of two weakly coupled superconducting rings. The nonlinear coupling leads to interference between the current states in each ring. It is manifested as the cooperative behavior of the rings in some region of the applied magnetic fluxes, which we call magnetic flux locking. We would like to remark that our macroscopic approach is not restricted by the special kind of the coupling through the crossed superconducting bridges. In fact, any mesoscopic 4-terminal weak link will produce a coupling similar to the U_J (2). For

example, it can be the experimental setup described in Ref. 6, namely the two-dimensional normal layer which is connected with four terminals instead of the two ones as studied in Ref. 7.

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1. E. D. Vol and A. N. Omelyanchouk, *Low Temp. Phys.* **20**, 87 (1994).
2. R. de Bruyn Ouboter, A. N. Omelyanchouk, and E. D. Vol, *Physica* **B205** 153 (1995); *Physica* **B239**, 203 (1997).
3. L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon press, New York (1963), Ch. 4.
4. A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect*, Wiley, New York (1982).
5. R. de Bruyn Ouboter and A. N. Omelyanchouk, to be published in *Physica* **B**; Los Alamos archive cond-mat/9805109.
6. J. P. Heida, B. J. van Wees, T. M. Klapwijk and G. Borghs, *Phys. Rev.* **B57**, R5618 (1998).
7. K. K. Likharev, *Radio Techn. Electr.* **20**, 660 (1975); *Rev. Mod. Phys.* **51**, 101, 146 and 147 (1979).