Fizika Nizkikh Temperatur, 1996, v. 22, No 6, p. 635–637

Critical currents and relaxation in Josephson media

A. I. D'yachenko

A. Galkin Donetsk Physico-Technical Institute, National Academy of Sciences of Ukraine, 72, R. Lyuksemburg St., Donetsk, 340114, Ukraine

J. Stankowski

Institute of Molecular Physics, Polish Academy of Sciences, 17, Smoluchowskiego, PL-60179, Poznan, Poland

The main aim of this paper is to show that Josephson critical current I_c stability to high magnetic fields $H >> H_{c1}$ tends to hysteretic relaxation rate of the Josephson media critical current $J_c(t)$ and produces a change in the Ferrell–Prange equation. In the framework of the discussed model, the reason for $J_c(t)$ vs. H hysteresis lies in the irreversible dependence of the granule surface current. It is shown that the effective intergranular field H_g in Josephson media plays a secondary role at these fields.

Introduction

Usually the magnetic field H considerably influences the critical current J_c of a Josephson medium at fields $H > H_0 \approx \Phi_0/S_0$, where Φ_0 is the magnetic flux quantum and the «quantization cell» $S_0 \sim \langle a\rangle^2$ ($\lambda_j > \langle a\rangle$); $S_0 \approx 2\lambda W$; $W < \lambda_j < \langle a\rangle$; $\langle a\rangle$ is the grain diameter; W is the Josephson contact width; λ, λ_j are the London and the Josephson penetration depths into the granules and intergranular space, respectively. The situation changes considerably if Abrikosov vortices enter into the granules [1,2]. It is generally believed that even in this case the critical current I_c vs. H dependence is controlled by the value of the intergranular field $H_g = H - n_g \times M_g$, where n_g is an effective demagnetizing factor; M_g is the average grain magnetization. But this is by no means always the case. What actually happens is the following: the Abrikosov vortices reduce the screening current density at the surface of the granules J_{sf}, thus stabilizing the critical current of Josephson contacts and therefore stabilizing the transport critical current density J_c of ceramics.

At large fields $H >> H_{c1}$ the surface current value J_{sf} in granules is determined by its equilibrium component J_{eq} , connected with the reversible magnetization M_{eq} , $J_{eq} \cong M_{eq}/\lambda$ and the irreversible one, J_{cg} , determined by the Abrikosov vortex pinning in the vicinity of the granule surface. The sign of J_{cg} depends on the history of the sample and therefore $J_{sf} = J_{eq} \pm J_{cg}$. Here the sign «+» corresponds to the external magnetic field increase, and the sign «−» corresponds to its decrease.

Actually, in the most general case the Josephson critical current $I_c(H)$ between adjacent grains in a local field H is determined by the magnitude of the surface current J_{sf}, $I_c(H) \sim 1/\lambda\, J_{sf}(H)$. Only in the Meissner phase of grains $J_{sf} \approx H/\lambda$ and therefore $I_c(H) \sim 1/H$. The basic idea of the model [1,2] is that the J_{sf} value is the sum of the equilibrium «Meissner» component J_m and the irreversible critical current of grains J_{cg} , $J_{sf} = J_m \pm J_{cg}$. The sign of J_{cg} depends on the magnetic history of the sample. Therefore $I_c(H) \approx I_{c0}/(1 + \Delta\varphi(H))$, where

$$\Delta\varphi = W(d\varphi/dx) =$$

$$= \{2\pi\mu_0 W/\Phi_0\}[2\lambda(J_m \pm J_{cg}) + H \times d_N] \qquad (1)$$

is the phase difference along the junction width W; d_N is the Josephson contact thickness. If there is a spatially inhomogeneous Josephson critical current density $j_c(x)$ then $W = r$, where r is the correlation radius of the spatial fluctuations of the j_c [3]. When the Abrikosov vortices penetrate into the granules the surface current $J_{sf} \approx H_{c1}/\lambda + J_{cg}$ is approximately constant and the Josephson medium critical current J_c vs. H dependence has a double-step behavior [4]. In the framework of the discussed model, the irreversible dependence of the granules surface current $\pm J_{cg}$ is responsible for the J_c vs. H hysteresis behavior. An additional contribution arises if the dipole field of the grains H_{rev} (or persistent loops) changes the intergranular field value H_g [5].

Intergranular field

Let $\mathbf{H}_{int}$ be the internal field, $\mathbf{H}_{ex}$ the external field, and $\mathbf{B}_{av}$ the average induction field in the sample. Then

$$\mathbf{H}_{int} = \mathbf{H}_{ex} - D \times \mathbf{M}_{av} \ , \ \mathbf{B}_{av}/\mu_0 = \mathbf{H}_{int} + \mathbf{M}_{av} \ , \quad (2)$$

$$\mathbf{H}_g = \mathbf{H}_{int} + D_g \times \mathbf{M}_{av} + \mathbf{H}_{rev} =$$

$$= \mathbf{H}_{ex} + (D_g - D) \times \mathbf{M}_{av} + \mathbf{H}_{rev} \ , \quad (3)$$

where D, D_g are the demagnetizing factors of the whole sample and of the average grain, respectively; $\mathbf{M}_{av} = \mathbf{M}_{av}(\mathbf{H}_{int})$ is the average magnetization of the Josephson media and $\mathbf{H}_{rev}$ is the dipole field of the grains (or persistent loops) which spills into the intergranular region, $\mathbf{H}_{rev} = -k_g \mathbf{M}_g$, $k_g \sim 0.1$. The average induction field $\mathbf{B}_{av}$ is related to the field within (subscript s) and between the granules (subscript n) by

$$\mathbf{B}_{av} = f_s \langle \mathbf{B}_s \rangle + f_n \langle \mathbf{B}_n \rangle =$$

$$= f_s(\mathbf{H}_g + \mathbf{M}_g + \mathbf{M}_{eq}) + (1 - f_s)\mathbf{H}_g = \mathbf{H}_g + f_s \times \mathbf{M} \ , \quad (4)$$

$f_n = 1 - f_s$, $\mathbf{M}(\mathbf{H}_g) = \mathbf{M}_{eq} + \mathbf{M}_g$ is the full grain magnetization; $\mathbf{M}_{eq}$ is the equilibrium part of the grain magnetization; $M_{eq} \sim -H_{c1g}$. From (2)–(4) one obtain the relation between internal $\mathbf{H}_g$ and applied $\mathbf{H}_{ex}$ fields:

$$\mathbf{H}_g = \mathbf{H}_{ex} + \frac{(D_g - D) \times f_s \times \mathbf{M} + (1 - D) \times \mathbf{H}_{rev}}{(1 - D_g)} \ , \quad (5)$$

therefore, the effective demagnetization factor is

$$n_g \approx \frac{(D - D_g)f_s + (1 - D)k_g}{(1 - D_g)} \leq 1 \ .$$

Relaxation

Kwasnitza and Widmer [6] have found that the relaxation rate of the intergrain critical current J_c is strongly hysteretic. They take it on trust that the value J_c is determined by the effective field $\mathbf{H}_g$ [5] between the grains. However, this model cannot explain the observed [6] large shift of the two J_c branches along the field H axis, as, according to (5), the magnetized grains M_g would have to generate nonrealistic high fields up to several Teslas.

On the other hand, even without grain demagnetization (5), the irreversible flux dynamics inside grains should contribute to the irreversible behavior of J_c. The reason is that the intragranular critical current density at the grain surface J_{cg} adds to or subtracts from the Meissner shielding current. With the knowledge of the hysteresis of intragrain current J_{cg} relaxation it is now possible to make detailed predictions as to how critical transport current J_c changes with time. The critical current of a Josephson contact I_c in the parallel field H can be written as follows: $I_c(H) = f(H_{eff}(H))$, where $f(H)$ is given by the real structure of the Josephson contact and $H_{eff} = (2\lambda^2 J_{sf} + d_N H)/(2\lambda + d_N)$. In the Meissner phase of grains the surface current density J_{sf} is $J_{sf} = H/\lambda$, so $H_{eff} = H$. For the derivation of the Josephson media transport-current relaxation $J_{c(t)} \sim I_c(t)$ we find the derivative

$$\frac{d \ln (J_c(t))}{d \ln t} = \frac{d \ln J_c(H)}{dH} \frac{dH}{dH_{eff}} \frac{dH_{eff}}{d \ln t} \ .$$

In the mixed state the derivative $dH_{eff}/d\ln t = \pm (2\lambda^2/d_N)\{dJ_{cg}/d\ln t\}$, $J_{cg} \sim M_g$ and at least

$$d \ln J_c / d \ln t =$$

$$= \pm (2\lambda^2/\gamma d_N \langle a \rangle)(d \ln J_c/dH)_{exp}(dM/d\ln t)_{exp} \ , \quad (6)$$

where the derivatives $(d \ln J_c/dH)_{exp}$, $(dM/d\ln t)_{exp}$ are the experimental values; (+) corresponds to the increasing and (–) to the decreasing in external field H. According to (6), the relaxation rate of the critical transport current is strongly hysteretic. The J_c relaxation rate dJ_c/dt can be *positive or negative* and even equal to zero as a function of the external magnetic field cycling (Fig. 1).

Ferrell–Prange equation

Finally, let us consider the modification of the Ferrell–Prange equation for high magnetic fields $H \gg H_{c1}$ when the $d\varphi/dx$ value is given by Eq. (1). It follows from Eq. (1) and the two-fluid equation $J_\perp = J_n + J_s$ that

$$\partial^2\varphi/\partial x^2 = (2\pi\mu_0/\Phi_0)\{d_N + 2\lambda^2(\delta J_{sf}/\delta H)\}J_\perp(\varphi) \ ,$$

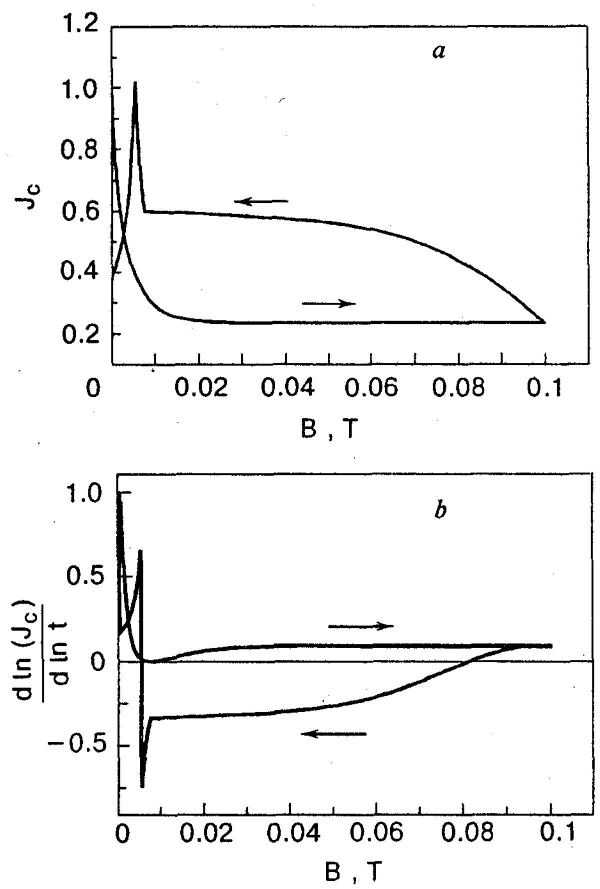

Fig. 1. The calculated $J_c(B)$ dependence for a Josephson contact; $J_{cg} = 3 \times 10^6\ \mathrm{A/cm^2}$, $W = 0.1\ \mu m$, $\lambda = 0.15 \mu m$, $H_{c1} = 80$ Oe (*a*). The hysteretic relaxation rate of the Josephson contact critical current $d \ln J_c / d \ln t$ vs. B (*b*).

therefore, the Ferrell–Prange equation should be given by

$$\Lambda_\omega \partial^2\varphi/\partial x^2 =$$

$$= \lambda_j^{-2} \exp(i\varphi)(\varepsilon - i) + c_0^{-2}\partial^2\varphi/\partial t^2 + \gamma\partial\varphi/\partial t \ ; \quad (7)$$

$$\Lambda_\omega = \frac{d_N + 2\lambda}{d_N + 2\lambda_\omega} \ ,$$

$$\lambda_\omega = \frac{\lambda_{\mathrm{eff}}[1 + i\omega\tau - (2i\lambda_c^2/\delta_{n\omega}^2)]}{1 + i\omega\tau + \lambda_c^2/\lambda^2} \ ;$$

$$\lambda_{\mathrm{eff}}^2 = \frac{\lambda^2 + \lambda_c^2/(1 + i\omega\tau)}{1 + 2i\lambda^2/\delta_{n\omega}^2} \ ;$$

$$\lambda_{\mathrm{eff}}^2 = \frac{\lambda^2 + \lambda_c^2(1 - i\omega\tau)^{-1}}{1 + 2i\lambda^2/\delta_{n\omega}^2} \ ;$$

$$J_s(\varphi) = \mathrm{Re}\{J_c(H)\exp(i\varphi)(\varepsilon - i)\} =$$

$$= J_c(H)(\sin\varphi + \varepsilon\cos\varphi) \ ;$$

λ_c is the Campbell penetration depth; $\lambda_c^2 = B\Phi_0/\mu_0 k_p$; $\tau = \eta/k_p$; η is the Abrikosov vortex viscous drag coefficient in the absence of flux creep; k_p is the Labusch parameter [7]; $\delta_{n\omega}$ is the normal fluid skin depth, $\delta_{n\omega}^2 = 2/\mu_0\omega\sigma_{n\omega}$; λ_j is the Josephson penetration depth;

$$\lambda_j^2 = \frac{\Phi_0}{2\pi\mu_0(2\lambda + d_N)J_c(H)} \ ,$$

$$c_0^{-2} = \frac{\mu_0\varepsilon_0\varepsilon_r(2\lambda + d_N)}{d_N} \ ;$$

ε_r is the dielectric constant of the intergranular media; $\gamma = \mu_0(2\lambda + d_N)/R_{sf}$, R_{sf} is the Josephson contacts resistance per square. According to (7), the J_c vs. H dependence can lead at high fields $H > H_{c1}$ to the well known magnetic-field hysteresis observed above H_{c1} and the *plateau region* in the microwave surface resistance of ceramic superconductors (see, for example, [8,9]).

In summary, we favor the idea that the hysteretic behavior of critical intergrain current $J_c(H)$ is caused by the change in the dirbction of the intragranular critical current density J_{cg}. The thermally activated motion of the Abrikosov vortices in the grains should be reflected in the surface current that occurs in the transport current $J_c(H) = J_c(J_m \pm J_{cg})$. This novel flux creep effect is a result of the decay of the induced intragrain current J_{cg} and supports the proposed model.

Acknowledgment

The work was supported by the State Committee on Science and Technology of the Ukraine (project «Sandwich»).

1. A. I. D'yachenko and V. V. Chabanenko, *Fiz. Nizk. Temp.* **18**, 826 (1992) [*Sov. J. Low. Temp. Phys.* **18**, 581 (1992)].
2. A. I. D'yachenko, *Physica* **C213**, 167 (1993).
3. I. K. Yanson, *Zh. Eksp. Teor. Fiz.*, **58**, 1497 (1970) [*Sov. Phys. JETP* **31**, 781 (1970)].
4. A. I. D'yachenko, *Mod. Phys. Lett.* **B8**, 1483 (1994).
5. J. E. Evetts and B. A. Glowacki, *Cryogenics* **28**, 641 (1988).
6. K. Kwasnitza and Ch. Widmer, *Physica* **C202**, 75 (1992).
7. M. Campbell and J. E. Evetts, *Critical Currents in Superconductors*, Barnes and Noble, New York (1972).
8. J. Stankowski, P. K. Kahol, N. S. Dalal, and J. S. Modera, *Phys. Rev.* **B36**, 7126 (1987).
9. K. W. Blazey, K. A. Muller, J. G. Bednorz, W. Berlinger, G. Amoretti, E. Buluggiu, A. Vera, and F. C. Matacotta, *Phys. Rev.* **B36**, 7241 (1987).