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Mobility of electrons in a quasi-one-dimensional conducting channel on the liquid helium surface in the presence of a magnetic field

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The influence of the transverse magnetic field on the electron mobility in a quasi-one-dimensional channel along the liquid-helium surface is investigated. The mobility calculations are carried out by using the Boltzmann kinetic equation and the criteria for the validity of this approach, which are different from those for two-dimensional systems, are established. Two different limiting regimes corresponding to different roles of the electron-electron interaction in the quasi-one-dimensional electron system are considered. The mobility is shown to be a decreasing function of the magnetic field. It is shown that the temperature dependence of the mobility in the presence of the magnetic field, as in the case of zero field, is a nonmonotonic function.

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In the last decades the investigation of low-density electron systems localized on the liquid-helium surface became one of the developing directions in the physics of systems with reduced dimensionality. In addition to the well-known features such as purity and homogeneity, these systems are very attractive because of their ability to vary their properties through external fields or constraints. The influence of a magnetic field is especially interesting because it drastically affects the energy spectrum of electrons and the character of their motion. In the case of a quasi-two-dimensional (2*D*) system of surface electrons (SE) over the flat surface of bulk helium, the energy spectrum in the presence of a magnetic field B along the z axis (electrons are located in the xy plane parallel to helium surface) is given by the Landau levels $(n + 1/2)\hbar\omega_c$, where $\omega_c = eB/mc$, in addition to the quantized energies Δ_l along the z direction [1].

In the last years quasi-one-dimensional (Q1*D*) electron systems over the liquid helium surface were predicted theoretically and realized experimentally [2]. In such a Q1*D* system formed due to the finiteness of the curvature radius R of liquid helium either along parallel channels on the surface of dielectric substrate with linear grooves or between two dielectric polymer sheets meeting at a sharp angle, electrons are confined across the channel near its bottom due to the holding electric field $E_\perp$ normal to the channel axis with a confinement frequency $\omega_0 = (eE_\perp/mR)^{1/2}$. In the presence of a transverse magnetic field (along the z direction) the energy spectrum of the electron in the Q1*D* channel can be written as [3]

$$E_{n,l} = \frac{\hbar^2 k_x^2}{2m^*} + (n + 1/2)\hbar\Omega + \Delta_l\,, \qquad (1)$$

where k_x is the wave number along the x direction, the hybrid frequency $\Omega = (\omega_0^2 + \omega_c^2)^{1/2}$, and the effective mass is $m^* = m\Omega^2/\omega_0^2$. The electron wave function is given as

$$\Psi_{n,l,k_x} = \frac{1}{L_x^{1/2}} \exp(ik_x x)\, \varphi_n(y)\, \chi_l(z)\,, \qquad (2)$$

where

$$\varphi_n(y) = \frac{1}{(2^n n! \pi^{1/2} l_c^*)^{1/2}} \exp\left(-\frac{(y-Y)^2}{2(l_c^*)^2}\right) H_n\left(\frac{y-Y}{l_c^*}\right). \tag{3}$$

Here the effective magnetic length is $(l_c^*)^2 = \hbar/m\Omega$, and $Y = -\hbar\omega_c k_x/m\Omega^2$ is the y coordinate of the center of the electron orbit; $H_n(x)$ is the Hermite polynomial, and L_x is the size of the system along the x direction.

The energy spectrum $E_{n,l}(k_x, B)$ for electrons localized in the Q1D channel reveals interesting peculiarities in the electron transport coefficients. Our aim in the present work is to calculate the electron mobility along the Q1D channel in the presence of B and to analyze the applicability of the Boltzmann kinetic approach in the same manner as the authors performed in Refs. 4 and 5. To begin with, we derive the criteria for the applicability of this approach in the case of Q1D electrons in magnetic fields. Here the equilibrium distribution function of the electron in the nth subband is approximated by the Boltzmann factor given by

$$f_{n,0} \propto \exp\left(-\frac{\hbar^2 k_x^2}{2m^*T} - \frac{n\hbar\Omega}{T}\right). \tag{4}$$

Note, however, that in the presence of magnetic field there are constraints to be imposed on k_x due to the structure of $\varphi_n(y)$ given by Eq. (2). The magnetic field mixes electron motions along the x and y directions and the y coordinate of the center of electron orbit depends explicitly on k_x. The electron which moves along the y axis can therefore escape at some values of k_x, from the region of the applicability of the parabolic confinement. This can modify significantly the conditions for the normalization of the distribution function $f_{n,0}(k_x)$ and for the wave function $\varphi_n(y)$ in comparison with the case of $B = 0$ considered in Refs. 4 and 5. To clarify this situation let us assume that the y coordinate of the electron satisfies the condition $|y| < L_y$, where L_y is the size of the system along the y axis. In order to make reliable the applicability of a parabolic-potential approximation for the electron confinement, which is suitable for $y << R$, L_y must satisfy the inequality $L_y << R$. The condition $|Y| < L_y$ must be also satisfied. We must therefore impose the upper bound $|k_x| < m\Omega^2 L_y/\hbar\omega_c$ in order to perform the k_x integration of the Boltzmann factor. However, only the values of k_x which satisfy $k_x^2 < k_T^2 = 2m^*T/\hbar^2$ contribute substantially and k_T plays the role of the maximum k_x in the integral of $f_{n,0}$. If k_T is significantly smaller than the upper bound of k_x, then that condition does not affect the integration of $f_{n,0}$ over k_x because the effective cutoff takes place at significantly lower wave numbers than the limiting value of k_x. The standard normalization of $f_{n,0}$ with infinite limits in the normalization integral is therefore valid if the condition $2m^*T/\hbar^2 << << (m\Omega^2 L_y/\hbar\omega_c)^2$ is satisfied. By substituting the value of m^*, which appears in Eq. (1), this inequality can be easily transformed into

$$T << \hbar\Omega\left(\frac{\omega_0^2}{2\omega_c^2}\right)\left(\frac{L_y}{l_c^*}\right)^2. \tag{5}$$

Assuming that the above condition is satisfied, we can write the normalized $f_{n,0}(T)$ as

$$f_{n,0}(T) = \left(\frac{2\pi\hbar^2}{m^*TL_x^2}\right)^{1/2} \frac{1}{Z_n} \exp\left(-\frac{\hbar^2 k_x^2}{2m^*T} - \frac{n\hbar\Omega}{T}\right), \tag{6}$$

where

$$Z_n = \sum_{n=1}^{\infty} \exp\left(-\frac{n\hbar\Omega}{T}\right) = \frac{1}{2}\left[1 + \coth\left(\frac{\hbar\Omega}{2T}\right)\right].$$

The quantity L_y must be larger than the scale of the electron localization across the Q1D channel, which is of the order of the effective magnetic length l_c^*. As shown in Ref. 3, l_c^* is always smaller than the scale of the electron localization $y_0 = (\hbar/m\omega_0)^{1/2}$ in the case of $B = 0$. For $y_0 \cong 10^{-6}$ cm (Ref. 4), the inequality $l_c^* < y_0 << R$ will than be well satisfied. This assures the validity of the approximation of a parabolic confinement potential along the y axis for $y << R$. For this reason, $L_y^2/(l_c^*)^2 >> 1$.

Another problem stems from the normalization of $\varphi_n(y)$. Considering the conditions under which the finiteness of the system along the y axis (finite value of L_y) does not influence the integration, we see from Eq. (3) that the range of y, which is relevant to the integration of $(\varphi_n(y))^2$, is limited by the condition $|y - Y| \leq l_c^*$. Since $l_c^* << L_y$, the effective cutoff in the integral takes place at values of y significantly smaller than L_y if, in turn, the condition $|Y| << L_y$ is satisfied. This can be easily seen if the integrand is rewritten in terms of the variable $|y - Y|/l_c^*$, after which the dependence of the normalization integral on Y moves into the limits of the integration. As a result, lower and upper limits of the integral can be extended to $\pm\infty$. Assuming $Y(k_T) \lesssim l_c^* << L_y$, we obtain the inequality which is formally the same as Eq. (5) but without the large factor $L_y^2/(l_c^*)^2$ on the right-hand side. If the inequality

$$T < \frac{\hbar\Omega\omega_0^2}{2\omega_c^2} \qquad (7)$$

is satisfied, then Eq. (5) is also satisfied and we obtain from Eq. (7) the following condition by substituting the expression of the hybrid frequency Ω:

$$\omega_c < A\omega_0 , \qquad (8)$$

where the coefficient A depends on the temperature and the frequency ω_0 as

$$A^2(T, \omega_0) = \frac{1}{8}\left(\frac{\hbar\omega_0}{T}\right)^2 \left\{1 + \left[1 + 16(\hbar\omega_0/T)^{-2}\right]^{1/2}\right\} . \qquad (9)$$

The Eq. (8) should be satisfied in order to have reliable expressions of $\varphi_n(y)$ and $f_{n,0}$. The inequality (8) gives the estimate of the upper value of ω_c , and consequently of B, under which we can employ the usual Boltzmann transport equation, as developed in Refs. 4 and 5. Note that in 2D electron systems the transition from the quantum limit where $\omega_c >> T/\hbar$ to the classical regime where $\omega_c << T/\hbar$ comes from the change in the occupation of the Landau levels with the temperature. In the Q1D charged system with parabolic-potential confinement across the channel, limitations for the applicability of the standard classical approach given by the condition (8) are based on other grounds, i.e., due to the constraints imposed on the normalization procedure of the electron wave function and the Boltzmann distribution function. The inequality (8) relates the upper bound of the cyclotron frequency which is given not only as a function of the temperature, as in the 2D case, but also a function of the confinement frequency ω_0 across the channel.

The range of values of the magnetic field and the clamping electric field, which satisfies the condition $\omega_c < A\omega_0$ for the validity of the classical Boltzmann approach to the electron mobility, is shown in Fig. 1, where the coefficient A was evaluated for the curvature radius $R = 5\cdot 10^{-4}$ cm. Note that for limited values of ω_c the separation $\hbar\Omega$ between the energy subbands is always larger than $\hbar\omega_0$ in the case $B = 0$. It was shown in Ref. 4, that $\hbar\omega_0$ is comparable to T for all reasonable values of holding electric fields. For this reason, the Q1D electrons in the presence of a magnetic field still form a multisubband system, and we must take into account

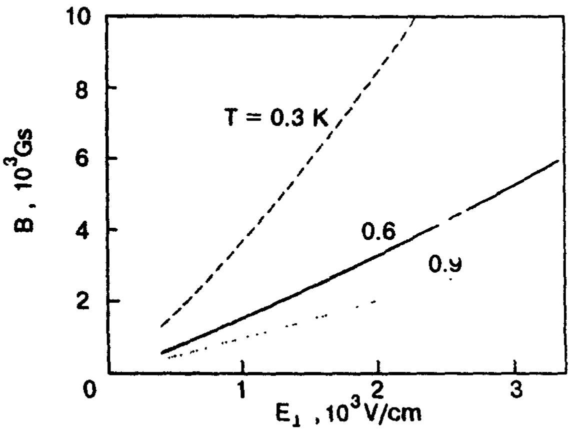

Fig. 1. Dependence of the magnetic field on the holding electric field $B(E_\perp)$, defined by the equation $\omega_c(B) = A\omega_0(E_\perp)$, for three temperatures. The curves show the region of the validity of the classical Boltzmann approach to the calculation of the electron mobility along the Q1D channel, given by inequality (8).

all subbands in the calculations of the electron mobility.

We have calculated the electron mobility in the Q1D channel in the single-particle approximation (SPA) and in the complete control approximation (CCA), where the influence of frequent two-electrons collisions is taken into account in the structure of the electron distribution function in the presence of a driving electric field $E_{||}$ along the x axis. The procedure is straightforward and the formulas are quite similar to those in Ref. 4 with few replacements due to the introduction of Ω and m^*. The calculation gives the following expression for the electron mobility in the Q1D system within SPA:

$$\mu(T, \Omega) = \frac{2e}{\sqrt{\pi}\, m^* Z_n(T, \Omega)}\left(\frac{\hbar\omega_0^2}{T\Omega}\right)^{3/2} \times$$

$$\times \sum_{n=0}^{\infty} \exp\left(-\frac{n\hbar\Omega}{T}\right)\int_0^{\infty} \frac{\sqrt{x}\ e^{-\hbar\omega_0^2 x/\Omega T}\, dx}{\nu_{er}^{(n)}(x) + \nu_{eg}^{(n)}(x)} , \qquad (10)$$

where $\nu_{er}^{(n)}(x)$ $[\nu_{eg}^{(n)}(x)]$ is the collision frequency of an electron in the nth subband due to electron-ripplon [electron-atom] scattering.

We assume in the complete control approximation (CCA) that the electron system can be characterized by a drift velocity u, when the frequency of electron-electron collisions is $\nu_{ee}^{(n)} >> \nu_{er}^{(n)}, \nu_{eg}^{(n)}$. Under this regime, the electron momentum is efficiently redistributed between the carriers, which leads to a *shifted* distribution function [6,7]. The advantage of the CCA is that the kinetic equation can be used for the calculation of electron mobility in the case of strong enough electron electron interaction without using an explicit form of such an interaction The role of electron-electron collisions in the Q1D system with a more complicated nature of the electron motion is not well established at this time One can hope, however, that this approximation would display the main features of the electron mobility in the correlated Q1D system in some range of the electron densities Note that for $B = 0$, the mobility calculated in CCA differs from the one-electron mobility, both qualitatively and quantitatively, since it is nearly three times smaller at $T < 1$ K [4,5] We also calculate the electron mobility in the CCA in the presence of B

The results of numerical calculations of the magnetic field dependence $\mu(B)$ in SPA are plotted in Fig 2 for $T = 0$ 6 K and for some values of the holding field $E_\perp$ As can be seen from Fig 2, the $\mu(B)$ is a decreasing function of B at a low enough electric field $E_\perp$ With an increase in $E_\perp$, the mobility at the high electric field becomes insensitive to B It is a consequence of the fact that, as we increase $E_\perp$, the frequency $\Omega = (\omega_c^2(B) + \omega_0^2(E_\perp))^{1/2}$ tends to ω_0 and $\mu(B)$ for fixed T becomes negligible and coincides with the mobility calculated for $B = 0$ In the region where the inequality is satisfied one can estimate that B cannot exceed 1000 Gs, and the corrections to the mobility due to the magnetic field are significant for $E_\perp < 500$ V/cm

Figure 3 shows the temperature dependence of mobility $\mu(T)$ in SPA, calculated for two values of the magnetic field and for $B = 0$ We see that $\mu(T)$

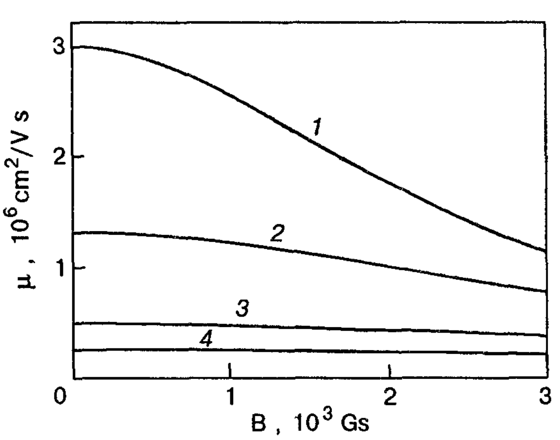

Fig 2 Electron mobility slotted as a function of the magnetic field calculated in the one electron approximation The curves are numbered *1* through *4* corresponding to the clamping electric fields $E_\perp = 500$ V/cm (*1*) 1000 V/cm (*2*) 2000 V/cm (*3*), and 3000 V/cm (*4*) The temperature is $T = 0$ 6 K

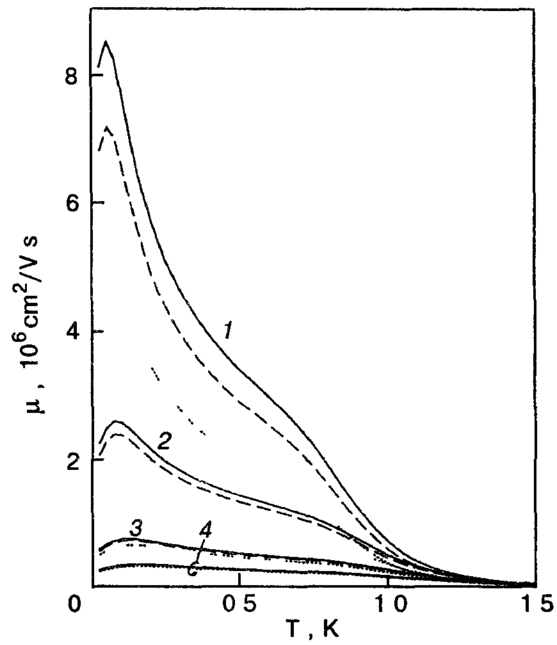

Fig 3 Temperature dependence of the electron mobility for $B = 0$ (solid line) $B = 1000$ Gs (dashed line), and $B = 2000$ Gs (dotted line) at $E_\perp = 500$ V/cm (*1*), 1000 V/cm (*2*), 2000 V/cm (*3*), and 3000 V/cm (*4*)

is qualitatively the same as in the case of $B = 0$, even though the values of the mobilities become small with increasing $E_\perp$ At higher $E_\perp$, the difference in the curves $\mu(T)$, calculated for different B, becomes small for the same reason as that described before We observe that maximum points appear in the curves of $\mu(T)$ for $T < 0$ 2 K The reason for this nonmonotonic temperature dependence of the mobility and the maximum in $\mu(T)$ are discussed in detail in Ref 4 for zero field

The low temperature expansion of Eq (10) can be written as

$$\mu_0 = \mu_\perp \left[6 \frac{\omega_0^2}{\Omega^2} + \frac{64}{\pi} \left(\frac{\omega_0^2 T}{\hbar \Omega^3} \right)^{1/2} \right] , \qquad (11)$$

where $\mu_\perp = \alpha\hbar/meE_\perp^2$ (α is the surface tension of liquid helium) This result explains the increase of $\mu(T)$ starting from zero until it reached the point where the contribution of the excited levels with $n > 0$ becomes dominant and leads to a decrease of $\mu(T)$ with increasing T As one can see, increasing B causes both the mobility at $T = 0$ [the first term in Eq (11)] and the temperature-dependent coefficient of the second term to become decreasing functions of Ω and, hence, of B For this reason, for higher values of B, the mobilities $\mu(T)$ reach lower

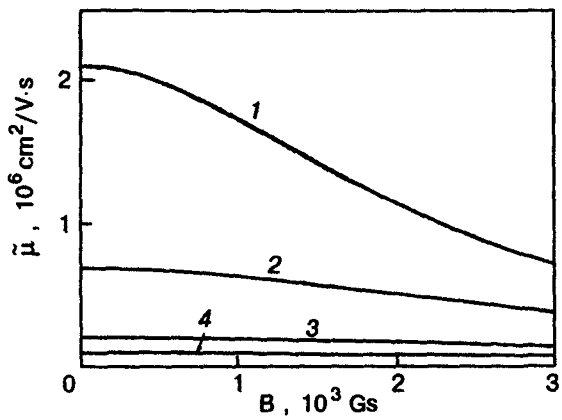

Fig. 4. The same as in Fig. 2 but in the complete control approximation.

values and the peak becomes broader. Such a behavior of $\mu(T)$ is observed in Fig. 3.

In Figs. 4 and 5 we present our results for $\tilde{\mu}(B)$ and $\tilde{\mu}(T)$ in the CCA. As one can see, the curves $\tilde{\mu}(B)$ and $\tilde{\mu}(T)$ are qualitatively similar to those shown in Figs. 2 and 3 in the case of SPA. However, the values of $\tilde{\mu}$ are lower than those of $\mu(T)$ and for $T < 1$ K the values of $\tilde{\mu}(T)$ are nearly three times smaller than the values of $\mu(T)$ for a given B. The maximum on the $\tilde{\mu}(T)$ curve occurs at temperatures higher than those at which the maximum occurs on the $\mu(T)$ curve. The reasons for the maximum on $\tilde{\mu}(T)$ are the same as in SPA. The mobility in the CCA at a very low temperature is given by

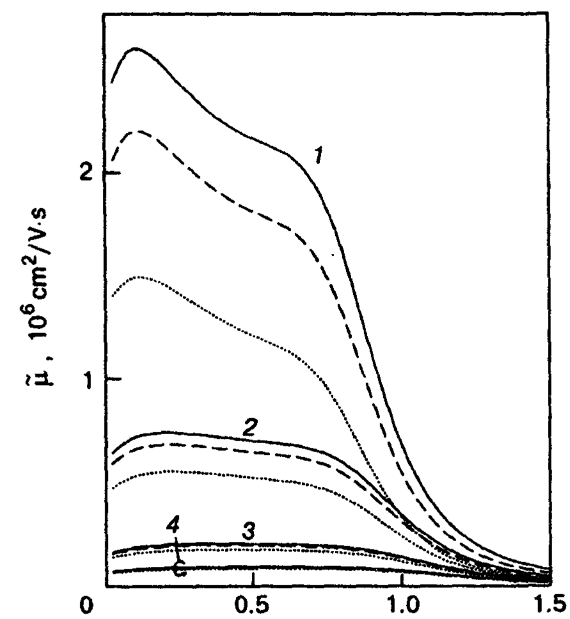

Fig. 5. The same as in Fig. 3 but in the complete control approximation.

$$\tilde{\mu}_0 = \mu_\perp \left[2\frac{\omega_0^2}{\Omega^2} + \frac{8}{\pi}\left(\frac{\omega_0^2 T}{\hbar\Omega^3}\right)^{1/2} \right] . \qquad (12)$$

Equation (12) gives the lower limit of the mobility at $T = 0$ [the first term in Eq. (12)] for $\tilde{\mu}(T)$, which is three times smaller in comparison with the lower limit in SPA. The coefficient of the second term in Eq. (12) is eight times smaller than that in Eq. (11). As a result, the maxima in $\tilde{\mu}(T)$ are significantly smoother than in $\mu(T)$.

In conclusion, we have investigated theoretically the influence of a magnetic field on the mobility of electrons localized in a Q1D channel on the liquid-helium surface. The dependence of the mobility on the magnetic field and temperature are calculated by using the classical Boltzmann approach in the framework of the usual SPA and by introducing the CCA, which takes into account the electron-electron interaction in an indirect way. The influence of the electron-electron interaction on the electron mobility seems to be more relevant in the Q1D case due to the more restricted nature of the electron motion. We hope, however, that the CCA would make it possible to describe the electron mobility under certain conditions. According to the results obtained in this study and in those of Refs. 4 and 5, $\mu(T)$ in the Q1D electron system in the complete control regime must differ both quantitatively and qualitatively from those conditions, under which the electron-electron interaction can be considered negligible. We should emphasize that the results obtained in our study can become invalid in the case of a sufficiently large magnetic field, for example, when the condition given by inequality (8) is not satisfied. In addition to the experimental studies [2], the study of the electron mobility in a wide range of B and T is very desirable. The experimental evidence for the deviation of the calculated mobilities obtained by us allows to confirm the region of B and T where the classical regime of electron system is reached.

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