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Quantum topological effects in a metallic ring with an Anderson impurity

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Spin and charge persistent currents in a metallic ring with an Anderson impurity are studied. It is shown that the impurity changes are caused by the external magnetic and electric fluxes — the Aharonov—Bohm and Aharonov—Casher type oscillations (the magnitudes and the periods) — in contrast with the normal metallic case. For instance, the Aharonov—Bohm oscillations look like the type II superconductor situation. These effects are a manifestation of the Luttinger liquid nature of the considered system.

In recent years physicists have been greatly interested in the quantum behavior of low-dimensional systems, especially at low temperatures, where the quantum effects play an essential role. Quantum fluctuation effects are known to show themselves most vividly in low-dimensional systems, where they are enhanced by singularities in the density of states. When describing the low-temperature behavior of low-dimensional many-body quantum systems one has to rely on the exact solutions of the Schrödinger equations because approximate methods like the mean-field theory and modifications or the perturbation theory can yield results that are incorrect even qualitatively. One of the most productive methods allowing the exact solution of the Schrödinger equation is the Bethe ansatz, or the algebraic quantum inverse scattering method [1]. There is a large number of $1D$ quantum models (spin or strongly correlated electron many-body systems), for which the exact solution (eigenfunctions, eigenvectors and correlation functions) has been found by this method. Among these systems one can distinguish the exact solutions for the quantum electron and spin systems with impurities, see, e.g. the review papers [2–4]. The thermodynamical properties (as the size of the system goes to infinity) of an electron gas with a single impurity remain the same as for the non-interacting electrons, and the interaction mainly changes the properties of the impurity. But the lowest excitations (charge and spin) are changed with respect to the electron gas situation.

It is well known that the nonforce topological effect of the electric and magnetic fields on charged particles with a magnetic moment manifests itself, e.g. in the quantum effects of Aharonov—Bohm (AB) [5] and Aharonov—Casher (AC) [6]. These effects are due to acquisition by the wave functions of particles with charges and magnetic momenta of phases proportional to the fluxes of the magnetic and electric fields, respectively. The gauge invariance of the electromagnetic field is the reason why the particles moving along closed trajectories exhibit this property.

During several last years the systems of the greatest interest in this connection are those in which the topological effect of the electric and magnetic fields reveals itself along with quantum features related to the nature of particle interactions in solids. The simplest way to see such topological effects in condensed matter is to study microscopically the persistent charge and spin currents in the $1D$ ring of interacting electrons. A number of theoretical papers have been published in which the persistent currents were studied with the help of exact methods, e.g. the Bethe ansatz [7–18] and the bosonization technique [19,20]. Development of the theory was enhanced by experiments studying the thermodynamical characteristics (the magnetic momenta, which are the another way of describing the persistent currents) in mesoscopic rings [21–24]. It was understood that the topological influence of the electromagnetic field fluxes cause the virtual creation and annihilation of the excitations of the system at the left or right Fermi points. For the ground state, this means the virtual processes for the lowest excitations forming the Dirac seas of the system for the gapless excitations, see, e.g. in [25]. The response of the system to the external magnetic and electric fluxes for the gapless situation is of mesoscopic scale $\propto L^{-2}$, where L is the number of sites. For the gap situation, the response is proportional to $\exp(-L\Delta/v_F)$, where Δ is the gap, and v_F is the Fermi velocity of the excitation. The same method is used to calculate the correlation functions for the strongly correlated $1D$ systems [9,26,27].

From this point of view it is interesting to examine theoretically the free electron model on a ring with the impurity, enclosed into the external magnetic flux

Φ or into the electric flux F of radially directed electric field generated by a charged string in the middle of the ring with $F = 4\pi\tau$, where τ is the linear density of the charge on the string. The theoretical description for the ring with the Kondo-like impurity was carried out in [28].

In this paper we study the topological effect of the magnetic and electric field on the electrons moving in the ring with the Anderson impurity. We show that the persistent current oscillations (both spin and charge) change their properties with respect to the system without impurity. The AB oscillations are characterized by two periods: Φ_0 and $\Phi_0/2$, where $\Phi_0 = hc/e$ is the ordinary metallic flux quantum, and the AC oscillations have the metallic period of $F_0 = hc/\mu$ where μ is the magnetic moment of an electron. The situation looks like a Hubbard chain with attraction between the electrons, where the electrons are bound in pairs. The ground state of the system is connected with the Dirac sea filled by unbound electrons and electron pairs, depending on the magnetic field value, like type II superconductors. The magnitude of spin and charge persistent current oscillations depends on the magnetic moment of the ring and the band filling. For nonzero temperature T, the spin and charge persistent currents in the system, as usual, are of order of $\exp(-L/\xi)$, because $T \neq 0$ causes a nonzero value of the correlation length ξ in the $1D$ system.

We can solve the quantum problem with the Bethe ansatz equations only for the dispersion law of the electrons linearized near the Fermi point (let the Fermi velocity be equal to unity). The ground state energy of electrons on the ring with the Anderson impurity is equal to the series in L^{-1}:

$$\frac{E_0}{L} = \varepsilon_\infty + \frac{\varepsilon_1}{L} + \frac{\varepsilon_2}{L^2} + \ldots , \tag{1}$$

where the first term describes the free «bulk» electrons in the thermodynamical limit, the second is connected with the behaviour of the impurity, and the third is connected with the excitations (or, what amounts to the same, with the topological fluxes effects, see, e.g., [8]).

The magnetic and electric field nonforce effects change the periodic boundary conditions for the quantum numbers k_j and λ_α, to the twisted ones [8,16] and the equations for the sets of quantum numbers k_j (the quasi-momenta) and λ_α (the real part of the pair rapidities) describing low-lying states are [3,16,28] (we study the case in which the impurity is at the site «0»)

$$k_j L = 2\pi\left(I_j + \frac{\Phi}{\Phi_0} + \frac{F}{F_0}\right) - \varphi(k_j) - $$

$$- 2\sum_{\beta=1}^{M} \operatorname{arctg} 2\left(g(k_j) - \lambda_\beta\right) , \tag{2}$$

$$-x(\lambda_\alpha)L + \operatorname{Re}\left(\varphi(x+iy)\right) = \pi\left(J_\alpha - \frac{2\Phi}{\Phi_0}\right) +$$

$$+ \sum_{j=2M+1}^{N} \operatorname{arctg} 2(\lambda_\alpha - g(k_j)) + \sum_{\beta=1}^{M} \operatorname{arctg}(\lambda_\alpha - \lambda_\beta) , \tag{3}$$

where N is the number of electrons in the ring; M is the number of paired electrons [29]; I_j and J_α are integer or half-integer numbers depending on the values of N and M:

$$\varphi(k) = 2\operatorname{arctg}\frac{\Gamma}{k - \varepsilon_d} ;$$

Γ is the resonance level width; ε_d is the energy of the d electron;

$$g(k) = (2U\Gamma)^{-1}(k - \varepsilon_d - U/2)^2 ;$$

$U > 0$ is the impurity level repulsion; and

$$x(\lambda) = \varepsilon_d + U/2 - (U\Gamma)^{1/2}[\lambda + (\lambda^2 + 1/4)^{1/2}]^{1/2} ;$$

$$y(\lambda) = (U\Gamma)^{1/2}[-\lambda + (\lambda^2 + 1/4)^{1/2}]^{1/2} .$$

The energy of N electrons with M bound electrons is equal to

$$E = \sum_{j=2M+1}^{N} k_j + 2\sum_{\alpha=1}^{M} x(\lambda_\alpha) . \tag{4}$$

One can see obviously, that Eqs. (2), (3) are periodic in Φ and F with the periods Φ_0 and F_0, respectively. That is why we can replace in Eqs. (2), (3) the phase shifts $(\Phi/\Phi_0) \to \{\Phi/\Phi_0\}$ and so on, where $\{x\}$ means the fractional part of x to the nearest I_j and J_α [14].

Using the method of the finite size corrections calculations, see, e.g. [30], one obtains for the energy correction connected with the electromagnetic field fluxes

$$2\varepsilon_2(\Phi, F) = v_{F1} \times$$

$$\times \left[z_{11}\left(D_1 + \left\{\frac{\Phi}{\Phi_0}\right\} + \left\{\frac{F}{F_0}\right\}\right) + z_{12}\left(D_2 - \left\{\frac{2\Phi}{\Phi_0}\right\}\right)\right]^2 +$$

$$+ v_{F2}\left[z_{21}\left(D_1 + \left\{\frac{\Phi}{\Phi_0}\right\} + \left\{\frac{F}{F_0}\right\}\right) + z_{22}\left(D_2 - \left\{\frac{2\Phi}{\Phi_0}\right\}\right)\right]^2 , \tag{5}$$

where the numbers $2D_1 = I_{\max} + I_{\min}$, $2D_2 = J_{\max} + J_{\min}$ determine the initial phase shift of the spin and charge persistent current [15]; $v_{F1,2}$ are the Fermi velocities of the unbound electrons and pairs, respectively:

$$2\pi\rho(k)v_{F1} = \frac{\partial\varepsilon}{\partial k} , \quad k = B , \tag{6}$$

$$2\pi\sigma(\lambda)v_{F2} = \frac{\partial\kappa}{\partial\lambda} , \quad \lambda = Q , \tag{7}$$

which are determined from the integral equations

$$\varepsilon(k) = k + \int_Q^\infty a_1(g(k) - \lambda)\kappa(\lambda)\, d\lambda , \tag{8}$$

$$\kappa(\lambda) = - 2x(\lambda) - \int_Q^\infty a_2(\lambda - \nu)\kappa(\nu)\, d\nu -$$

$$- \int_{-\infty}^B g'(k)a_1(\lambda - g(k))\varepsilon(k)\, dk , \tag{9}$$

where the kernels are

$$a_n(z) = \frac{2n}{\pi(n^2 + 4z^2)} . \tag{10}$$

The densities of the quantum numbers k and λ can be calculated from the equations [31,32]

$$\rho(k) = \frac{1}{2\pi} + g'(k) \int_{+Q}^\infty a_1(g(k) - \lambda)\sigma(\lambda)\, d\lambda , \tag{11}$$

$$\sigma(\lambda) + \int_Q^\infty a_2(\lambda - \nu)\sigma(\nu)\, d\nu =$$

$$= - \frac{x'(\lambda)}{\pi} - \int_{-\infty}^B a_1(\lambda - g(k))\rho(k)\, dk , \tag{12}$$

and the first term in Eq. (1), ε_∞, is equal to

$$\varepsilon_\infty = \frac{E}{L} = \int_{-D}^B k_\rho(k)\, dk + 2 \int_Q^{Q_1} x(\lambda)\sigma(\lambda)\, d\lambda . \tag{13}$$

We have omitted in Eqs. (8)–(12) the terms of order of L^{-1} because we are interested in the response of the «bulk» electrons but not of the impurity. The limits of integrations yields from

$$\frac{N}{L} = \int_{-D}^B \rho(k)\, dk + 2 \int_Q^{Q_1} \sigma(\lambda)\, d\lambda , \tag{14}$$

$$\frac{S^z}{L} = \frac{1}{2} \int_{-D}^B \rho(k)\, dk . \tag{15}$$

Note that in the last equations one has to integrate over the finite limits, in order to avoid divergences, and there are no divergences in Eqs. (8)–(12) [32].

With regard to $z_{j,k}$ ($j, k = 1, 2$) one can say that they are the dressed charges of the excitations at the Fermi points

$$z_{ij} = \xi_{ij}\Big|_{\substack{k=B \\ \lambda = Q}} , \quad i, j = 1, 2 ,$$

see [9,30]. To determine the dressed charges for the system with an Anderson impurity we have to solve the set of integral equations:

$$\xi_{11}(k) = 1 + \int_Q^\infty a_1(g(k) - \lambda)\xi_{21}(\lambda)\, d\lambda , \tag{16}$$

$$\xi_{12}(k) = \int_Q^\infty a_1(g(k) - \lambda)\xi_{22}(\lambda)\, d\lambda , \tag{17}$$

$$\xi_{21}(\lambda) = - \int_Q^\infty a_2(\lambda - \nu)\xi_{21}(\nu)\, d\nu -$$

$$- \int_{-\infty}^B g'(k)a_1(\lambda - g(k))\xi_{11}(k)\, dk , \tag{18}$$

$$\xi_{22}(\lambda) = 1 - \int_Q^\infty a_2(\lambda - \nu)\xi_{22}(\nu)\, d\nu -$$

$$- \int_{-\infty}^B g'(k)a_1(\lambda - g(k))\xi_{12}(k)\, dk . \tag{19}$$

One can see that for free electron gas without impurity the dressed charges are equal to the bare ones: $z_{11} = z_{22} = 1$, $z_{12} = z_{21} = 0$.

One can obviously see from Eq. (5) that the ground state energy is a periodic function of fluxes, and, therefore, the oscillations of the charge and spin persistent currents arise in the ring with the magnetic and electric fluxes, respectively. It is seen that spin persistent current induced by the electric flux oscillates with period F_0 , the ordinary «metallic» period of the oscillations. It is connected with the movement of virtual unbound electron excitations from the one Fermi point to the another one, caused by the electric flux. As to the charge persistent current oscillations, induced by the magnetic flux, we can see that these oscillations consist of two periods: the first one (Φ_0) is connected with the movement of the virtual unbound electron excitations, as for the spin persistent current but the second, one ($\Phi_0/2$) arises from the movement of the virtual pair excitations. $\Phi_0/2$ — periodic oscillations mean, obviously, that the charge of the pair is $2e$. The situation is analogous to the Hubbard model with the attraction between electrons, and looks like a «toy» model for the type II superconductor. Note that we are in $1D$, and thus even for the ground state the real order parameter for our system is equal to zero.

So, we face with the «strange» situation: the thermodynamical properties of the «bulk» electrons are as of the free electron gas, but the mesoscopic response reveals the effective electron-electron interaction, caused by the coupling with single impurity. That «strange» property is, naturally, the manifestation of the Luttinger liquid nature of the $1D$ quantum impurity system [33]. For our case the persistent currents are connected with Haldane's topological numbers J [33].

The magnitudes of all types of the oscillations depend strongly on the impurity level filling and the magnetization of the system. If the unbound electrons or pair excitations had gaps, the mesoscopic response $\propto L^{-2}$ would not exist, and the persistent currents would be of the order of $\exp(-L)$.

For the situation with the opposite sign of U, $U < 0$, [29], the ground state is described by two Dirac seas one for unbound electrons, and another, instead of pairs, for the spinons. That is why the AB period of charge persistent current oscillations in this case is equal to Φ_0, as it must be, and the AC oscillations contain, two harmonics. The first period, as in the former case is equal to F_0, and is caused by the unbound electron excitations. The second one, $F_0/2$, is connected with the excitation of a spinon (with the magnetic moment 2μ). That situation is analogous to the Hubbard model with the repulsion between the electrons [14].

In the Kondo limit, we naturally see that the AB oscillations have only one period Φ_0, because the «bulk» electrons are coupled to the impurity only with the magnetic degrees of freedom, and the AC oscillations resemble the repulsive Hubbard model [14,28]. The charge persistent current is the same as is the orbital magnetic moment of the total ring, so we could see the «underscreened» or «overscreened» situation for the system under consideration similarly the situation for the multichannel Kondo problem [34]. But the calculation of the influence of the external electromagnetic fluxes on the impurity is of higher order in the $1/L$ effect and needs special consideration.

The values of $\sigma(B)$ and $\rho(Q)$ can be obtained from the Refs. 31,32. One can solve the set of equations (16)–(19) numerically, depending on the number of unbound spins and the number of pairs in the system. The simple analytic solutions we can obtain for the nonmagnetic situation $H = 0$, $B = -\infty$, $\rho = 0$, $v_1 |_{k=B} = 0$, where H is the magnetic field value. We have for the symmetric case, where $2\varepsilon_d = -U$, and $Q = \infty$: $\xi_{11} = 1$, $\xi_{12} = (1/2)^{1/2}$, $\xi_{21} = 0$, $\xi_{22} = = (1/2)^{1/2}$.

Thus, for this case we have only $\Phi_0/2$ periodicity of the charge persistent current, because the ground state is formed by the Dirac sea filled with electron pairs only. It is easy to see that the Fermi velocity of the pairs is equal to 1 in this limit.

To conclude, in present paper we have studied the topological effect of the fluxes of the electric and magnetic fields on the spin and charge persistent currents in a metallic ring with an Anderson impurity. We have shown that the electron-impurity interaction caused electron-electron coupling of a Luttinger liquid nature, which is the origin for the «nonmetallic» oscillations of the persistent currents in the ring. These currents are connected with the topological virtual excitations of the Dirac seas and are of the mesoscopic scale L^{-2}.

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