Fizika Nizkikh Temperatur, 1996, v. 22, No 5, p. 494–498

Temperature dependence of the magnetic field penetration depth in $YBa_2Cu_3O_{7-\delta}$: effect of impurity scattering

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Temperature dependence of the anisotropic magnetic field penetration depth of a two-band superconductor is calculated in the model of strong s-wave pairing interaction in one of the bands (S band) and induced superconductivity in the second band (N band). The results of the calculations are compared with experimental data on the (ab) plane and c axis penetration depth in $YBa_2Cu_3O_{7-\delta}$. It is shown that all recent measurements over the entire temperature range are consistent with the two-band model.

1. The measurements of the temperature dependences of the magnetic field penetration depth $\lambda(T)$ in YBaCuO have revealed an increased slope of $\lambda^{-2}(T)$ near the critical temperature T_c and nonexponential behavior of $\lambda(T)$ at $T < T_c/2$. These deviations could be, in principle, explained in terms of the model of strong electron-phonon interaction with isotropic superconducting order parameter. However, the results of microwave measurements of $\lambda(T)$ [1–3] obtained on higher quality YBaCuO samples cannot be described by this model. According to Refs. 1–3 the temperature dependence of $\Delta\lambda_{ab}(T) = \lambda_{ab}(T) - \lambda_{ab}(0)$ in the ab plane at $T < T_c/4$ changes from exponential [3] to linear [1] and then to quadratic dependence [2], mainly with an increase in oxygen deficit in a sample. Subsequent microwave measurements [4–6] have confirmed the odd linear low-temperature behavior of $\Delta\lambda_{ab}(T)$.

The existence of crossover from the linear to quadratic temperature dependence of $\Delta\lambda(T)$ has been demonstrated by calculations made in the framework of the d wave pairing model with account of impurity scattering [7]. Such a crossover has actually been observed in Ref. 4 upon doping of YBaCuO samples with Zn or Ni. However, it is impossible to reconcile the d wave model with activation exponential behavior of $\Delta\lambda_{ab}(T)$. Moreover, the experimentally measured slope of $\lambda_{ab}^{-2}(T)$ dependences near T_c is much larger than that predicted by this model [8]. The qualitative difference between the c axis penetration depth $\lambda_c(T)$ behaviors predicted by d and s pairing models was also pointed out in Ref. 8. Very recently the low-temperature dependence of $\Delta\lambda_{ab}$ was calculated in the framework of s wave two-band model [9] first proposed in Ref. 10. It was shown in Ref. 9 that the crossover from the exponential to power law temperature dependence occurs with increasing the concentration of magnetic impurities.

Following this approach, the quantitative comparison between the calculations of $\lambda_{ab}(T)$ within the two-band model in the temperature range $0 < T < T_c$ and recent measurements in YBaCuO [3–6] is given in the present paper. We assume strong electron-phonon interaction in the S band (CuO_2 planes), while superconductivity in N band (CuO chains) is induced due to the interband proximity effect. We show that for temperatures not too close to T_c the value of $\Delta\lambda_{ab}(T)$ is mainly determined by both the intraband scattering on magnetic impurities in N band and the usual scattering in N and S bands. Furthermore, good quantitative agreement is found between the calcula-

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tions and low-frequency measurements of the penetration depth $\lambda_c(T)$ on aligned ultrafine YBaCuO powders from Ref. 11.

2. In the case of strong pairing interaction in S band the Eliashberg strong-coupling theory is a starting point for discussing anisotropy effects, as it properly takes into account the effects of retardation and decay of quasiparticle excitations. The straightforward generalization of Eliashberg equations for the many-band case leads to the system of coupled equations:

$$\Delta'_{i,n}(i\omega_n) \equiv \Delta_{i,n} Z_{i,n} = \pi T \sum_j \sum_{n'} \left\{(\lambda_{ij} D_{n,n'} - \mu^*) + (\gamma_{ij} - \gamma^s_{ij})\right\} \frac{\Delta'_{j,m}}{\sqrt{(\Delta'_{j,m})^2 + (\omega'_{j,m})^2}}, \tag{1}$$

$$\omega'_{i,n}(i\omega_n) \equiv \omega_n Z_{i,n} = \omega_n + \pi T \sum_j \sum_{n'} \left\{\lambda_{ij} D_{n,n'} + (\gamma_{ij} + \gamma^s_{ij})\right\} \frac{\omega'_{j,m}}{\sqrt{(\Delta'_{j,m})^2 + (\omega'_{j,m})^2}}. \tag{2}$$

Here $D_{n,n'} = \sum_m \Omega^2_m[(\omega_n - \omega_{n'})^2 + \Omega^2_m]^{-1}$ is the phonon Green function and Ω_m are phonon frequencies which we choose in the model form as $\Omega_1 = 200$ K, $\Omega_2 = 5\Omega_1 = 10^3$ K. We note that the function $D_{n,n'}$ in Eqs. (1), (2) could hold for any other nonphonon interaction (except for spin interaction). Further, a particular choice of Ω_i does not change our conclusions because the coupling constants λ_{ij} are the most important parameters. The intraband coupling constants λ_{ii} describe the strength of electron-phonon interaction within ith band. The physical meaning of interband coupling constants λ_{ij} $(i \neq j)$ is that they describe phonon mediated transitions between different bands. In the two-band case considered below this process leads effectively to Cooper pair formation in the N band, provided that the coupling constant $\lambda_{21} \neq 0$. The terms γ_{ij} denote the scattering rates from band i into band j due to nonmagnetic impurities; γ^s_{ij} are the magnetic scattering rates; $Z_{i,n}$ is a renormalization function; $\omega_n = \pi T(2n + 1)$ are Matsubara frequencies. It is important to note that the interband scattering rates γ_{ij} $(i \neq j)$ lead to Cooper pair tunneling between the corresponding bands and therefore play the same role as λ_{ij} $(i \neq j)$ in the case of two bands N and S. For a weak coupling $\lambda_{ij} \ll 1$ Eqs. (1), (2) are reduced to the equations of BCS model.

The choice of parameters of the model is motivated by the situation in YBaCuO. We assume that the planes are characterized by large coupling constants and form an S band ($\lambda_{11} = 3$), whereas the chains form an N band ($\lambda_{22} = 0$). A nonzero order parameter in the chains is induced through interband interactions $\lambda_{12} = \lambda_{21} = 0.2$. The above parameter set is consistent with $T_c = 90$ K in YBaCuO.

We assume that the interband scattering rates γ_{ij} in Eqs. (1), (2) are small γ_{12}, $\gamma_{21} \ll T_c$ and we set $\gamma^s_{12} = \gamma^s_{21} = \gamma^s_{11} = 0$. Account is taken of the impurity scattering due to usual and magnetic impurities in N band (γ_{22} and γ^s_{22}, respectively) and of the nonmagnetic scattering in the S band γ_{11}.

The origin of magnetic scattering is that upon decreasing the oxygen content the oxygen atoms move preferentially out of the chains. As a result, the chain Cu atoms develop magnetic moments that act as pair breakers. Thus, the magnetic scattering γ^s_{22} in the N band is the only free parameter of the model related to oxygen deficit in $YBa_2Cu_3O_{7-\delta}$.

The choice of elastic scattering rates in the N and S bands should be made consistent with the absolute values and anisotropy of resistivities above T_c. The resistivity of the corresponding band (S, N) at T_c is estimated as

$$\rho_s(T_c) = 8\pi^2\hbar^{-1}\omega^{-2}_{p,1} k_B T_c \sum_i \left[\lambda^{tr}_{1i} + \frac{(\gamma_{1i} + \gamma^s_{1i})}{\pi k_B T_c}\right],$$

$$\rho_n(T_c) = 8\pi^2\hbar^{-1}\omega^{-2}_{p,2} k_B T_c \sum_i \left[\lambda^{tr}_{2i} + \frac{(\gamma_{2i} + \gamma^s_{2i})}{\pi k_B T_c}\right], \tag{3}$$

were λ^{tr}_{ij} are transport coupling constants; $\omega_{p,1}$ and $\omega_{p,2}$ are the plasma frequencies of S and N band, respectively. With plasma frequencies 3.6 eV for planes and 3.2 eV for chains [12] and the elastic scattering rates in the N and S bands $\gamma_{11} = \gamma_{22} = (2-4)T_c$ (16–32 meV) the model (1)–(3) is consistent with the absolute values of resistivity 50–100 $\mu\Omega\cdot$cm at 100 K.

The electromagnetic response of a two-band superconductor can be calculated by generalization of the

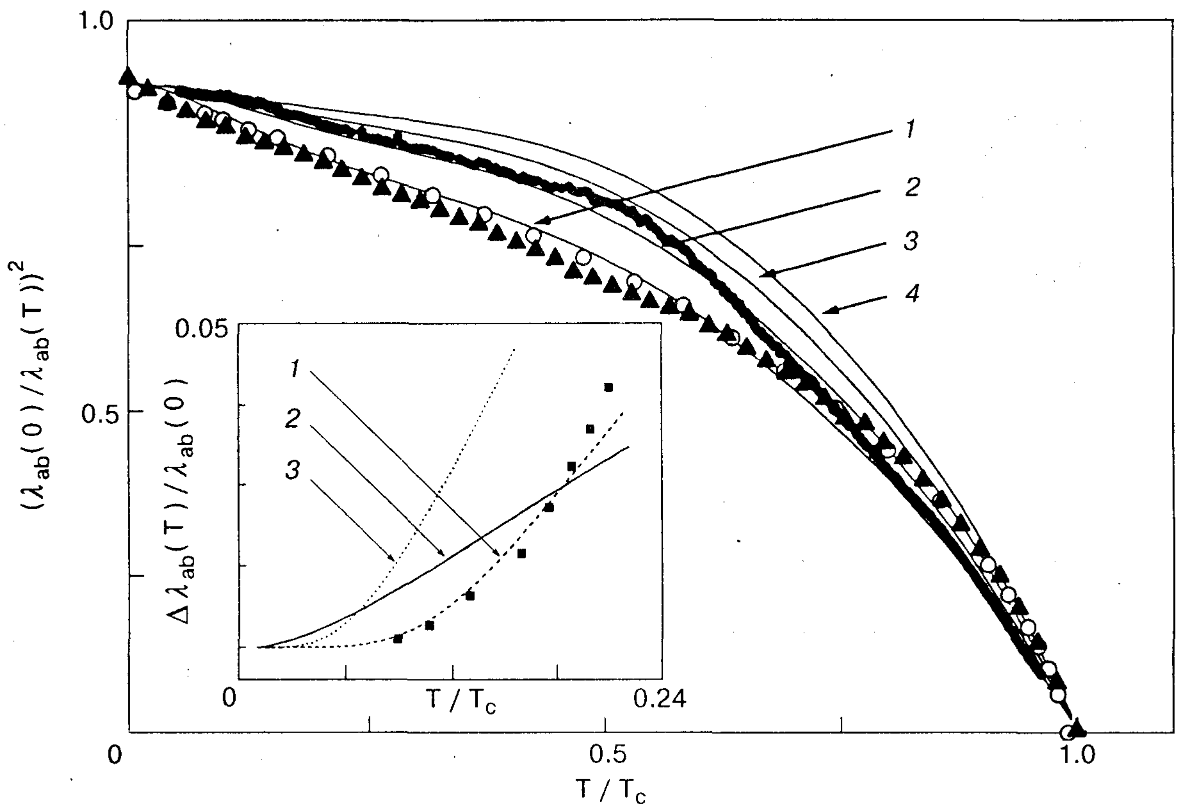

Fig. 1. Calculated curves ($\gamma^S_{22} = 1.6$ meV, $\gamma_{12} = 0$) compared with data from Ref. 5 (▲), Ref. 4 (○) and Ref. 6 (●). *1* — $\gamma_{22} = 16$ meV, *2* — $\gamma_{22} = 32$ meV, *3* — $\gamma_{22} = 64$ meV, *4* — $\gamma_{22} = 160$ meV. Inset: The crossover from exponential (curves *1*, *2*) to linear (curve *3*) behavior of $\Delta\lambda_{ab}(T)$ for $\gamma_{22} = 16$ meV and $\gamma^S_{22} = 0$ (*1*), 0.8 (*2*) and 3.2 meV (*3*). Solid squares: data from Ref. 3.

standard approach developed for strongly-coupled superconductors [9,13]. The penetration depth in *ab* plane is given by

$$\frac{\lambda_{ab}^{-2}(T)}{\lambda_{ab}^{-2}(0)} = \frac{Q_1(T) + Q_2(T)}{Q_1(0) + Q_2(0)} , \quad (4)$$

where the corresponding kernels for two bands are

$$Q_{1,2}(T) = \omega^2_{p;1,2}\, \pi T \sum_{\omega_n = 0}^{\infty} \frac{\Delta^2_{1,2}(i\omega_n)}{[\omega_n^2 + \Delta^2_{1,2}(i\omega_n)]^{3/2} Z_{1,2}(i\omega_n)} . \quad (5)$$

We assume that the conductivity in the *c* direction is nonmetallic [14], i.e. transport along *c* direction occurs via incoherent hopping. In this limit the penetration depth λ_c is given by the Josephson interaction between neighboring pairs of planes and chains and is determined by

$$\frac{\lambda_c^{-2}(T)}{\lambda_c^{-2}(0)} = 8\gamma^* \frac{T}{T_c} \sum_{\omega_n = 0}^{\infty} \frac{\Delta_1(i\omega_n)\Delta_2(i\omega_n)}{\sqrt{\omega_n^2 + \Delta_1^2(i\omega_n)}\sqrt{\omega_n^2 + \Delta_2^2(i\omega_n)}} \quad (6)$$

where $\gamma^* \simeq 1.78$ is Euler's constant.

3. With the parameters specified above, the Eliashberg equations are solved numerically for two bands. A set of $\lambda_{ab}^{-2}(T)$ curves for $\gamma^S_{22} = 1.6$ meV with different values of $\gamma_{11} = \gamma_{22}$ is shown in Fig. 1. The single crystal data from Refs. 4,5 ($\rho_{ab} \simeq 50\,\mu\Omega\cdot$cm) with the data on an electron-beam evaporated YBaCuO film ($\rho_{ab} \simeq 80\,\mu\Omega\cdot$cm) [6] are in good agreement with the calculations. The linear term at $T < 0.5T_c$ is well described by the model.

It was demonstrated explicitly in Ref. 15 that a small gap exists in the *N* band for $\gamma^S_{22} = 0$ and decreases rapidly with an increase of γ^S_{22}. As a result, at sufficiently large γ^S_{22} a gapless state develops in the system. The inset to Fig. 1 displays the results of calculations of $\Delta\lambda_{ab}(T)$ for different γ^S_{22} values. As a consequence of the development of the gapless state the crossover takes place at low *T* from the exponential temperature dependence for $\gamma^S_{22} = 0$ to the linear temperature dependence at larger γ^S_{22}. The data of

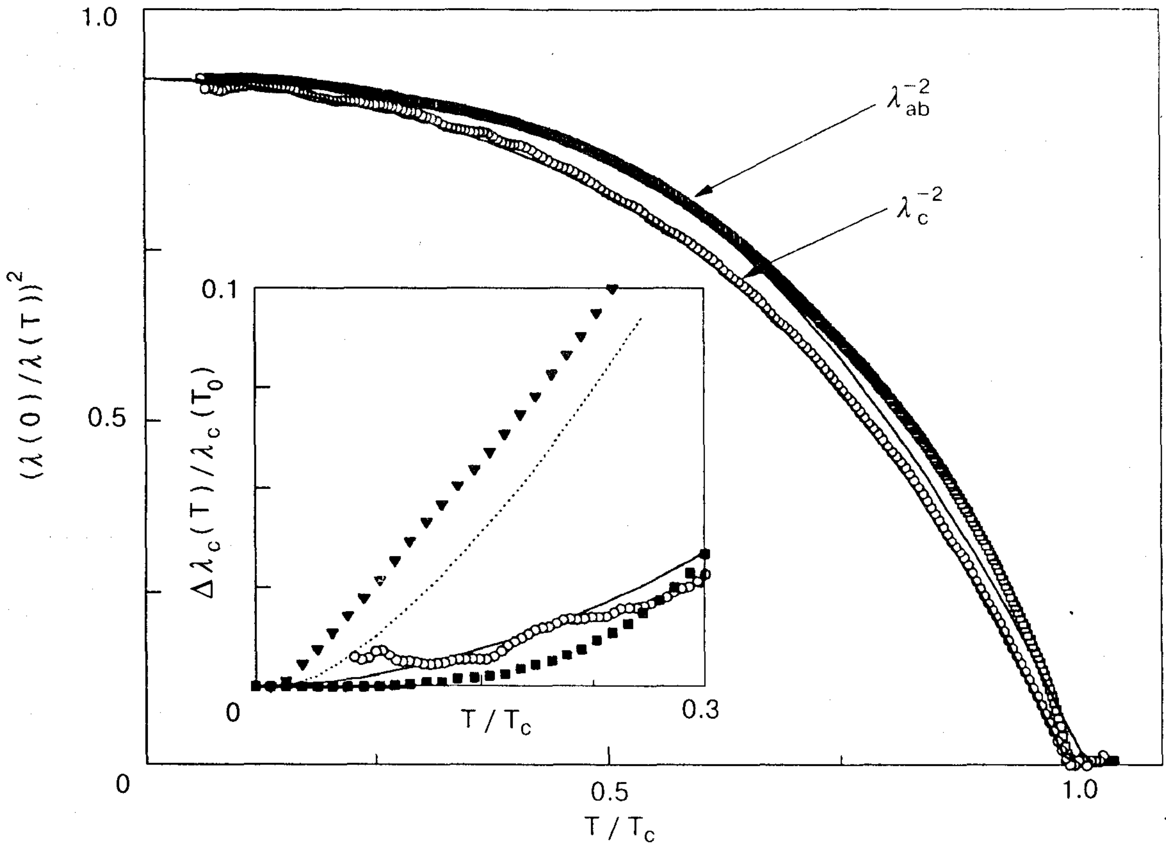

Fig. 2. $\lambda^{-2}(T)$ vs. T/T_c for $\lambda_c(T)$ and $\lambda_{ab}(T)$ from Ref. 11. Solid: two-band model with $\gamma_{22}^s = 6.4$ meV, $\gamma_{12} = 1.6$ meV, $\gamma_{22} = 24$ meV. Inset shows the results of two-band model with $\gamma_{22}^s = 6.4$ meV, $\gamma_{12} = 1.6$ meV, $\gamma_{22} = 24$ meV (solid) and $\gamma_{22}^s = 1.6$ meV, $\gamma_{12} = 0$, $\gamma_{22} = 16$ meV (dotted). Open circles: low T part of the data Ref. 11. $T_0 = 1.35$ K. d wave model from Ref. 8 (▲); s wave model from Ref. 8 (■).

Ref. 3 are compared with our results for $\gamma_{22}^s = 0$. Good agreement between the theory and experiment points out that at low T the exponential temperature dependence can be attributed to the absence of magnetic impurities in fully oxygenated YBaCuO films [3].

With an increase of γ_{22}^s and γ_{22} the contribution of the N band to the penetration depth vanishes and the behavior of $\lambda_{ab}(T)$ becomes closer to that predicted by the isotropic strong-coupling model. Such a crossover illustrated in Figs. 1 and 2 correlates well with the increase in the resistivity above T_c. The computed $\lambda(T)$ curves alongside the data on YBaCuO powders [11] are shown in Fig. 2. At low T the experimental dependence $\Delta\lambda_{ab}^{-2}(T) \sim T^2$ is in agreement with theory. A fit of the calculated $\lambda_c(T)$ curve to the experimental data [11] is presented in Fig. 2. Our results for different sets of parameters γ_{22}^s and γ_{22} point to the absence of linear behavior of $\lambda_c(T)$, which is in contradiction to the predictions of d wave model [7,8].

So, the influence of impurity scattering on the magnetic field penetration depth in YBaCuO manifests itself in the following way. As discussed in Refs. 9,10, the magnetic scattering rate γ_{22}^s in N band may serve as a quantitative measure of oxygen deficit in $YBa_2Cu_3O_{7-\delta}$. In fully oxygenated $YBa_2Cu_3O_7$ films [3] the magnetic pair-breaking in chains (N band) is absent, i.e. $\gamma_{22}^s \simeq 0$. In this case the two-band model and measurements [3] demonstrate an exponential temperature dependence at low T with exponent given by a small proximity-induced energy gap in the chains. Even for relatively small concentration of magnetic impurities $\gamma_{22}^s \geq 0.2T_c$ the superconducting state in chains becomes gapless. The gaplessness results in a linear term in the contribution of chains to $\Delta\lambda_{ab}(T)$ at low T. The data of Refs. 4, 5 obtained in $YBa_2Cu_3O_{6.95}$ single crystals indeed shows a linear dependence of $\Delta\lambda_{ab}(T)$ in accordance with the model. When going from oxygen-deficient single crystals [4,5] to oxygen-deficient $YBa_2Cu_3O_{7-\delta}$ thin films [6] and powders [11] the usual and magnetic scattering in the chains increase. This leads to the suppression of the superconductivity in chains and to a decrease in chain contribution to the total penetration depth. As a result, the crossover at low T from the linear to quadratic [11] temperature dependence of $\Delta\lambda_{ab}$ takes place. This behavior correlates well with the experimental data corresponding to different values of resistivity above T_c.

In conclusion, we have demonstrated for the first time that all recent measurements of $\lambda_{ab}(T)$ in YBaCuO are consistent with the two-band model. The approach [9,10] for calculation $\lambda_{ab}(T)$ is extended to the entire temperature range $0 < T < T_c$ at arbitrary concentration of nonmagnetic impurities. The model correctly describes the disappearance of the linear term in $\lambda_{ab}(T)$ with increasing sample resistivity. The temperature dependence of λ_c is calculated within the two-band model in the approximation of Josephson interaction between the layers. In this approach the linear term never appears in $\lambda_c(T)$ at low T. The key experiments to check the two-band description for YBaCuO would be measurements of $\lambda_c(T)$ on YBaCuO single crystals as well as those of $\lambda_{ab}(T)$ in oxygen depleted and Fe- or Ga-doped YBaCuO, when superconductivity of CuO chains is destroyed.

We are grateful to G. M. Eliashberg, N. Klein, V. Z. Kresin, E. G. Maksimov, G. Müller, S. Orbach-Werbig, and S. Hensen for useful discussions. The work was supported by the Programme «High Temperature Superconductivity» under Grant No 93194 and, in part, by INTAS under Grant No 93-2154 and by Grant No REY300 from the International Science Foundation and the Russian Government.

1. W. N. Hardy, D. A. Bonn, D. C. Morgan, et al., *Phys. Rev. Lett.* 70, 3999 (1993).
2. Z. Ma, R. C. Taber, L. W. Lombardo, et al., *Phys. Rev. Lett.* 71, 781 (1993).
3. N. Klein, N. Tellmann, H. Schulz, et al., *Phys. Rev. Lett.* **71**, 3355 (1993).
4. D. A. Bonn, S. Kamal, Kuan Zhang, et al., *Phys. Rev.* **B50**, 4051 (1994).
5. Jian Mao, D. H. Wu, J. L. Peng, et al., *Phys. Rev.* **B51**, 3316 (1995).
6. S. Orbach-Werbig, A. A. Golubov, S. Hensen, et al., *Physica* C235–240, 1823 (1994).
7. P. J. Hirschfeld, W. O. Puttika, and D. J. Scalapino, *Phys. Rev.* B50, 10250 (1994).
8. R. A. Klemm and S. H. Liu, *Phys. Rev. Lett.* **74**, 2343 (1995).
9. S. D. Adrian, M. E. Reeves, S. A. Wolf, and V. Z. Kresin, *Phys. Rev.* B51, 6800 (1995).
10. V. Z. Kresin and S. A. Wolf, *Phys. Rev.* **B46**, 6458 (1992); *ibid.* 51, 1229 (1995).
11. A. M. Neminskii and P. N. Nikolaev, *Physica* **C212**, 389 (1993).
12. I. I. Mazin and O. V. Dolgov, *Phys. Rev.* **B45**, 2509 (1992).
13. S. B. Nam, *Phys. Rev.* 156, 487 (1967).
14. D. G. Clarke, S. P. Strong, and P. W. Anderson, *Phys. Rev. Lett.* 74, 4499 (1995).
15. A. A. Golubov, O. V. Dolgov, E. G. Maksimov, et al., *Physica* C235–240, 2383 (1994).